Understanding Liquidity Shortages During Severe Economic Downturns

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Abstract

One feature of economic recessions is the appearance of aggregate liquidity shortages that can exacerbate the economic downturn. We develop a model in which the demand for liquidity arises suddenly in response to continued funding needs of partially completed investment projects whose outcomes are subject to idiosyncratic shocks and moral hazard. When the economy experiences an adverse aggregate productivity shock, incentive constraints that underlie equity contracts may bind, provided the shock is severe enough. In this case, credit-rationing appears, and the heightened demand for liquidity coincides with a greater reluctance to take on equity positions or deepen investments in on-going investment projects. The consequence is a reduction in new investment and termination of on-going projects due to a lack of liquidity, thereby worsening the economic slowdown.

Keywords: Credit-rationing, moral hazard, occasionally binding constraints.

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1 Introduction

Absent from most DSGE macroeconomic models of the business cycle is the important role that private information and financial contracts can play in affecting short-run economic fluctuations, particularly during severe economic downturns. Exceptions, based on the early work of Diamond (1984), include Williamson (1986) and Bernanke and Gertler (1989), in which incentive constraints were expressly incorporated into loan agreements between investors (lenders) and firms (borrowers). These models exhibit equilibrium credit rationing through which adverse economic shocks are seen to exacerbate economic downturns and increase bankruptcies. In Williamson (1987), heightened uncertainty over the future outcome of funded projects is also seen to induce an economic slowdown, even if the mean expectation of project returns is unchanged. These results generally rely on a financial accelerator that operates through a procyclical net worth position of the firm which affects the ability of a firm to acquire working capital to fund new investments. These papers focus on debt-financing through financial intermediaries and do not address liquidity issues per se, i.e., when funds are suddenly needed to meet unanticipated expenditures associated with ongoing operations.[1]

Holmstrom and Tirole (1998) have investigated these liquidity issues in three-period, partial equilibrium models. They find that limited pledgeable future income generated by funded projects requires that incentive constraints be present in original loan contracts, which lead to "suboptimal" funding of socially valuable projects. Consequently, adverse shocks to individual firms may result in termination of ongoing projects and worsen the state of an already weakened economy. They examine conditions under which an inadequate provision of liquidity arising in the private sector may provide a rationale for an enhanced supply of liquidity by the government.

[1] One branch of this literature that deals with the liquidity issues of financial institutions is represented by the bank runs model of Diamond and Dybvig (1983) and models in which financial fragility serves as a commitment mechanism as in Diamond and Rajan (2001). These models rely on adverse selection associated with investor types and are not the subject of this paper which is more concerned with how the interaction of liquidity shortages and moral hazard during economic downturns can heighten the economy's contraction.

Kiyotaki and Moore (2005, 2008) have examined insufficient aggregate liquidity and its consequences for the business cycle in a different context. They focus on a combination of liquidity constraints – one in which the supply of new equity issues by entrepreneurs is bounded by entrepreneurs' inalienable human capital, and a second constraint in which existing equity shares are not fully marketable. Both constraints limit the value of equity in the financing of investment opportunities, and give rise to a demand for money. Their models are structured to capture some asset-pricing anomalies and to demonstrate how monetary policy may offset liquidity shortages through open market operations that take place in the equity market.[2]

In this paper, we build a stylized DSGE model with entrepreneurs raising funds in an equity market to undertake risky multi-period projects with a positive expected social value. However, as in Holmstrom and Tirole (1997), moral hazard is present due to the private information possessed by the entrepreneurs whose actions bear on the equilibrium outcomes of the projects. Therefore, equity contracts are premised on incentive constraints designed to induce desirable actions on the part of the entrepreneur that ensure a positive expected return on the funded project. However, these provisions in the contracts do not affect the economy-wide supply of liquidity unless the economy experiences a sufficiently negative aggregate (productivity) shock, in which case, credit rationing may result.

