

LEVELING THE NET SINGLE PREMIUM

CHAPTER OBJECTIVES

- Review the assumptions used in calculating the net single premium.
- Describe what is meant by the present value of future benefits.
- Explain the mathematics involved in leveling the net single premium for life insurance.
- Explain the mathematics involved in leveling the net single premium for annuity contracts.

INTRODUCTION

Most consumers purchase life insurance in installments rather than paying one premium at the inception of the contract. A male age 30 would have to pay a lump sum of \$20,918 to purchase \$100,000 of whole life insurance.¹ This large premium would make the purchase of any practical amount of life insurance impossible for most people. Instead of paying one large amount or purchasing term insurance annually, people may purchase life insurance by paying equal periodic installments - the level premium. This chapter describes the process for leveling the net single premium. Using this method, the policyowner "overpays" the mortality expense in the early years of the contract and in the later years of the contract, the mortality expense is "under paid." This chapter describes the logic underlying the development of the level payment plans and the mathematics to calculate its amount.

¹This amount is the net single premium, assuming the 1980 CSO Table and a 4 percent interest rate. Refer to the previous chapter for calculation procedures. Gross premiums would include loading for expenses, taxes, and additions to surplus, all actually paid to the life insurance company. Using the 1958 CSO and 4 percent, the amount would be approximately \$26,500.

AN OVERVIEW OF THE LEVEL PREMIUM

As stated, most people cannot afford the purchase of large amounts of life insurance on a single premium basis. An alternative includes purchasing a series of one-year renewable term insurance contracts. With this alternative, as premiums increase, the insured will begin to question the need for owning the life insurance contract, especially if he or she expects to continue in good health. And clearly the insured will feel a need to renew the contract if he or she perceives a possibility of premature death. This self-selection process produces adverse selection. Less healthy people will tend to remain in the insurance pool while the healthy will tend to select themselves out. As a result of self-selection, the mortality rate of the remaining insured group will be higher than originally expected and the premiums will be inadequate to pay death claims as they become due. One solution to this problem is to adjust the mortality table to reflect this antiselection process. Simply adjusting the mortality table is not an adequate solution to the problem of adverse selection since affordable insurance coverage will still not be available at older ages and self-selection will continue.

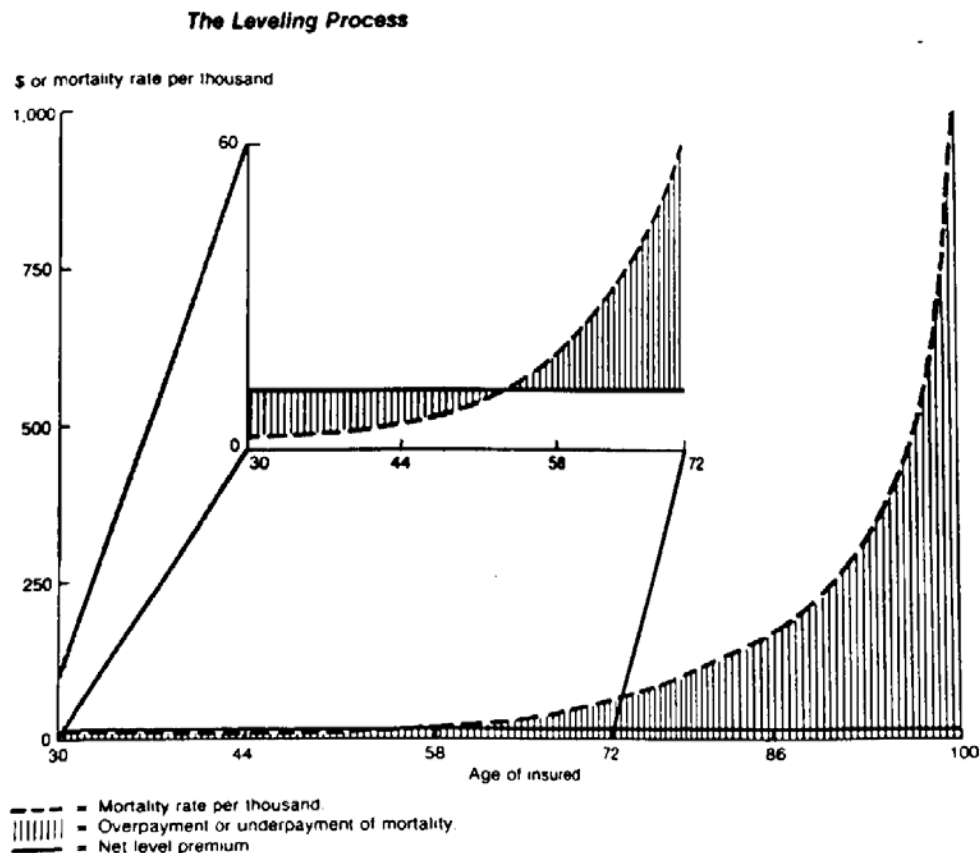
The insurance industry, seeking a solution to the dual problems of affordability at older ages and adverse selection, devised a method to level the premiums to make the ownership of life insurance practical in the later years.² The level premium may be thought of as an annuity paid to the life insurance company to purchase a life insurance contract that will pay a claim whenever death occurs.³ Once the net single premium is calculated for the desired contract (see previous chapter), a periodic payment can be calculated so the net level premium exceeds the mortality cost in the early years of the

²The process is actually self-selection. The difference between self-selection and adverse selection is that in adverse selection, people are seeking coverage when losses are likely; Self-selection occurs when people remove themselves from or stay in a group that is already insured.

