Problem Set 1 **Due Jan. 27**

1) One of your friends just started work in a lab that cultures bacteria. His adviser told him to prepare some solutions to use to grow the bacteria. He was told to make up solutions including:
   - 2 L of 100 mmol L^{-1} glucose ($C_6H_{12}O_6$)
   - 100 mL of 1 mol L^{-1} thymidine ($C_{10}H_{14}N_2O_5$)
   - 1 L of 10 mmol L^{-1} leucine ($C_6H_{13}NO_2$)

   Your friend has access to a scale, but needs to know how much glucose, thymidine, and leucine to weigh out. Please explain to him how he could use the dot product and vector multiplication to figure out how much of each of the chemicals he needs.

2) You have a friend who is a meteorologist and is interested in determining how wind speed during a storm affects the amount of damage that occurred in the storm. Unfortunately, she gets all of her data from a meteorological reanalysis product that reports wind as east-west and north-south vectors and she needs to know the absolute wind speed, since it is absolute wind speed that likely determines the amount of damage done.

	East-west velocity (km h^{-1})	North-south velocity (km h^{-1})
Storm 1	10	51
Storm 2	36	-47
Storm 3	-52	55
Storm 4	-49	32
Storm 5	81	-23
Storm 6	-64	-12
Storm 7	28	72

3) You're studying penguin colonies on two islands in the Western Antarctic Peninsula. On Anvers Island, the Adelie penguin population has been decreasing drastically, while the Gentoo penguin population has been increasing over time. On Charcot Island, there are no Gentoo Penguins and the Adelie population has been increasing. Your adviser wants you to investigate whether these patterns are related to the food availability for each of these species. Both species feed on both adult and larval Antarctic krill (*Euphausia superba*), ice krill (*Euphausia crystallophorias*), and silverfish (*Pleuragramma antarctica*). You've decided on a sampling plan to let you estimate the amount of carbon, nitrogen, and phosphorus that they are getting from their diets. First you collected a bunch of the prey species from their natural environment using net tows. You found that larval Antarctic krill have an average of 102 mg C, 12 mg N, and 1.0 mg P each. Adult Antarctic krill have an average of 1.1 g C, 130 mg N, and 10 mg P each. Ice krill have 250 mg C, 40 mg N, and 2.5 mg P each. Silverfish have 10 g C, 2.3 g N, and 190 mg P each. You also caught 5 adult penguins of each species in each colony, forced them to eject their stomachs, and counted the prey items in their stomachs. Please calculate the average amount of carbon, nitrogen, and phosphorus that the species are getting in each respective colony.

Adelies on Anvers Island				
	Larval *superba*	Adult *superba*	Adult *crystallophorus*	Silverfish
Penguin 1	16	10	3	1
Penguin 2	12	11	5	0
Penguin 3	5	8	9	0
Penguin 4	20	4	4	2
Penguin 5	6	9	1	0

Gentoos on Anvers Island				
	Larval *superba*	Adult *superba*	Adult *crystallophorus*	Silverfish
Penguin 1	11	12	156	0
Penguin 2	7	13	212	0
Penguin 3	12	10	135	0
Penguin 4	15	15	648	0
Penguin 5	1	11	312	0

Adelies on Charcot Island				
	Larval *superba*	Adult *superba*	Adult *crystallophorus*	Silverfish
Penguin 1	468	8	0	5
Penguin 2	636	5	0	4
Penguin 3	405	16	0	6
Penguin 4	1944	4	0	1
Penguin 5	936	6	0	2

4) You have a friend who is a chemist. Secretly you suspect that he really wanted to be an environmental scientist, but he only went into chemistry because he doesn't have the mathematical skills that you rely upon. As a result, he often comes to you for help. This time he's trying to figure out the stoichiometric balance of a series of chemical reactions. Please use matrix algebra and elementary row operations to balance the following chemical equations for him.