Technically, the incentive constraints are not always binding, but when they bind, they affect the ability of the entrepreneur to raise funds for new investment projects and make it more difficult to bring these projects to completion should unexpected expenses suddenly arise. These unplanned expenses give rise to a demand for liquidity. However, the supply of liquidity decreases with adverse aggregate shocks, and if the shock is strong enough, the expected profitability threshold that these projects must meet if they are to receive additional injections of new funds required

[2] In a related literature, Gertler and Kiyotaki (2010) and Curdia and Woodford (2010) construct models with exogenous financial frictions that are able to simulate the observed behavior of selected key financial and macroeconomic variables during the recent credit crisis in the United States in order to examine appropriate monetary policy responses.

to bring these projects to fruition is raised. This greater reluctance of investors to take equity positions or to deepen their current investments in a weak economy is shown to exacerbate an economic downturn and to alter the cyclical properties of the macroeconomy by influencing investment decisions of households.[3]

The mechanism that we describe, through which financing requirements affect economic activity, differs from the financial accelerator described, for example, in Holmstrom and Tirole (1997) and Bernanke and Gertler (1989). In this model, net worth plays no role in limiting the need for external finance. Rather, when negative aggregate shocks are sufficiently strong, they can exacerbate the significance of firm-specific idiosyncratic shocks and lead to an excess demand for liquidity that causes credit rationing to limit overall economic activity. Some continuing projects that would otherwise be funded are terminated; while fewer new investment projects receive funding.[4]

To focus attention on the importance of the incentive constraints in dealing with the moral hazard issue, we calibrate the model such that the magnitude of the moral hazard problem is parametrically set sufficiently low that the incentive constraints never bind. The model is then re-calibrated with the importance of moral hazard increased sufficiently that the incentive constraints occasionally bind. Simulation results are then provided that illustrate how a lack of aggregate liquidity can "kick in" after a severe negative productive shock and exacerbate the subsequent downturn in the economy.

[3]We note in passing that this feature of an occasionally binding constraint is one-sided, in that it only binds during sharp negative aggregate shocks. This could be a factor helping to explain the asymmetry of business cycle fluctuations, in which economic recoveries are more gradual than economic contractions, as documented, for example, by Van Nieuwerburgh and Veldkamp (2006).

[4]We note that the credit rationing that occurs in this model may be present after the initial financing of the project takes place, when a demand for liquidity exceeds the expected liquidity needs of ongoing projects and requires additional funding.

2 The Model

With the principal focus on the aggregate consequences of moral hazard issues at the firm level, the following assumptions are made to ensure perfect risk-sharing within a representative household setting. The economy is populated by a continuum of households of measure one. Each household consists of an investor and continuums (of measure one) of workers and entrepreneurs. The entrepreneurs run projects which require both external finance and labor services supplied from outside the household. The workers offer labor services to other households' entrepreneurs and receive labor income for their efforts. The investor is responsible for managing the household's assets that consist of a risk-less bond, a liquid real asset called 'money,' and shares in the projects of entrepreneurs from other households. With all projects *ex ante* identical, their shares command the same price and are traded in a unified equity market. This market is perfectly competitive as individual projects have measure zero. The risk-averse household diversifies idiosyncratic project risk through this asset market by investing equally in all projects.[5]

In our setup, the household does not invest in its own projects nor does it provide labor services to those projects. The members of the household separate at the beginning of each period and at the end of the period, they reunite, pool their resources, and consume together. This structure ensures equity and labor markets in which moral hazard issues may be present.

2.1 Project Implementation and Financing

The projects require investment one period in advance which the entrepreneur finances by issuing shares. The projects are subject to a liquidity shock at the beginning of the next period when they can potentially produce. The entrepreneur does not have the

[5] Households could achieve the same diversification through financial intermediaries (banks). In that case, it would be more natural to consider financing the projects with bank loans. As in Tirole (2006, p. 119), these two financial arrangements are equivalent in our model. To simplify the exposition, we assume that the diversification of idiosyncratic risk is achieved through equity markets.

funds to finance the second period liquidity shock. The outside investors decide at that time if they would finance the liquidity shock. If the liquidity shock is financed, the entrepreneur goes ahead with the project. The entrepeneur can affect the likelihood of a successful project (described below) through work effort. The project succeeds with probability p_H if the entrepreneur does not shirk; otherwise, the probability of success falls to p_L. This timing of decisions and resolution of uncertainty with respect to projects is shown Figure 1.

Each period, the entrepreneurs from each household start new projects that have a measure of one. The projects are indexed by $i \in [0, 1]$. A project's output depends on the amount of labor employed in the first period. Thus, the output of a project i started in period t, if successfully implemented, is

$$y_{t+1}^{i}\left(\theta_{t+1}\right) = \theta_{t+1}(n_{1,t}^{i})^{\alpha}, \tag{1}$$

where $n_{1,t}^{i}$ is the outside labor employed by the household in period t and θ is the random aggregate productivity parameter. Thus, the project output is random and depends on the realization of θ at the beginning of time $t + 1$.