³The annuity is also contingent upon surviving and the desire to continue the life insurance contract.

contract; in the later years the mortality cost will exceed the net level premium (see Figure 1). The overpayments (with respect to the mortality cost) in the early years, plus interest added, pay for the underpayments in the later years of the contract.⁴

FIGURE 1



At the beginning of each policy year, level premium payments plus balances remaining from the prior year are invested at a positive rate of interest. And if the mortality rate and the investment rate have been correctly calculated, the insurance company will have sufficient funds available to pay all death claims at the end of the

⁴The survivor benefit also provides funds to pay the benefits promised by the contract. The survivor benefit will be discussed later.

year.⁵ The leveling process produces what are commonly called reserves, and in the discussion to follow, the link between the premium leveling process and the reserves generated by life insurance contracts will be explained.

NET SINGLE PREMIUM ASSUMPTIONS: REVISITED

In the previous chapter several assumptions were made in order to develop the mathematics of the NSP calculation and reduce its difficulty. All of these assumptions are violated in practice. Because several of the assumptions will be relaxed in this chapter, it is necessary to review them so that their influence on the level premium is understood.

The first assumption made for the NSP calculation is that the net single premium is paid in full at the beginning of the contract term. The net single premium is the present value of the future benefits to be provided by the contract or the actuarial present value. The financial promises made by the life insurance contract are discounted for the time value of money and the probability of a death claim occurring. When the premium payment equals the net single premium, the insured need never pay another premium, and the life insurance company will have sufficient funds to fulfill the actuarial obligations promised by the policy.

The second assumption used to simplify the NSP calculation is that all death proceeds are paid at the end of each policy year and all premiums are collected at the beginning. If the insured buys the contract at the beginning of the year, and dies during the year, the life insurance company has time to earn investment income for the entire year until the claim is paid at the end of the year.

⁵The insurance company will also have the overpayments plus accrued interest called reserves. The accrued overpayments plus future premiums collected plus interest will be sufficient to pay all claims.

The third assumption used to simplify the NSP calculation is that the actual mortality rate experienced is equal to the rate predicted by the table selected.⁶ This third assumption is required so no risk exists in predicting the financial outcome due to mortality considerations.

The final assumption is that amounts paid to the life insurance company are used to pay only for mortality costs; no additional amounts are included for operating expenses, additions to surplus, or taxes.

With respect to an interest rate assumption one must recognize that investments made by life insurance companies are conservative because all future claims under all insurance policies must be paid. Therefore relatively long-term, low-risk (and consequently, relatively low return) investments are sought. Financial disaster would occur if the life insurance company discounted the financial promise(s) at a high interest rate and subsequently could not earn the projected rate, unless there happened to be an offsetting underwriting gain. In addition to the possibility of not achieving target investment income, a company may lose principal amounts when risky investments are made.

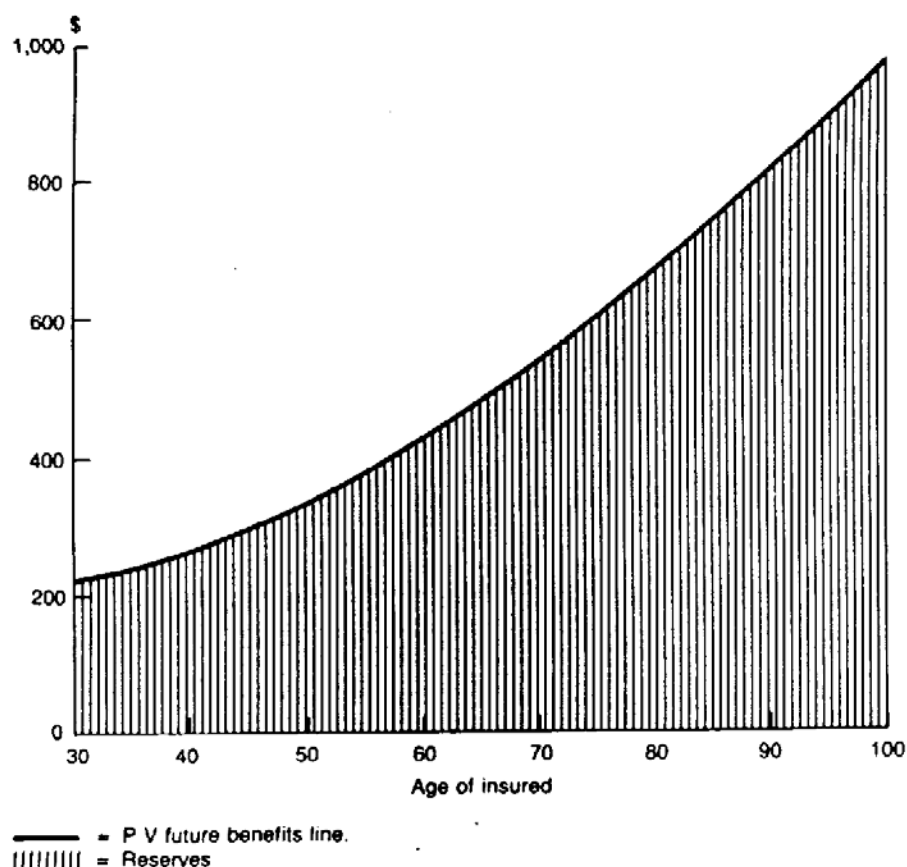
In practice, life insurance companies do not operate within these restrictive assumptions relating to their financial and operating environment. Life insurance companies pay claims and receive premium payments throughout the calendar year. They also do not adhere exclusively to the 1980 CSO Mortality Table. In calculating rates various mortality tables are used and there are wide fluctuations in rates of return on investments. However, the above assumptions are still used in order to simplify matters so that reasonably accurate and conservative calculations can be made.

⁶The 1980 CSO Table is used for the life insurance examples in this chapter.

