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Introduction:

The present book is prepared in 11 chapters for the course of General Mathematics (1) in the fields of Mathematics, Statistics and engineering fields. In the first chapter, complex numbers and their properties are expressed. In the second chapter, relation and function, their properties and different types of functions are reviewed. In chapters three, four and five the concepts of limit, derivative and continuity are presented by using functions and different examples. In chapter six, derivative applications, maximum and minimum questions and the related theorems are presented.

In chapter seven, the concept of indefinite integral and methods of integration has been introduced. Definite integrations and their applications are presented in chapters eight and nine. Chapter ten includes polar coordinates and their properties, calculation of arc length, volume and area. In chapter 11 the concepts of sequences and series have been expressed by presenting some questions. For better comprehension, some examples have been presented in each chapter and at the end of each chapter there some questions that solving them shall help in better understanding of the concepts. It is hoped that this book would meet the students' needs.

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Chapter One –Complex Numbers

In solving equations, it is claimed that some equations like $x^2+1=0$ has no real answer. The necessity to find a solution for these types of equations resulted in introduction of a wide range of numbers called *complex numbers*. In the present chapter, these numbers along with some of their properties have been introduced.

Definition: the complex number Z is an ordered pair and is shown in the form of $Z = (a, b)$.

The first component of the complex number i.e. (a) is called the *real part* ($\text{Re}Z$) and the second component i.e. (b) is the *imaginary part* ($\text{Im}Z$) of the complex number. The set of all complex numbers is denoted by ϕ .

(1.1) Addition and subtraction of two complex numbers

In order to add or subtract the complex numbers, the real parts shall be added or subtracted together and the imaginary parts shall also be added or subtracted together.

$$Z_1 = (a_1, b_1) \qquad Z_1 + Z_2 = (a_1 + a_2, b_1 + b_2)$$

→

$$Z_2 = (a_2, b_2) \qquad Z_1 - Z_2 = (a_1 - a_2, b_1 - b_2)$$

(1.2) Multiplication of two complex numbers

If $Z_1 = (a_1, b_1)$ and $Z_2 = (a_2, b_2)$, then multiplication of two complex numbers of Z_1 and Z_2 is defined as the following:

$$Z_1 \times Z_2 = (a_1 a_2 - b_1 b_2, a_1 b_2 + a_2 b_1)$$

(1.3) Multiplication of a real number in a complex number

If $\alpha \in \mathbb{R}$ and $Z \in \phi$, then αZ is as the following:

Formula 1

Since the set of complex numbers is greater than the real numbers and this set also contains the numbers $\mathbb{R} \in \phi$, then it can be concluded that each real number is a complex number. Therefore, each real number can be shown as a complex number. Contractually, the real number of a is shown in the form of $a = (a, 0)$, which is a complex representation of real number.

Among the complex numbers, there is a significant number i.e. $i = (0, 1)$. The most significant property of this number is that its square is equal to -1 .

$$i^2 = (0, 1)(0, 1) = (-1, 0) = -1$$

It shows the main difference between real numbers and complex numbers. By introducing this complex number, a new form of complex numbers is introduced which is so-called standard form of a complex number.

Formula 2

By using this standard form, the set of complex numbers shown by ϕ shall be as the following:

Formula 3

In this chapter, the standard form of complex numbers is used instead of ordered pairs. Based on the standard form of complex numbers, addition, subtraction and multiplication of the standard numbers shall be as the following:

Formula 4

(1.4) Complex numbers conjugate

If $Z = a + ib$, then the conjugate of Z which is denoted by \bar{Z} , is introduced as ----- . Conjugate properties of a complex number is represented in the following theorem.

Theorem (1.1):

Note (1.1): in case of using complex numbers, if i appears in the denominator, the best of removing i in the denominator is to multiply and divide the expression in the denominator's conjugate.

(1.5) Inverse of a complex number

The inverse of non-zero real numbers can be easily computed and the inverse of a complex number of $Z \neq 0$, is denoted by $\frac{1}{Z}$, if $Z = a + ib$, and shall be computed as the following:

Formula 5

Then: Formula 6

(1.6) Properties of addition in complex numbers

1. Addition of two complex numbers has the property of displacement. In other words,

$$Z_1 + Z_2 = Z_2 + Z_1$$

2. If, $w = 0 + 0i$, this member is introduced as the neutral member in addition.

$$Z + w = w + Z = Z$$

3. If $Z = a + ib$, the symmetry of Z is as the following:

$$(\text{Symmetry of } Z) \quad -Z = -a - ib$$

$$Z - Z = (a - a) + i(b - b) = w$$