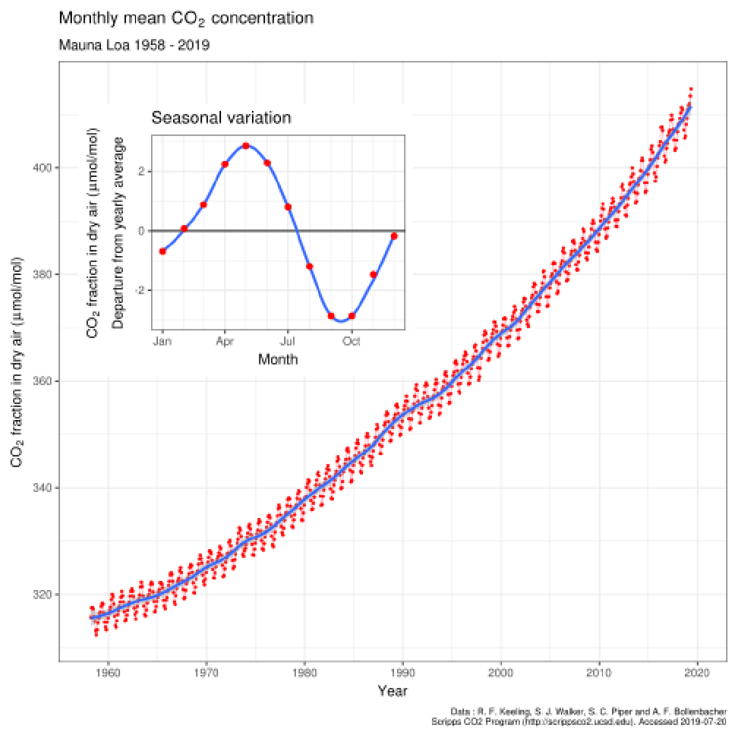
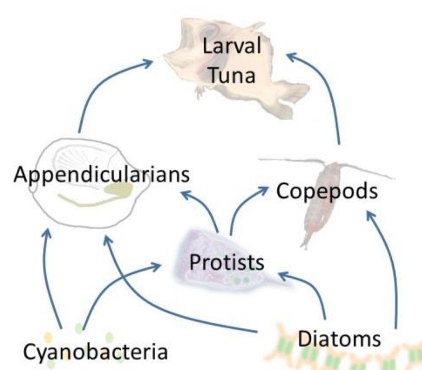
Problem Set 3 Due: Mar. 2

1) In 1958, Dr. Charles David Keeling started continuous measurements of carbon dioxide at the Mauna Loa Observatory on the island of Hawaii. This location was chosen because it was far from any point sources of carbon dioxide, and hence was likely to reflect the average concentrations of carbon dioxide in the troposphere in the Northern Hemisphere. Keeling's measurements showed clear evidence for rising global carbon dioxide levels. In addition to a continual increase in atmospheric carbon dioxide as a result of fossil fuel emissions, the results also show a clear seasonal cycle in carbon dioxide that is related to seasonal patterns of photosynthetic activity of (mainly terrestrial) plants. A colleague of yours is studying the impact of carbon dioxide on plant growth and hence wants a function to use to predict carbon dioxide as a function of time. Please write such a function that approximates both the seasonal cycle of carbon dioxide as well as the steady rise of carbon dioxide over the past 60+ years.



2) In food web studies, ecologists are often interested not only in how much an organisms' diet comes directly from some other species, but also in what proportion of its diet comes indirectly from another species after trophic steps through intermediate trophic levels. For instance, one of my graduate students is studying larval Bluefin tuna in the Gulf of Mexico. This very important species is found throughout the Atlantic Ocean, but spawns only in some very nutrient-poor regions of the Gulf of Mexico and Mediterranean. We are worried about the future of this species, because climate change will likely make its ecosystem more stratified, which will mean that there will be less nutrient input to support large algae, especially diatoms. To try to understand the foodwebs in which these organisms live, we went out and investigated the dietary

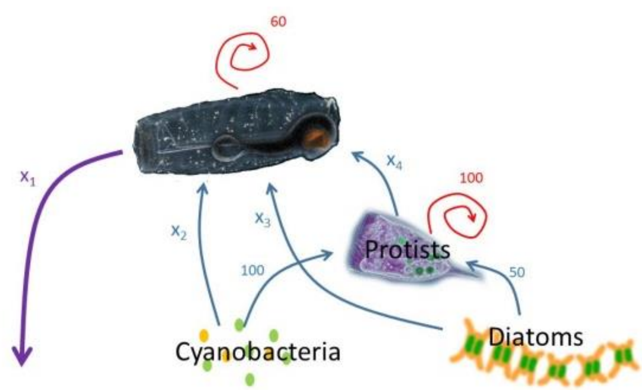


preferences of the larval tuna, as well as trophic interactions within the planktonic ecosystem that supports them. A basic depiction of the foodweb is shown in the picture here. Our main goal was to figure out, for all of those arrows, how important each one was to the diet of any particular organism. We were able to quantify what is called the normalized production matrix (G). G quantifies the proportion of any groups diet that comes from any other group. I've put the values of this matrix in the table below:

		To					
		Cyanobacteria	**Diatoms**	**Protists**	**Copepods**	**Appendicularians**	**Larval Tuna**
	Cyanobacteria	0	0	0.9	0	0.6	0
	Diatoms	0	0	0.1	0.5	0.3	0
	Protists	0	0	0	0.5	0.1	0
From	**Copepods**	0	0	0	0	0	0.7
	Appendicularians	0	0	0	0	0	0.3
	Larval Tuna	0	0	0	0	0	0

For instance, as we look at this table, we can see that larval tuna diet is comprised of 70% copepods and 30% appendicularians. But what we're really interested in, is how much of the larval Bluefin diet comes *indirectly* from diatoms, since diatom abundance is expected to decline in the future. Foodweb theory (and linear algebra concepts) have taught us that we can determine what proportion of an organisms' diet is supported indirectly by every other trophic group by computing the inverse of the matrix that you get when you subtract the normalized production matrix from the identity matrix (or in other words you need to compute the inv(I_n – G)). Please determine how much of the larval Bluefin diet is currently derived indirectly from diatoms.