THE PRESENT VALUE OF FUTURE BENEFITS

The net single premium for a whole life insurance contract is calculated by summing the present value of future benefits to be provided by the contract (see Chapter 21). For our hypothetical male age 30 to purchase a \$1,000 whole life contract, the net single premium was calculated at \$209.18, assuming a 4 percent interest rate. If a net single premium of 209.18 is paid at the beginning of the contract period, \$8.37 ($209.18 \times .04$) is added to the net single premium, due to the investment income during the year; and \$1.73 [$(16,573 / 9,579,998) \times \$1,000$] is subtracted from that sum due to the insured's contribution to mortality costs. Figure 2 shows that if one continues this calculation for a hypothetical insured who survives through age 99, the cash amount accumulated increases over time. Amounts under the curve are commonly referred to as the cash value, the nonforfeiture value, or the reserve amount.

FIGURE 2



Note that the line drawn from \$209.18 at age 30 to \$1,000 at age 100 is called the present value of future benefits line. If an individual desires to purchase a life insurance policy, the contract would be purchased for an amount equal to the present value of the future benefits for the given age of the individual, otherwise known as the net single premium. The net single premium amount (\$209.18) was calculated for the male age 30. For males age 35 and 40 a net single premium of \$246.82 and \$290.81 respectively, would have to be paid. At age 99 the present value of future benefits would

simply be \$1,000 discounted for the time value of money since death is assumed to be a certainty during the year.⁷

One must realize that if the life insurance company accepts premium payments over the life of the insurance contract, several of the above assumptions are violated. The first violation is that the entire net single premium will not be received at the beginning of the contract term. In lieu of the NSP, the life insurance company accepts a series of payments. Therefore, the insurer will not generate interest earnings on the whole NSP from the inception of the contract. Second, since most insureds will not survive the premium-paying period, the life insurance company must forgo collecting any additional premiums from these individuals, reducing the funds flowing to the insurer. Thus the level premium plan produces two sources of "loss" relative to the NSP: (1) loss resulting from the premature termination of premium installments and (2) loss arising from lower investment income.⁸

Because of "missing" premiums and interest, the life insurance company cannot divide the net single premium by the number of premium payments desired. The problems associated with these "missing" amounts are solved by the leveling process. In addition two other problems are solved. First, if insureds pay a relatively small periodic premium for each \$1,000 of life insurance, they will be able to afford larger face amounts. Second, the problem of adverse selection at advanced ages is minimized since the premium stays constant over the life of the payment term, creating no inducement for the healthy individuals to leave the pool.⁹

⁷ $\$1,000 \times (10,757 / 10,757) \times 1/1.04)^1 = \$961.54$

⁸A third loss of funds arises from lapses - policyowners not continuing the contract. When a lapse occurs, however, the contractual obligation is modified or terminated.

⁹Actually the problem continues but not to the extreme extent encouraged by paying ever increasing premiums.

THE LEVELING PROCESS

The net level premium is the amount of money paid to the life insurance company on a periodic basis to purchase a life insurance contract that will pay a death claim regardless of the age at death.¹⁰ The number of periodic payments depends on the contract chosen and the number of years the insured desires to pay premiums.¹¹ It is interesting to note that, everything being equal, the life insurance company should be indifferent as to the premium payment plan selected.

An Analogy To develop the mathematics for leveling the net single premium, the mortgage problem similar to the one in Chapter 13 will be used. Suppose Mr. Worth is contemplating the purchase of a \$40,000 house. Mr. Worth wants to know the annual payment amount for a 4 percent, 20-year amortization schedule. To solve this problem, time value-of-money techniques are used. The present value of the stream of payments to the bank is equal to \$40,000 at 4 percent. Formula 1 is used to solve for the annual payment.

$$PV = PMT \times PVAF \text{ at 4 percent for 20 years} \quad (\text{Formula 1})$$

where

PV = Present value

PMT = Yearly payment amount

PVAF = Present value annuity factor of \$1

$$\$40,000 = PMT \times 13.590$$

$$PMT = \frac{\$40,000}{13.590} = \$2,943.34$$

¹⁰This applies to the whole life contract. Term policies are leveled but the death benefit is only available during the contract's term. The level premium may be payable for the term of the contract or for a shorter period of time.

¹¹The frequency of payment is assumed to be once per year although the premium for life insurance policies can be paid on a more frequent basis.

13.590

The bank is indifferent as to whether it receives the principal now (not lending the money) or later through annual installments.¹² This is the case because the borrower's installment payments compensate the bank for the interest that could have been earned on an alternative investment. In a similar fashion a life insurance contract can be purchased on an installment basis, except that two modifications must be made to the above payment determination.

Modification 1. In the mortgage example, payments are assumed to be made at the end of each year, and a present value of an annuity factor was used. On the other hand, premiums paid to a life insurance company are made at the beginning of each year, requiring a different mathematical adjustment for the time value of money. Instead of using the present value of an annuity factor, an annuity due factor must be substituted.

$$PV = PMT \times PVAF_d \text{ at 4 percent for 20 years} \quad (\text{Formula 2})$$

$$PVAF_d = \text{Present value annuity due factor of \$1}$$

The $PVAF_d$ factor is calculated by summing 20 present value factors (Table 1). The first factor is equal to 1.00 since payments are made at the beginning of the year instead of at the end of the year, as in the ordinary annuity. The reader should note that the individual factors in Table 1 may be calculated by the use of an appropriate formula, or one may find the factors in the appendix to this book. If a present value of an ordinary annuity table is used, the sum of the individual factors must be shifted one

¹²Most mortgages are paid on a monthly basis. In this example the rate would be .04 / 12 and for 240 (12 x 20) periods.

year. The present value of an annuity due factor can also be calculated by adding 1.00 to the present value of an annuity factor for 19 years.¹³

TABLE 1 Present Value of an Annuity Due
Calculation at 4 Percent: 20 Years

Year	Factor	Year	Factor
1	1.000	11	0.676
2	0.962	12	0.650
3	0.925	13	0.625
4	0.889	14	0.601
5	0.855	15	0.577
6	0.822	16	0.555
7	0.790	17	0.534
8	0.760	18	0.513
9	0.731	19	0.494
10	0.703	20	0.475
	Present Value of an Annuity Due		14.137