In addition to the aggregate shock, each project started in period t also experiences, at the beginning of time $t + 1$, a project-specific liquidity shock ρ_{t+1}^{i} with a known distribution $F\left(\rho\right)$ and corresponding density $f\left(\rho\right)$. As a result, the entrepreneur needs to make an additional investment in period $t + 1$ for the project to potentially succeed. To be precise, the liquidity shock results in the need to hire an additional $n_{2,t+1}^{i}$ outside workers, or

$$n_{2,t+1}^{i} = \rho_{t+1}^{i}. \tag{2}$$

The reason this shock is labeled a liquidity shock is that the shock must receive external funding with a liquid asset.

The entrepreneurs do not have funds to finance either the first-period wage bill or the second-period liquidity shock. They issue equity in the first period to outside investors to meet the wage bill. The liquidity shock at the beginning of the second period is also financed by the outside investors and they are aware of this fact when they decide to invest. With all costs already paid when the project actually produces, the entire revenue proceeds are profits that are distributed among the shareholders at the end of the second period on completion of the project. We normalize the total shares of a project to 1. The entrepreneur sells s_t^i shares to finance the wage bill, so that

$$s_t^i p_t = w_t n_{1,t}^i, \tag{3}$$

where p_t is the price of the share of a project started in period t in the period of issue, t, and w_t is the wage rate in period t.[6] Thus, s_t^i represents the "outside equity" (see Tirole, 2006, p. 119) in the project.

The investors realize they will need to finance the liquidity shock for which they carry a liquid asset from period t to $t+1$. However, not all projects have their liquidity needs financed by investors. After observing ρ_{t+1}^i, they compare their expected benefit from financing with the cost of financing. The benefit from the project is uncertain even after the liquidity need is met as not all projects finally succeed in producing output. Yet, conditional on being financed, the expected benefit to the investor is the same for all continued projects. Thus, there exists a threshold value of $\rho_{t+1}^* (\theta_{t+1})$ such that all projects with lower liquidity needs than this threshold are financed in period $t+1$. The functional dependence of this cutoff value on θ_{t+1} arises from the fact that both the project revenue, conditional on the liquidity need being financed, and the liquidity need itself depend on the aggregate shock.

[6] In an appendix available from the authors, an expanded version of the model is solved in which the entrepreneur may choose to withhold a portion of the first-period funds to absorb the liquidity shock. These precautionary holdings increase the probability of obtaining the requisite second-period funding to complete the project. However, they also reduce the size of the project, and hence the entrepreneur's expected profits. We find numerically that the latter (cost) always outweighs the former (benefit), and the entrepreneur would always choose to maximize the size of the project and never carry any excess liquidity.

2.2 The Household sector

In this section, the decisions of the representative household – excluding the entrepreneurs decisions – are described. For added focus, the entrepreneur's problem is treated separately in the next subsection of the paper.

The representative household maximizes lifetime utility, with period utility, $U(C, L)$, defined over consumption and leisure. The varieties produced by different projects are perfect substitutes in consumption. Hence, the aggregate good, C, is a linear aggregate of different varieties, or

$$C_t = \int_0^1 c_t^k dk, \tag{4}$$

where c_t^k is the consumption of variety or good k. The household consumes jointly but the members of the household – the investor, the entrepreneurs, and the workers – specialize in different income-earning activities. Based on the consumption-leisure decision of the household, the worker provides the labor, n_t, which is one source of household income. The entrepreneurs start new projects in each period and retain shares in the projects denoted $(1 - s_t^i)$. Maturing projects yield profits in the amount Π_t^l, thus providing another source of income for the household.

The final source of income is from the household's assets. These assets are managed by the investor who determines the household's optimal consumption-saving decision and makes the portfolio allocation decision for investing the household's savings in three assets. The investor buys B_{t+1} units of a risk-less bond, each unit of which provides one unit of aggregate output in the next period. Second, it decides to buy s_t^j shares of projects externally operated by other households, where $j \in [0, 1]$.[7] As the number of shares of each project is normalized to 1, s_t^j shares entitle the household to a corresponding fraction of the gross revenue from sales of the project's output

[7] We note that the households in the economy are of measure one, and each household starts projects of measure one. Thus, a double continuum of projects are started every period. However, in the spirit of the representative agent assumption and to simplify notation, we avoid the double index notation for the projects. Furthermore a single index notation is sufficient for our purposes as a single continuum of projects allows the household to completely diversify idiosyncratic risk.

in period $t+1$, provided the project is eventually successful and produces the output. A necessary condition for the project to produce output is that its random liquidity need at the beginning of period $t+1$ is financed. This liquidity need arises from the fact that the entrepreneur needs to pay for unanticipated extra costs of operations in period $t+1$ before the revenue from the output becomes available. The provision of this liquidity is the third investment option for the household. In particular, the household carries M_{t+1} units of liquid assets (the composite good which is assumed to be costlessly storable), which are held across periods but yield zero net return.