3) Of course, in order to compute the problem above, one of the most important things that we need to do is actually figure out all of those flows through the ecosystem. In one of the projects that my graduate students are working on, we are trying to figure out the importance of salps in transporting carbon dioxide into the deep ocean and hence sequestering it for periods of time ranging from decades to millennia. Salps produce rapidly-sinking fecal pellets that are responsible for this carbon transport, and hence a proportion of the carbon they consume when they feed on their prey winds up exported into the deep ocean. However, salps are not the only herbivorous zooplankton in the ocean. They compete with single-celled protists for their prey. We can draw a simple picture of the ecosystem that looks like this. In this diagram each arrow represents the amount of carbon going from one part of the ecosystem to another. The blue arrows are the energy flow from one organism to another. The red squiggly arrows are the energy lost through respiration of the different heterotrophic groups. The purple arrow is the amount of

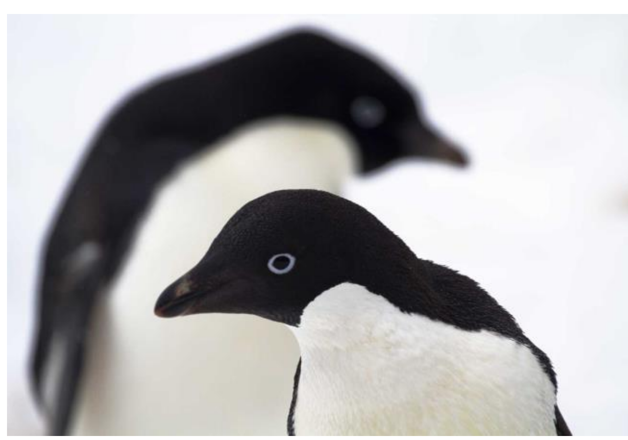


carbon transported into the deep ocean by the salps. Some of these arrows we can actually measure at sea. For instance, we can use a method known as the dilution method to estimate that the protists were consuming 100 mg C m^{-2} d^{-1} of cyanobacteria and 50 mg C m^{-2} d^{-1} of diatoms. We can also use relationships between temperature, size and biomass to estimate that the protists were respiring 100 mg C m^{-2} d^{-1} and the salps were respiring 60 mg C m^{-2} d^{-1}. However, some of the arrows we can't measure directly, and hence I've given them variable names. We can however, use some other measurements to further constrain the system. For instance, we could measure that the total primary productivity, which is equal to the total amount of net carbon produced by the cyanobacteria and diatoms together, is equal to 220 mg C m^{-2} d^{-1} and we can also use lipids to determine that cyanobacteria comprised 5/12s of the diet of salps. Since all of the carbon eaten by an organism has to go somewhere, if we assume that the ecosystem is at steady-state we can also write mass balance equations for the salps and for the protists that state that all the arrows coming in have to equal all the arrows going out. We can thus put together the following linear system of equations.

$x_2 + x_3 + 100 + 150 = 220$	Total primary productivity
$x_2 = \frac{5}{7}x_3 + \frac{5}{7}x_4$	Proportion of cyanobacteria in salp diet
$100 + 50 = x_4 + 100$	Mass balance for protists
$x_2 + x_3 + x_4 = x_1 + 60$	Mass balance for salps

Please solve this system of equations using the determinant approach to figure out how much carbon the salps are transporting to depth (in other words, figure out what x_1 is).

4) You have a friend who studies Adelie penguins in the western Antarctica Peninsula. He explains to you that their populations are declining precipitously in the regions he studies as a result of climate change. After all, the western Antarctica Peninsula is the part of our planet that is warming the fastest and the Adelies are part of an ice-dependent ecosystem. He tells you that he's having some trouble with math in the field, because he can't take out a powerful computer. "When I'm out doing my fieldwork for days on end," he explains, "I don't have any power outlets to run laptops for any heavy computing. However, I still need to be able to quantitatively investigate what is driving the patterns with my penguin colonies. It is really frustrating to have to wait until the end of the field season to actually work up my data. In particular, I think that the distance between the colony and the ice edge is really important for penguin survival. You see, the adults like to feed along the ice edge, so if the ice edge is farther from the colony it is much more difficult for them to get food. Unfortunately, when I GPS tag the penguins what I get is their position in latitude and longitude. However, it is not easy for me to figure out from their



latitude and longitude how far they are from the ice edge, because the western Antarctic peninsula doesn't run north-south and the ice edge is usually perpendicular to the coast." He pauses then and draws you a quick picture of the peninsula. "I wish there was some easy way that I could just rotate my whole study region," he says. Please help him define one (or more) coordinate transformations to transform latitude and longitude into a coordinate system in which one of the coordinates is distance from the coast and the other is perpendicular to the coast (and hence aligned with the ice edge).