When changing the timing of the payments (in the above mortgage example) from the beginning of the year to the end of the year, the payment is determined by substituting the annuity due factor for the annuity factor.

$$\$40,000 = \text{PMT} \times 14.137$$

$$\text{PMT} = \frac{\$40,000}{14.137} = \$2,829.45$$

The yearly payment is reduced to \$2,829.45 from \$2,943.34. The payment declined because (1) the borrower does not have use of all the money the first year

¹³For example: if one needs the present value of an annuity due for 25 years at 5%, PVA 24 years at 5% equals 13.799. $13.799 + 1 = 14.799$. The present value of an ordinary annuity may alternatively be multiplied by $(1 + i)$. PVA 25 years at 5% equals 14.094. $14.094 \times 1.05 = 14.799$. The PVAD factor for 25 years at 5% is 14.799.

since the first payment is made right away, and (2) in subsequent years payments are made at the beginning of each year instead of the end of each year. This calculation is similar to the leveling calculation of the life insurance contract except there is one additional modification.

Modification 2. If the insured dies during the premium-paying period, subsequent payments will cease. When this happens, the assumption of collecting the NSP at the beginning of the contract period is violated because the payments cease. Thus an additional adjustment must be made for missing premiums. The first and second adjustments are made by substituting a life annuity due factor for the annuity due factor. The nature of this second adjustment is explained in the following section.

LEVELING THE TERM INSURANCE CONTRACT

Switching to the life insurance transaction, Mr. Worth decides to purchase a five-year term insurance contract. The net single premium for the five-year term insurance contract is equal to \$8.183 (see Chapter 21). Instead of paying \$8.183 for the contract, Mr. Worth wants to pay in equal annual installments and only if he is alive. Dividing the net single premium by five does not provide the correct level premium. If the net single premium is divided by an ordinary annuity factor, the timing of the payments (beginning versus the end of the period) is not correct, and it does not account for missing premiums. An appropriate adjustment is made for the timing of the payments and the missing premiums and interest when a temporary life annuity due factor (Table 2) is substituted for the annuity due factor.

$$PV = PMT \times PVTLAD \text{ at 4 percent for five years, age 30} \quad (\text{Formula 3})$$

$$NSP = NLP \times PVTLAD$$

$$NLP = NSP / PVTLAD$$

where

PV = Present value amount (NSP)

PMT = Yearly payment or net level premium (NLP)

PVTLAD = Present value of a temporary life annuity due

$$\$8.183 = PMT \times 4.615$$

$$PMT = \$8.183 / 4.615 = 1.774$$

The net level premium for the five-year term policy is equal to 1.774 (\$8.183 / 4.615) which Mr. Worth will pay for each \$1,000 of life insurance at the beginning of each contract year if he is alive to do so.

TABLE 2 Temporary Life Annuity Due Calculation

Year	Interest Factor	x Probability of Survival	= PVTLAD Factor
1	1.000	9,579,998 / 9,579,998	1.000
2	0.962	9,563,425 / 9,579,998	0.960
3	0.925	9,546,402 / 9,579,998	0.922
4	0.889	9,528,932 / 9,579,998	0.884
5	0.855	9,510,732 / 9,579,998	0.849
		Temporary Life Annuity Due	4.615

Male, Age 30, Five Payments, 1980 CSO, 4 Percent

The above calculation is mathematically similar to the mortgage problem but is adjusted for the problems associated with violating several of the original restrictive assumptions.

Before it is shown that Formula 3 produces the correct net level premium, the nature of the temporary life annuity due (TLAD) calculated in Table 2 needs to be discussed. A life annuity due (LAD) factor is calculated by taking the sum of an interest discount factor for \$1 times a survivor factor for the five years. In the example, the number 4.615 is called a temporary life annuity due factor. It is called temporary because the number is not calculated for the whole life of the individual, and the word

life is used because the sum takes into account the probability that the individual will survive to pay the premium. Therefore this factor adjusts for the lack of premium payments and interest thereon when the insured dies.

A whole life annuity due factor must be calculated for leveling the net single premiums on whole life insurance contracts that are paid each and every year (through age 99). When life insurance premiums are not paid when due, the contract lapses. The process of leveling the premium must also account for these lapsed contracts people who survive but do not pay. Adjusting the life insurance premium for the lapse of insurance contracts is discussed in Chapter 23.

Below is proof that \$1.774 is the correct leveled premium. If 1,000 insureds, all age 30, each purchase a five-year term insurance contract, the following financial results occur:

Leveling the Net Single Premium

<u>Year 1:</u>	
1,000 Insureds pay 1.774	1,773.50
Interest at 4 %	70.94
Death Claims	
16,573 / 9,579,998	
x 1,000 x \$1,000	1,729.96
Ending Balance	114.48
Reserve per surviving Policy	0.11
<u>Year 2:</u>	
998.270 Insureds pay 1.774	1,770.43
Beginning Balance	114.48
Interest at 4 %	75.40
Death Claims	
17,023 / 9,579,998	
x 1,000 x \$1,000	1,776.93
Ending Balance	183.38
Reserve per surviving Policy	0.18
<u>Year 3:</u>	
996.493 Insureds pay 1.774	1,767.28
Beginning Balance	183.38
Interest at 4 %	78.03
Death Claims	
17,470 / 9,579,998	
x 1,000 x \$1,000	1,823.59
Ending Balance	205.09
Reserve per surviving Policy	0.21
<u>Year 4:</u>	
994.670 Insureds pay 1.774	1,764.05
Beginning Balance	205.09
Interest at 4 %	78.77
Death Claims	
18,200 / 9,579,998	
x 1,000 x \$1,000	1,899.79
Ending Balance	148.11
Reserve per surviving Policy	0.15
<u>Year 5:</u>	
992.770 Insureds pay 1.774	1,760.68
Beginning Balance	148.11
Interest at 4 %	76.35
Death Claims	

Leveling the Net Single Premium

19,021 / 9,579,998	
x 1,000 x \$1,000	1,985.49
Ending Balance	(0.35)
Reserve per surviving Policy	(0.00)

At the beginning of each year, the surviving insureds pay the required premium of \$1.774 for \$1,000 of life insurance. During each year the remaining balances plus the new premium payments earn 4 percent interest. At the end of each year the expected number of death claims are paid. Dividing the ending balance each year by the number of survivors results in the reserve per policy. If 1,000 insureds purchase the five-year term contract, \$1,773.50 will be collected in net premiums in the first year. This amount is invested at 4 percent, and investment income of \$70.94 will be added to the premium amount.