In addition to making the investment decisions for the next period, the household's investor also determines which of the on-going projects will have their liquidity needs financed. This decision is made after observing the current period aggregate shock (θ_t) and the individual realization of ρ_t^k, where the index k is used for the projects in which the household had invested in period $t-1$. Note that this notation contrasts with the use of the superscript j for the projects being started in period t. As discussed earlier, this decision would take the form of a cut-off value of the liquidity shock, ρ_t^*.

The liquidity need per share, denoted $m_t^k\left(\rho_t^k\right)$, that the household must choose whether to fund, given that the number of shares s_{t-1}^k that was determined in the previous period. Then, in equilibrium, the total liquidity need for project k becomes:

$$m_t^k\left(\rho_t^k\right)s_{t-1}^k = \rho_t^k w_t. \tag{5}$$

The household's total income, Y_t, is

$$Y_t = w_t n_t + \int_0^1 \Pi_t^l dl + \int_0^1 p_H \hat{R}_t^k(\theta_t) s_{t-1}^k I_{[\rho_t^k \leq \rho_t^*]} dk. \tag{6}$$

The consumption-based price index for the aggregate goods is

$$Q_t = \min_k q_t^k, \tag{7}$$

which in equilibrium will imply that

$$q_t^k = q_t = Q_t = 1, \tag{8}$$

for all varieties k that are produced in equilibrium as the composite good is the numeraire.[8]

Thus, the household's budget constraint is

$$C_t + \int_0^1 p_t s_t^j dj + \int_0^1 m_t^k(\rho_t^k) s_{t-1}^k I_{[\rho_t^k \leq \rho_t^*]} dk + M_{t+1} + \frac{B_{t+1}}{R_t} \leq M_t + B_t + Y_t, \tag{9}$$

where the right-side has the total funds available to the household: the liquidity carried from the last period, the revenues from maturing bonds, and the income described in (6). The left-hand side is the use of those funds: consumption, the purchase of shares in new projects, funds needed to meet the liquidity needs of existing projects, provision for the liquidity needs for the next period, and the investment in risk-less bonds, where R_t is the risk-free rate. In addition to this overall funding constraint, the ability to meet the current liquidity needs is constrained by the liquidity carried over from the previous period

$$\int_0^1 m_t^k(\rho_t^k) s_{t-1}^k I_{[\rho_t^k \leq \rho_t^*]} dk \leq M_t. \tag{10}$$

At the beginning of period 0, the household takes as given its initial asset holdings that includes shares in its own projects $(M_0, B_0, s_{-1}^k, s_{-1}^l)$ and solves the following problem:

$$\max_{\{C_t, L_t, n_t, M_{t+1}, B_{t+1}, s_t^i, s_t^j, \rho_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{11}$$

subject to

$$n_t + L_t \leq 1, \tag{12}$$

and $(9 - 10)$. For the economy as a whole $n_{1,t-1}$ is also given.

If we reformulate the optimization problem as a dynamic program, the household-

[8] We can generalize this to the case with less than perfect substitution in consumption but in that case the firms with have pricing power which they do not have now.

specific state variables are $\left(M, B, s_{-1}^{l}, s_{-1}^{k}\right)$, where s_{-1}^{l} represents shares of projects started by the household in the previous period, s_{-1}^{k} represents shares of projects of other households in which the representative household had invested in the previous period. Shares of projects undertaken by the household in the current period are denoted s^{i}, and shares of projects begun by other households in which the representative household is investing in the current period are denoted s^{j}. In addition, there are aggregate state variables θ and $n_{1,-1}$. Thus, the problem can be written as

$$V\left(M, B, s_{-1}^{l}, s_{-1}^{k}; \theta, n_{1,-1}\right) = \max_{C,L,n,M',B',s^{i},s^{j},\rho^{*}} \left\{U\left(C, L\right) + \beta E_{\theta'}\left[V\left(M', B', s^{i}, s^{j}; \theta', n_{1}\right)\right]\right\}, \tag{13}$$

which is again maximized subject to $(9-10)$ and (12).

