Problem Set 2 Due: Feb. 10

Functions of Matrices

1) In toolbox 2.3 I discussed salps. Salps have an interesting life history that includes both a solitary "oozooid" stage and a colonial "blastozooid" stage. Every generation each oozooid gives birth to a chain of 135 blastozooids. And every generation each blastozooid gives birth to one oozooid. (Please note that here we are defining "generation" as length of time between births). In the toolbox, we assumed that newborn oozooids and blastozooids are almost immediately able to reproduce. However, this is definitely not the case.

a) If we assume that it takes an average of three generations for an oozooid or a blastozooid to reach sexual maturity, please write a matrix that can be used to compute the number of young oozooids, young blastozooids, mature oozooids, and mature blastozooids from the current population structure of young oozooids, young blastozooids, mature oozooids, and mature blastozooids. If you start with 100 young oozooids and 100 mature oozooids, how many of each type will you have in the next generation? And the one after that?

b) The generation time and duration that it takes to reach maturity are not actually constant. They vary with temperature. Assume that you can calculate the generation time from the equation:

$$generationlength = 2.5 \times {Q_{10}}^{-T/10}$$

and that you can calculate the time to maturation from the equation:

$$timetomaturation = 8 \times {Q_{10}}^{-T/10}$$

In both equations T is the temperature (a variable) and Q_{10} is the temperature coefficient, which we will assume is equal to 2 for these salps. Please write a function that calculates the population structure of salps one day from now as a function of the population of salps now and the temperature.

c) The analyses above assume that there is no mortality for the salps. Please write another, similar function, that computes the population structure of salps one day from now as a function of the population of salps now and the temperature if we assume that there is 20% mortality per day for young salps (both oozooids and blastozooids) and 10% mortality per day for mature salps (both oozooids and blastozooids). This is what we refer to as density *independent* mortality. It means that the likelihood of salps dying does not depend on the abundance of salps.

d) In ecosystems, density *independent* mortality is not the norm. Instead, organisms often experience density *dependent* mortality. The most common form of density dependent mortality involves higher mortality rates when the population is more abundant. This usually occurs because there are usually more *predators* of a species, when that species is more abundant. This means that during high abundance times, there are more predators and each individual of the species is more likely to get eaten. Please re-write your function from (c) using density dependent mortality. Assume that for each group of salps, daily morality is equal to 0.01 times the population size of that salp group.

e) Assuming that you start with 100 young oozooids and 100 mature oozooids and that the temperature is 10°C, please use the function from (d) to calculate the number of salps 1 day later and then 2 days later. Then bring your function up to me and I will help you graph your function for longer periods of time to see whether or not the population structure will stabilize over time.