At the end of the year, 1.729 (16,573 / 9,579,998) insureds are expected to die out of each 1,000 insureds. Since the pool starts with 1,000 insureds, 1.729 death claims are expected to be paid. The amount paid as death claims (\$1,729.96) reduces the sum of the net premium plus investment income to \$114.48. If the ending balance is divided by the number of insureds surviving (998.270), an amount results equal to the reserve per surviving policy. This reserve amount can be thought of as the overpayment plus interest added in the first year to partially compensate for the underpayment in the later years. The calculations for the five years demonstrate that the leveling process is at work. In the early policy years the insureds are overpaying the predicted mortality rate so that underpayments may be made in the later years. After the end of the fifth year, all contractual promises have been met by the life insurance company, and all sums have been liquidated.

Leveling the Whole Life Insurance Contract

Determining the level premium for the whole life insurance contract is similar to leveling the premium for the term policy. In reviewing the procedure for calculating the

level premium for the one-year and the five-year term contract, one will notice that the TLAD is calculated for the maximum length of time the premiums are to be paid. Notice that the one-year term net single premium is already leveled since the TLAD is always equal to 1.00. Also note that the possibility of purchasing the one-year term contract and not paying the premium does not exist. In the five-year term policy, the TLAD was calculated for the full five years. Logic dictates that in leveling the whole life premium the LAD calculation must be performed for each year a premium payment potentially can be made. For our hypothetical male age 30, 70 yearly payments are possible. The first payment is made at the beginning of the year the insured is age 30 and the last at the beginning of the year the insured is age 99. Age 99 is the last year for a premium payment because the 1980 CSO Table predicts that all insureds who reach age 99 will die before age 100. Thus the probability of surviving and paying a premium for the next year (age 100) is zero. The LAD calculation for our hypothetical 30-year-old male is calculated for 70 years. Table 3 shows the LAD calculation for the first five and the last five years. The first five lines in Table 3 are the same as the first five lines in Table 2.

TABLE 3 Life Annuity Due Calculation

Age	Year	Interest Factor	x Probability of Survival	= LAD Factor
30	1	1.000	9,579,998 / 9,579,998	1.000
31	2	0.962	9,563,425 / 9,579,998	0.960
32	3	0.925	9,546,402 / 9,579,998	0.922
33	4	0.889	9,528,932 / 9,579,998	0.884
34	5	0.855	9,510,732 / 9,579,998	0.849
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
95	66	0.078	146,721 / 9,579,998	0.001
96	67	0.075	98,309 / 9,579,998	0.001
97	68	0.072	60,504 / 9,579,998	0.000
98	69	0.069	31,450 / 9,579,998	0.000
99	70	0.067	10,757 / 9,579,998	0.000
			Life Annuity Due	20.563

Male, Age 30, 70 Payments, 1980 CSO, 4 Percent

The sum of the 70 calculations starting at age 30 and ending at age 99 provides the LAD figure required for leveling the whole life contract for the 30-year-old male. The last line of the table may be thought of as the present value of the contingent promise to pay \$1.00 in 70 years if the insured is still alive. Dividing the NSP (\$209.185) by the life annuity due (LAD) factor results in the leveled premium.

$$\text{NLP} = \text{NSP} / \text{LAD}$$

$$\text{NLP} = \frac{\$209.185}{20.563} = \$10.172$$

Leveling the NSP for ages other than 30 requires calculating a new net single premium for the selected age and an LAD figure commensurate with the age of issue. When the net single premium was calculated in the previous chapter, the iterative calculations were made for the number of years a benefit could possibly be paid. In this chapter, the LAD calculation is made for the number of payments (years) scheduled. Thus the number of years used for the iterative calculation is the duration of time that a benefit payment or a premium payment could possibly be made for the type of contract specified.

Leveling the Premium for an Endowment Policy

The logic presented above is also used to determine the level premium for the endowment policy. The net single premium for the five year endowment policy for our 30-year-old male is equal to \$822.53. This net single premium is composed of \$8.183 to pay for death benefits and \$814.35 to pay a survivor benefit.¹⁴ The survivorship

¹⁴Please refer to the previous chapter where the net single premium was calculated for the endowment contract.

promise occurs at the end of the fifth year. If five installments are needed for this endowment contract the number of years of potential benefits and the number of potential premium payments are equal.

The five-year TLAD (4.615) that is used to level the five-year term contract is the same TLAD required to level the five-year endowment contract. The net single premium for the endowment contract is divided by the temporary life annuity due (4.615), and the result is the net level premium (\$178.23) required for the five-year endowment contract.

$$NLP = NSP / TLAD$$

$$NLP = \frac{\$822.536}{4.615} = \$178.23$$

The net single premium for a 10-year endowment (30-year-old male) is equal to \$678.14 (calculations are not shown). The TLAD for 10 years is equal to 8.368, and the level premium is \$81.04. For comparison and instructional purposes, the student should investigate why the 10- year endowment's net single premium and level premium are less than the 5-year endowment's net and level premiums.¹⁵

LIMITED PAYMENT PLANS

Limited payment plans are generally associated with the whole life insurance product. These payment plans allow the insured to pay the life insurance company an amount equivalent to the net single premium in a shorter length of time than the whole of life. Purchasing a whole life insurance contract with a single premium is the most extreme case of a limited payment plan. The single-pay life insurance contract pays a

¹⁵Using the 1980 CSO and 4 percent, the NSP, TLAD, and NLP are \$822.53, 4.61407, and \$178.2269 respectively for the five year endowment policy issued to a male age 30.

benefit when the insured event occurs, the promise extends over many years, and the total premium is paid to the life insurance company with a single payment when the policy begins.