2.3 Entreprenuer's Problem

As the project under management by the entrepreneur is subject to moral hazard, the probability of success of the project depends on the effort of the entrepreneur. If the entrepreneur exerts effort, the probability of success is p_H, and if he shirks, the probability falls to $p_L < p_H$. Shirking provides an exogenous benefit to the entrepreneur. Investors are aware of this possibility and limit funding to the point where their expected return equates to the return from alternative investment options.

We assume that the entrepreneur maximizes his expected profits subject to his incentive compatibility constraint and his first-period funding needs. He is the residual claimant to the fraction $(1 - s_t^i)$ of period $t+1$ gross revenues (which are the same as profits) that are realized if the project succeeds. It is assumed that the loss from shirking is high enough that it is always optimal to incentivize the entrepreneur to exert effort so that in equilibrium the probability of success, conditional on the liquidity need being financed, is p_H.

If successful, the revenue from the project is

$$\hat{R}^{i}\left(\theta_{t+1}\right) \equiv q_{t+1}^{i} y_{t+1}^{i}\left(\theta_{t+1}\right) = q_{t+1}^{i} \theta_{t+1} (n_{1t}^{i})^{\alpha}, \tag{14}$$

where q_{t+1}^i is the price of good i produced by entrepreneur i's project.

Coming back to the entreprenuer's profit maximization, his profits are $(1 - s_t^i)\,\hat{R}_{t+1}^i$ with probability p_H and zero otherwise. Thus, the entrepreneur's objective becomes

$$\max_{s_t^i, n_{1t}^i} E_{t,\theta} \left\{ \beta \frac{U_{C_{t+1}}}{U_{C_t}} \left[(1 - s_t^i) p_H \hat{R}^i \left(\theta_{t+1}\right) F\left(\rho_{t+1}^*\right) \right] \right\}, \tag{15}$$

where the profits are discounted back to time t using the household's stochastic discount factor and $E_{t,\theta}$ denotes expectation over θ_{t+1} conditional on information at date t. Recall, ρ_{t+1}^* is the maximum liquidity need that is financed by the investor.

The incentive compatibility constraint for the entrepreneur is

$$p_H \left(1 - s_t^i\right) \hat{R}_{t+1}^i \left(\theta_{t+1}\right) \geq p_L \left(1 - s_t^i\right) \hat{R}_{t+1}^i \left(\theta_{t+1}\right) + J s_t^i, \tag{16}$$

where the total benefit from shirking, $J s_t^i$, $J > 0$, is an increasing function of outside equity, s_t^i.[9] Note that there is one incentive compatibility constraint for each aggregate state. Thus, the maximization of (15) is subject to (3) and (16).

3 Solving the Model

We begin by solving the household's problem followed by that of the entrepreneur. Of particular interest will be the binding nature of the incentive compatibility constraint imbedded in the entrepreneur's problem. When it binds due to a significant adverse shock, the liquidity needs for continuation of the project may not be funded. Such a condition will be shown in the next section to exacerbate economic downturns.

[9] While there is no standard way to model the benefit from shirking, we assume that the entrepreneur's motivation diminishes as his stake in the outcome falls with an increase in equity shares issued, s_t^i. An alternative would be to assume that shirking increases as the factor input, n_{1t}, rises. In this model, there is no difference between these two choices since labor input in the initial period is a linear function of the number of shares issued.

3.1 Solution to Household's Problem

The first-order conditions for the household's problem yield familiar Euler equations for the household's labor-leisure choice and consumption-savings decisions[10]

$$w_t U_{C_t} = U_{L_t} \tag{17a}$$

$$U_{C_t} = \beta R_t E_{t,\theta} \left[U_{C_{t+1}} \right] \tag{17b}$$

In addition, we have the optimality conditions for the choice of liquidity (M_{t+1}), levels of investment in projects $\left(s_t^j\right)$, and the decision to finance the liquidity needs of previous-period projects (ρ_t^*) which are

$$U_{C_t} = \beta E_{t,\theta} \left[U_{C_{t+1}} \left\{ \frac{p_H \hat{R}_{t+1}\left(\theta_{t+1}\right)}{m_{t+1}\left(\rho_{t+1}^*\right)} \right\} \right], \tag{17c}$$