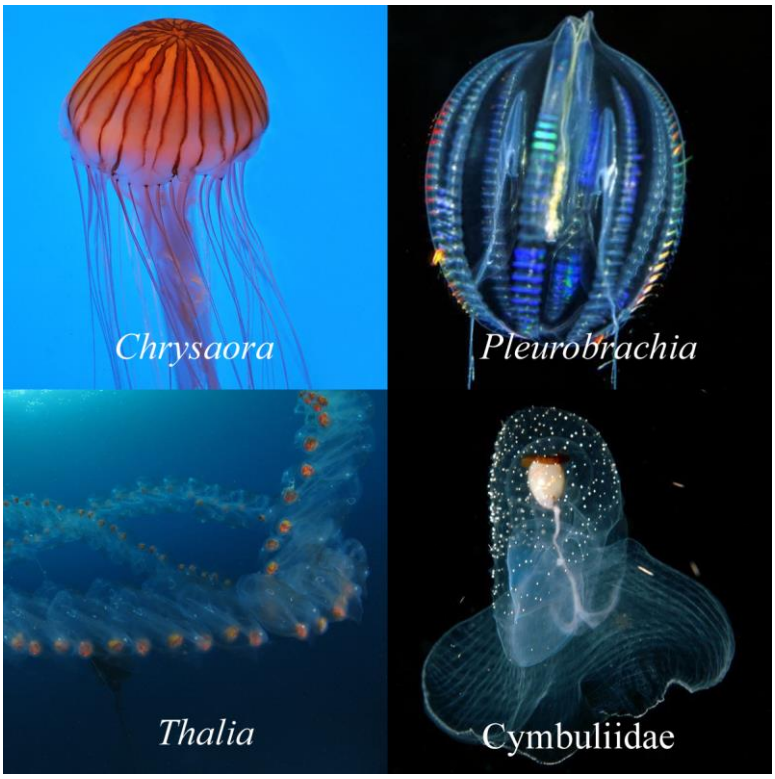
$x_1NH_3 + x_2O_2 \rightarrow x_3NO + x_4H_2O$

$y_1NH_4^+ + y_2O_2 + \rightarrow y_3NO^{2-} + y_4H^+ + y_5H_2O$

$z_1CaCN_2 + z_2H_2O \rightarrow z_3NH_3 + z_4CaCO_3$

$w_1HIO_3 + w_2FeI_2 + w_3HCl \rightarrow w_4FeCl_3 + w_5ICl_2 + w_6H_2O$

5) You've been tasked with studying the diets of leatherback turtles living out in the open ocean. Unfortunately, the simple approach (analyzing the prey inside their guts) doesn't work very well for open ocean turtles, because these turtles consume primarily gelatinous zooplankton and those gelatinous zooplankton are usually not recognizable inside their guts. You believe that the leatherbacks are feeding on 4 different types of zooplankton: *Chrysaora* (a cnidarian jellyfish), *Pleurobrachia* (a ctenophore or comb jelly), *Thalia* (a salp), and Cymbuiliidae (a family of pteropods or sea butterflies). In order to quantify the proportion of these organisms in the leatherback's diet, you decide to use carbon, nitrogen, and sulphur isotopes. This approach is commonly used in ecology, because organisms' isotopic composition reflects the isotopic composition of their prey. Generally an organism's carbon and sulphur isotopic composition are a direct reflection the average carbon and sulphur isotopic composition of their prey. However, their nitrogen isotopic composition is approximately 3 higher than their prey, because organisms fractionate nitrogen while they make their amino acids. To determine the relative proportions of these prey in the leatherback's diet, you went out and conducted net tows to collect zooplankton. You then sorted them out to genus and measured the isotopic composition of each prey group using an isotope ratio mass spectrometer. You also measured the isotope composition of some biopsy samples you collected from leatherbacks that you caught (and then gently released) in the same region. Now it's time to analyze the data and figure out what the ratios of the prey items in their diets are.



	Chrysaora	*Pleurobrachia*	*Thalia*	Cymbuliidae	Leatherback
$\delta^{13}C$	-19	-21	-21	-20	-20.1
$\delta^{15}N$	11	8.8	6.3	5.3	12
$\delta^{34}S$	15	10	11	9.1	12.2

6) You are studying pollutants in an estuary. The sources of water entering the estuary include:
   - A river with three different tributaries
   - Groundwater from the aquifer
   - The ocean

   In order to investigate the sources of these pollutants to the estuary, you have gone out and measured the salinity, DDT concentration, Atrazine concentration, and mercury levels in the three tributaries, the surrounding ocean water, and in the groundwater.