One can envision individuals requiring life insurance coverage but anticipating not being able to nor wanting to pay a premium over the whole contract term. One illustration might involve an individual planning to retire at age 65. Since wage income is expected to cease upon retirement, the retiree may not want to continue premium payments. Such an insured might choose a limited payment plan that allows for terminating insurance premiums at age 65. Upon death at any age, benefits still will be paid. There are many such premium payment plans available. Individuals may want to pay up their life insurance contract in one payment (single-pay), in 5 payments (5-pay), in 10 payments (10-pay), or in any practical length of time. A straight whole life insurance contract would not be included in this limited-pay category since payments are made for all of one's life.

Mathematically, determining the periodic premium under a limited payment plan is no different than leveling the net single premium over the entire potential life of the contract. The net single premium for the policy is the same regardless of the premium payment plan. The periodic payment must be calculated so that the life insurance company is indifferent to receiving the premium in one lump sum at the inception of the contract or to receiving a level periodic amount over the premium payment term. The main difference between leveling for the entire contract term and for a limited length of time is that in the limited-payment leveling calculation, the number of periodic payments used to determine the TLAD is less than the number of years benefits could possibly be paid.

Suppose a male age 30 needs to purchase a whole life insurance contract that is paid up in 10 years instead of paying for the contract for all of his life. The net single premium for the whole life insurance contract, already calculated, is \$209.185. In order

to calculate the net level premium for paying the net single premium over 10 years, an appropriate temporary life annuity due must be determined. The new TLAD amount is calculated in Table 4.

Dividing the net single premium by the temporary life annuity due factor provides the level premium for the payment plan selected.

$$NLP = NSP / TLAD$$

$$NLP = \frac{209.185}{8.370} = \$24.99$$

TABLE 4 Temporary Life Annuity Due Calculation

Year	Interest Factor x	Probability of Survival x	= PVTLAD Factor
1	1.000	9,579,998 / 9,579,998	1.000
2	0.962	9,563,425 / 9,579,998	0.960
3	0.925	9,546,402 / 9,579,998	0.922
4	0.889	9,528,932 / 9,579,998	0.884
5	0.855	9,510,732 / 9,579,998	0.849
6	0.822	9,491,711 / 9,579,998	0.814
7	0.790	9,471,683 / 9,579,998	0.781
8	0.760	9,450,466 / 9,579,998	0.750
9	0.731	9,427,785 / 9,579,998	0.719
10	0.703	9,403,461 / 9,579,998	0.690
		Temporary Life Annuity Due	8.370

Male, Age 30, 10 Payments, 1980 CSO, 4 Percent

The 30-year-old male will pay a net level premium equal to \$24.99 for each \$1,000 of whole life insurance. When 10 annual payments are made the policy is said to be paid up, and the insured need not pay another premium. Of course, if the insured dies within the 10-year premium payment period, benefits will be paid, and further premiums will not be collected.

If the insured wants to pay up this whole life insurance contract in 5 years instead of 10, we would expect the annual installment to increase. This result occurs because the new 5-year TLAD is less than the 10-year TLAD. Dividing the same NSP by a smaller TLAD results in a larger net premium. The TLAD required is the same one calculated in Table 2. One should notice that the TLAD (4.615) is equal to the sum of the first five lines in Table 4.

$$NLP = NSP / TLAD$$

$$NLP = \frac{209.185}{4.615} = \$45.33$$

If one recognizes the simplicity of the above calculation, other premium payment plans can be derived by adjusting the TLAD figure according to the number of desired payments.

LEVELING ANNUITY CONTRACTS

The process of determining a periodic payment to purchase an annuity contract is not the same one used to calculate the level premium for life insurance products. Leveling the premium of a deferred annuity contract is also treated differently from an immediate annuity contract.¹⁶

Immediate Annuity Contracts

A life annuity contract pays equal periodic installments to the annuitant for a specified length of time or "till death do us part." If the annuitant receives a rent payment

¹⁶An immediate annuity pays rent on the next payment date. The deferred annuity at least skips the next payment date. Annuities may be deferred for short or long periods of time.

on the next payment date after the contract is started, the annuity is referred to as an immediate annuity; otherwise a deferred annuity contract's rent starts after at least one payment date.

For example, Rebecca Wade, who is age 60 and has no dependents or spouse, is concerned about allocating her savings over the remainder of her life. If Rebecca deposits all of her life savings in an immediate annuity contract, she would expect to receive the first monthly rent payment on the next payment date. If the contract is purchased on January 7, 1992, and all rent payments from the insurance company occur on the 15th, Rebecca will receive a rent payment on January 15, 1992. However, if a five-year deferred contract was purchased, Rebecca would have to survive until January 15, 1997, to receive any rent payments.

Because of the nature of the immediate annuity contract, accumulation of funds over a period of time or on an installment basis is not possible. The present value of the annuity contract (NSP) must be paid to the life insurance company at the inception of the contract, before rent payments begin; therefore it is not possible to level the payment for an immediate annuity.

SINGLE PREMIUM DEFERRED ANNUITY CONTRACTS

Deferred annuity contracts can be purchased on a lump-sum basis, or periodic installments can be made. Consider the lump-sum purchase first.