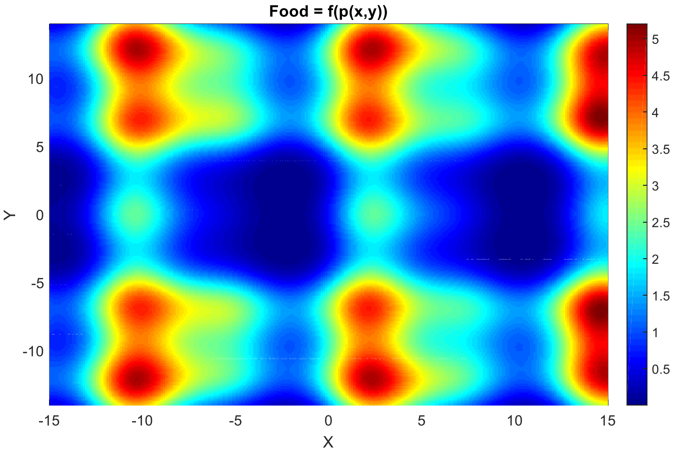
Trigonometry and functions

- Marine bacteria do not have much in the way of brain power for making decisions. They are also pretty limited in how well they can sense their environment – most importantly, they are so small that they cannot use differences in the concentration of their food between different sides of their bodies to detect which direction has more food. However, they *have* evolved very simple behavioral mechanisms that allow them to engage in what is known as chemotaxis – or concerted movement to try and maximize the amount of food they find. The capabilities of different bacterial taxa can be quite different, so it is interesting to consider what sort of behavioral patterns they might follow in order to find more prey. Many bacteria use what is known as a “run-and-tumble” movement pattern. This means that they basically travel in a straight line for a little while and then periodically “tumble” which means essentially spinning around, after which they travel in a new, random direction. The main thing that these bacteria can control is not the direction they turn, but just how far they travel between turns. So let’s consider some different decision making patterns that the bacteria could make.
    - Bacteria A has no memory. All it can do is decide to either increase or decrease its next run length, based on what the food availability is. It evolves to take longer straight line runs if food availability is >1 μmol L^{-1}. Specifically, if the food availability is >1 μmol L^{-1} the next run will be twice as long as the previous run. On the other hand, if food availability is <1 μmol L^{-1} its next run will be only half as long as the previous run.
    - Bacteria B also has no memory. However, it evolves the opposite strategy of taking shorter straight line runs if food availability is >1 μmol L^{-1}. Specifically, if the food availability is >1 μmol L^{-1} the next run will be only half as long as the previous run. On the other hand, if food availability is <1 μmol L^{-1} its next run will be twice as long as the previous run.
    - Bacteria C has a very simple memory and puts it to use. It evolves to increase its run length if the food availability increased during the last run. Specifically, if the food concentration at the end of one run is higher than the food concentration at the end of the previous run, then its next run will be twice as long as its previous run. On the other hand, if the food concentration at the end of one run is lower than the food concentration at the end of the previous run, then its next run will be only half as long as its previous run.
    - Bacteria D also has a simple memory, but evolves the opposite strategy. It decreases its run length if the food availability increased during the last run. Specifically, if the food concentration at the end of one run is higher than the food concentration at the end of the previous run, then its next run will be only half as long as its previous run. On the other hand, if the food concentration at the end of one run is lower than the food concentration at the end of the previous run, then its next run will be twice as long as its previous run.

Your task is to simulate these behaviors and determine which one is likely to be ecologically advantageous and hence which taxon is likely to outcompete its competitors (if all else is equal). I've written a function to mimic a very patchy food environment. The function, which we will call f for food is: $f(p) = \left(\frac{p}{6}\right)^2$ where p is a function of the position, such that:

$$p(x, y) = \sin(x) + \cos(y) + \sin\left(\frac{x}{2.5}\right)\cos\left(\frac{y}{2.5}\right) + 3\cos\left(\frac{y}{3} + 180°\right) + 3\sin\left(\frac{x}{2}\right)$$

Just to show you what this looks like here is a graph of f(p(x,y))


Food = f(p(x,y))

Alright, here's how the simulation will work. Every group will have a bacteria that starts off at x=0 and y=0. It will also start off with an initial run length of 0.2. First you'll need to calculate how much food you have at time t_0. Then you'll need to determine a random angle for your bacterium to head off in. You'll determine this by rolling three die (a blue one, a red one, and a green one). After you have rolled the die, you will determine your angle using the function $\theta(r, g, b) = 60r + 10g + b$ where *r* is the number you rolled with the red dice, *g* is the number you rolled with the green dice, and *b* is the number you rolled with the blue dice. Please record the roll you got with each die and the angle you calculated. Once you've calculated your angle use your initial run length of 0.2 to compute where you wind up after your next time step. Record your new x and y location. Compute the food available at your new location and record this information. Determine from your bacterium's behavior whether the next run should be longer or shorter and record your computed run length. Then roll a new angle and repeat the process. Do this 20 times and develop a table that looks like this:

Time step	x	Y	Food	Run length	Red	Green	Blue	θ
t_0	0	0	???	0.2				
t_1								
t_2								
…								

Now take the average of all of your food concentrations. Once every group has finished this first question, compare your food concentrations to those of all the other groups. Which strategy worked best?