	Salinity	DDT	Atrazine	Mercury
Ocean	35 ppt	0.2 ng m^{-3}	0.1 µg L^{-1}	4.1 ng L^{-1}
Groundwater	0.5 ppt	0.3 ng m^{-3}	2.0 µg L^{-1}	5.2 ng L^{-1}
Tributary 1	0.1 ppt	0.5 ng m^{-3}	0.8 µg L^{-1}	15 ng L^{-1}
Tributary 2	0.2 ppt	0.4 ng m^{-3}	6.2 µg L^{-1}	6 ng L^{-1}
Tributary 3	0.1 ppt	3.0 ng m^{-3}	1.3 µg L^{-1}	4.8 ng L^{-1}
Estuary	21.1 ppt	0.32 ng m^{-3}	1.33 µg L^{-1}	5.13 ng L^{-1}

   Please calculate the percentage of each pollutant that is coming from each source so that you can help managers determine what management decisions they might want to take to improve the health of the estuary.

7) Oxygen, carbon dioxide (and bicarbonate and carbonate), and nutrient concentrations in the ocean are all linked, because when phytoplankton (algae) produce organic matter by photosynthesis they take up carbon dioxide and nutrients and produce oxygen. Conversely, when the organic matter created by phytoplankton is released due to respiration and remineralization by zooplankton and bacteria, oxygen is consumed and carbon dioxide and nutrients are produced. Because most phytoplankton have very similar carbon:nitrogen:phosphorus ratios, it is possible to estimate things like carbon dioxide sequestration or nitrogen fixation rates solely by looking at patterns of oxygen and nutrients in the deep ocean. However, things are a bit more complicated than this, because sometimes phytoplankton use NO_3^- for their nutrients and sometimes they use NH_4^+. One of your friends is struggling to understand these differences. Please help her balance the full equations for photosynthesis based on NO_3^- and NH_4^+.

   $x_1CO_2 + x_2NO_3^- + x_3PO_4^{3-} + x_4H_2O + x_5H^+ \rightarrow x_6C_{106}H_{175}O_{42}N_{16}P + x_7O_2$
   $y_1CO_2 + y_2NH_4^+ + y_3PO_4^{3-} + y_4H_2O + y_5OH^- \rightarrow y_6C_{106}H_{175}O_{42}N_{16}P + y_7O_2$

   Next please write a vector equation to show how to calculate the ratio of O2 produced to CO2 consumed, from the proportion of the phytoplankton nitrogen that comes from NO_3^-.

8) Your friend wants to predict what the growth rates will be for some phytoplankton that he wants to grow in an incubator set up on top of the new EOAS building, and has turned to you for help. He has found an equation from the manuscript Morrow et al. (2018):

$$PrimaryProductivity = V \times \left(1 - e^{-a \times PAR/V}\right) e^{-b \times PAR/V}$$

In the manuscript, the authors determined that V = 64 mg C m^{-2} d^{-1} / mg Chl, a = 1.0, and b = 0.049. PAR is photosynthetically active radiation and varies throughout the day. Please combine some of the elementary functions to write a function that can approximate the light levels that we measured on the roof. Then write a function for primary productivity as a function of time.

9) In 1958, Dr. Charles David Keeling started continuous measurements of carbon dioxide at the Mauna Loa Observatory on the island of Hawaii. This location was chosen because it was far from any point sources of carbon dioxide, and hence was likely to reflect the average concentrations of carbon dioxide in the troposphere in the Northern Hemisphere. Keeling's measurements showed clear evidence for rising global carbon dioxide levels. In addition to a continual increase in atmospheric carbon dioxide as a result of fossil fuel emissions, the results also show a clear seasonal cycle in carbon dioxide that is related to seasonal patterns of photosynthetic activity of (mainly terrestrial) plants. A colleague of yours is studying the impact of carbon dioxide on plant growth and hence wants a function to use to predict carbon dioxide as a function of time. Please write such a function that approximates both the seasonal cycle of carbon dioxide as well as the steady rise of carbon dioxide over the past 60+ years.