TABLE 5 Ten Year Temporary Annuity
Age 65, Deferred Five Years: Refund During Accumulation

Age	Year	Probability of Survival x	\$ Payment	x Interest Factor	=
65	1	8,577,575 / 8,577,575	100	1.000	100.000
66	2	8,467,345 / 8,577,575	100	0.962	94.964
67	3	8,347,117 / 8,577,575	100	0.925	90.015
68	4	8,215,925 / 8,577,575	100	0.889	85.152
69	5	8,072,853 / 8,577,575	100	0.855	80.469
70	6	7,917,079 / 8,577,575	100	0.822	75.870

71	7	7,747,883 / 8,577,575	100	0.790	71.358
72	8	7,564,669 / 8,577,575	100	0.760	67.025
73	9	7,366,997 / 8,577,575	100	0.731	62.783
74	10	7,154,570 / 8,577,575	100	0.703	58.637
		Temporary Annuity Benefit			786.274
		Deferral Factor			x 0.822
		Deferred Net Single Premium			\$646.317

Male, 1983 Annuity Table, 4 Percent

Table 5 shows the NSP calculation for a 10-year temporary annuity deferred for 5 years. In this table the net single premium of \$646.32 is the amount of money required by the life insurance company if the annuity contract is purchased on a single-premium deferred basis. The NSP of \$646.32 is the result of discounting the NSP for an immediate 10-year temporary annuity for 5 years at 4 percent (0.822).¹⁷ During the accumulation period \$646.32 will grow to \$786.27 in five years at a 4 percent interest rate ($646.32 \times 1.04^5 = \$786.27$). To provide the \$100 annuity for 10 years, \$786.27 is required when the distribution period begins. Thus the annuitant can pay \$646.32 for a contract deferred five years or a larger amount if deferred for a fewer number of years.

LEVEL PREMIUM DEFERRED ANNUITY CONTRACTS

In addition to purchasing the annuity contract with one deferred lump sum, the annuitant can purchase the contract by making installment payments. The amount of the net single premium and the net level premium is a function of two factors. The first factor is the discount rate, and the second is whether a refund provision exists during the accumulation period.

If the contract refunds savings during the accumulation period, no life contingencies are involved during the accumulation process. Thus, to calculate the

¹⁷Instead of using the deferral factor of 0.822, the calculation may be made by adjusting the interest factor for the appropriate length of time.

annual installment to purchase the annuity contract, an annuity due payment needs to be calculated that will provide \$786.27 to the life insurance company on the annuity's starting date.

$$FV = FVA_d \times PMT$$

$$PMT = \frac{\$786.27}{5.4162} = \$145.17$$

The future value amount (\$786.27) is divided by the future value of an annuity due factor of 4 percent. If \$145.17 is deposited at the beginning of each year for five years, \$786.27 will be available when the fifth payment is made. The growth of the installment payments with interest added is shown in Table 6. Notice that in the accumulation process, payments are made at the beginning of the year and that as the last payment is made, the required NSP (\$786.27) has been accumulated. The life insurance company then, on the next payment date, pays to the annuitant the first \$100.00 rent payment.

TABLE 6 Accumulation Period: Deferred Installment Contract

Year	Beginning Balance	Payment	Invested Funds	Interest Added	Ending Balance
1	\$0.00	\$145.17	\$145.17	\$5.81	\$150.97
2	150.97	\$145.17	\$296.14	\$11.85	\$307.99
3	307.99	\$145.17	\$453.15	\$18.13	\$471.28
4	471.28	\$145.17	\$616.45	\$24.66	\$641.10
5	641.10	\$145.17	\$786.27	-	-

TABLE 7 Ten Year Temporary Annuity
Age 65, Deferred Five Years: No Refund During Accumulation

Age	Year	Probability of Survival	x \$ Payment	Interest Factor	=
65	1	8,577,575 / 9,013,785	100	1.000	95.161
66	2	8,467,345 / 9,013,785	100	0.962	90.368
67	3	8,347,117 / 9,013,785	100	0.925	85.659
68	4	8,215,925 / 9,013,785	100	0.889	81.031
69	5	8,072,853 / 9,013,785	100	0.855	76.575
70	6	7,917,079 / 9,013,785	100	0.822	72.199
71	7	7,747,883 / 9,013,785	100	0.790	67.905
72	8	7,564,669 / 9,013,785	100	0.760	63.782
73	9	7,366,997 / 9,013,785	100	0.731	59.745
74	10	7,154,570 / 9,013,785	100	0.703	55.800
		Temporary Annuity Benefit			748.223
		Deferral Factor			x 0.822
		Deferred Net Single Premium			\$615.040

Male, 1983 Annuity Table, 4 Percent

When there is no refund during the accumulation period, the same options as above exist. The annuitant may purchase the contract on a deferred single premium basis for \$615.04).¹⁸ Table 7 presents the single premium, deferred calculation when there is no refund during accumulation. In addition to the above option, the annuitant may purchase the contract on an installment basis. The calculation for the installment must consider the possibility of the annuitant paying some (or all) of the premium payments and the annuitant dying and forfeiting the accumulated amount before any rent payments begin. The periodic payment is calculated using Formula 4.

$$FV = FVLAD \times PMT \quad (\text{Formula 4})$$

FV

¹⁸Instead of using the deferral factor, the interest factor may be adjusted for the appropriate length of time.

$$\text{PMT} = \frac{\text{FVLAD}}{\text{FVLAD}}$$

Table 8 shows the calculation of the future value of a temporary life annuity due. Notice that this calculation is similar to the PVTLD used to level life insurance contracts. However, the numerator and denominator for the probability of survival is taken from the annuity table starting at age 60, the year the first payment is made, instead of the year of the first possible benefit. Also note that the time value factor is a future value factor. The factor for the first year is 1.04 to the fifth power (1.217), and in consecutive years the power is reduced by one.

TABLE 8 Future Value of a Temporary Annuity Due Calculation
Age 60, 4 Percent, 1983 Annuity Table

Age	Year	Probability of Survival x	FVF	=FV
65	1	9,013,785 / 9,013,785	1.217	1.217
66	2	8,938,628 / 9,013,785	1.170	1.160
67	3	8,858,332 / 9,013,785	1.125	1.106
68	4	8,772,052 / 9,013,785	1.082	1.053
69	5	8,678,805 / 9,013,785	1.040	1.001
		Future Value of TLAD		5.537

Table 9 shows the process of collecting \$135.15 (748.22 / 5.577) in premiums from 1,000 initial insureds that accumulate interest over the deferral period resulting in an accumulation of \$748.22 for each insured who started the accumulation period. Thus, for each insured starting the accumulation period, \$748.22 is available at the beginning of the payment period when there is no refund during accumulation.

To illustrate that the above calculation is correct, Table 10 is presented to show how the ending balance (\$748,224) in Table 9 is distributed among the surviving annuitants. The probability of being alive at each age is calculated relative to the number of insureds starting the cohort at age 60. The beginning balance is reduced by the number of insureds who have survived, multiplied by the rent payment for each insured.

TABLE 9 Year-by-Year Analysis Installment Payment: No Refund

Year	Number of Insureds	Probability of Survival	Dollars Added	Beginning Balance	Interest	Ending Balance
1	1,000	1.000	135,152	135,152	5,406	140,558
2	1,000	0.992	134,025	274,583	10,983	285,567
3	1,000	0.983	132,821	418,388	16,736	435,123
4	1,000	0.973	131,527	566,651	22,666	589,317
5	1,000	0.963	130,129	719,446	28,778	748,224

$$\text{\$ } 748,224 / 1,000 = \text{\$ } 748.22$$

Note: Required installment = NSP / FVTLAD.

Premium = \$ 748.223 / 5.537 = \$135.15

Dollars added is equal to 1,000 x probability of surviving x \$135.15.

Beginning balance equal to previous year's ending balance + dollars added.

TABLE 10 Year-by-Year Analysis of Distribution Period

Year	Age	Number Living	Beginning Balance	\$100 Payment x Number Alive	Invested Funds	Interest Added	Ending Balance
1	65	951.794	748,224	95,179	653,045	26,122	679,166
2	66	939.562	679,166	93,956	585,210	23,408	608,619
3	67	926.221	608,619	92,622	515,996	20,640	536,636
4	68	911.664	536,636	91,166	445,470	17,819	463,289
5	69	895.788	463,289	89,579	373,710	14,948	388,658
6	70	878.503	388,658	87,850	300,808	12,032	312,840
7	71	859.728	312,840	85,973	226,868	9,075	235,942
8	72	839.399	235,942	83,940	152,002	6,080	158,082
9	73	817.464	158,082	81,746	76,336	3,053	79,389
10	74	793.893	79,389	79,389	0	0	0

Numbers are rounded.

Number living is equal to the numbers surviving to receive a payment divided by the number starting at age 60 multiplied by 1,000 (the numbers starting the group).

The rent payment, paid at the beginning of the year, reduces investable funds.

The remaining funds are then invested at the assumed interest rate of 4 percent.

Investment earnings are then added to the invested amount to equal the balance at the end of the year. Since the contract is a 10-year temporary annuity, only 10 years of rent

payments are possible. One can see that as the last rent payment is made to the surviving annuitants, the pool of funds is exhausted and all obligations under the temporary annuity contract terminate.

CHAPTER REVIEW

- Chapter Objectives
- Introduction
- An Overview of the Level Premium
- Net Single Premium: Revisited
- The Present Value of Future Benefits
- The Leveling Process
 - Mortgage Analogy and Modifications
 - Leveling the Term Insurance Contract
 - Leveling the Whole Life Insurance Contract
 - Leveling the Endowment Policy
 - Limited Payment Plans
 - Leveling Annuity Contracts

REVIEW QUESTIONS

1. Explain the term present value of future benefits. Use a mathematical example to demonstrate its calculation.
2. The leveling process compensates for missing premiums and missing interest. Explain what is meant by this statement.
3. Calculate the temporary life annuity due factor to level a 10-year term life insurance policy (1980 CSO Table, 6 percent, male age 22).
4. Refer to Question 3 above. Show the last five years of the calculation if a life annuity due amount is required to level a whole life contract for a male age 22 (1980 CSO Table, 6 percent).

5. How does the calculation in Question 3 above change if the policy being leveled is a 10-year endowment?

6. What is meant by a limited payment plan? At what point does the insurer consider the policy paid up?

7. Refer to the NSP calculations actually made in the end-of-chapter questions for Chapter 21. Calculate the level premiums required for the term and endowment contracts. Set up mathematically the leveling process for the whole life contracts.

8. Calculate the net single premium required to purchase (at age 55) a five-year temporary annuity with a refund during accumulation (1983 Annuity Table, 6 percent). What would the net single premium be if the contract promised deferred benefits (purchased at age 50, deferred five years)? Calculate the installments necessary to purchase the contract over the five-year accumulation period.

9. How does the calculation in Question 8 above change when there is no refund of accumulated amounts allowed? Show all calculations.

10. What differences exist between calculating the level premium for a deferred temporary annuity and a deferred life income annuity?

BIBLIOGRAPHY

Batten, Robert W. Mortality Table Construction. Englewood Cliffs, N.J.: Prentice-Hall, 1978.

Harper, Floyd S., and Lewis C. Workman. Fundamental Mathematics of Life Insurance. Homewood, Ill.: Richard D. Irwin, 1970.

Huebner, S. S., and Kenneth Black, Jr. Life Insurance. 11th ed. Englewood Cliffs, N.J.: Prentice-Hall, 1987.

Jordan, C. W. Life Contingencies. 2d ed. Chicago: Society of Actuaries, 1967.

Pedoe, Arthur, and Colin E. Jack. Life Insurance, Annuities and Pensions. 3d ed.
Toronto: University of Toronto Press, 1978.