$$U_{C_t} = \beta E_{t,\theta} \left[U_{C_{t+1}} \left\{ \frac{p_H \hat{R}_{t+1}(\theta_{t+1}) F\left(\rho_{t+1}^*\right)}{p_t} \right\} \left\{ \frac{\hat{R}_{t+1}^j(\theta_{t+1})}{\hat{R}_{t+1}(\theta_{t+1})} - \frac{\bar{m}_{t+1}\left(\rho_{t+1}^*\right)}{m_{t+1}\left(\rho_{t+1}^*\right)} \right\} \right], \tag{17d}$$

$$U_{C_t} + \lambda_t = U_{C_t} \frac{p_H \hat{R}_t\left(\theta_t\right)}{m_t\left(\rho_t^*\right)}. \tag{17e}$$

where λ_t is the Lagrange multiplier on the liquidity constraint (10) and in ($17d$)

$$\bar{m}_{t+1}\left(\rho_{t+1}^*\right) = \int_0^{\rho_{t+1}^*} m_{t+1}(\rho_{t+1}) \frac{f(\rho)}{F\left(\rho_{t+1}^*\right)} d\rho, \tag{20}$$

is the average liquidity need, conditional on the need being financed.

In each of equations ($17c - 17e$), the left-hand side is the (current) marginal (utility) cost of the choice and the right-hand side the expected discounted (future) marginal benefit. Consider ($17c$). The term in curly braces is the gross one-period (marginal) return to liquidity and hence the right-hand side is the expected discounted (future) marginal benefit.

Equation ($17d$) after imposition of symmetry across project simplifies to

[10] The details of derivations of first-order conditions are available from authors upon request.

$$U_{C_t} = \beta E_{t,\theta}\left[U_{C_{t+1}}\left\{\frac{p_H \hat{R}_{t+1}(\theta_{t+1}) F\left(\rho^*_{t+1}\right)}{p_t}\right\}\left\{1 - \frac{\bar{m}_{t+1}\left(\rho^*_{t+1}\right)}{m_{t+1}\left(\rho^*_{t+1}\right)}\right\}\right]. \tag{17d'}$$

The term in the first curly braces is the gross return on share in absence of liquidity shock in the second period. The term in the second curly braces captures the reduction in gross return caused by the need for second-period liquidity financing. This term is also quite intuitive. For example, consider the case where average liquidity need, $\bar{m}_{t+1}\left(\rho^*_{t+1}\right)$, is zero, in that case, gross return from shares in unaffected. Overall, $(17d)$ determines the price of shares of the project based on the household's preferences and the projects' characteristics.

Finally, $(17e)$, the first-order condition for ρ^*_t on simplification yields

$$\rho^*_t\left(\theta_t\right) = \frac{1}{1 + \frac{\lambda_t}{U_{C_t}}} \frac{p_H s^k_{t-1} \hat{R}^k_t\left(\theta_t\right)}{w_t}. \tag{21}$$

This condition on financing the liquidity need is very intuitive. When liquidity is in abundant supply, λ_t is zero and we have

$$\rho^*_t\left(\theta_t\right) w_t = p_H s^k_{t-1} \hat{R}^k_t\left(\theta_t\right), \tag{22}$$

where the left side is the liquidity need of the marginal firm and the right side is the expected revenue accruing to the investor, conditional on the liquidity need being financed. The liquidity need of a project will be financed up to this amount because the past investment decision is not relevant for liquidity financing. In addition, since the investor is diversified over a large number of identical projects, he is risk-neutral with respect to any single project. When liquidity is limited, λ_t is positive and the amount of liquidity supplied to firms is reduced accordingly as shown by (21).

3.2 Solution to Entrepreneur's Problem

Begin by noting that the incentive-compatibility constraint (16) holds in every aggregate state, and if it binds for a realization of $\theta_{t+1} = \bar{\theta}$, then it will also bind for all $\theta_{t+1} < \bar{\theta}$. We assume that shirking is extremely costly (p_L is very low) so that it is never optimal for the investor to let the entrepreneur shirk. This implies that the relevant incentive-compatibility constraint is the one corresponding to a value, θ_{Lt}, which denotes the lowest possible draw for θ_{t+1}, to be specified in section 4.1, given current θ_t. The incentive-compatibility constraint can therefore be written as

$$p_H \left(1 - s_t^i\right) \hat{R}_{t+1}^i \left(\theta_{Lt}\right) \geq p_L \left(1 - s_t^i\right) \hat{R}_{t+1}^i \left(\theta_{Lt}\right) + J s_t^i, \tag{16^L}$$

Using (3) and (21) the entrepreneur's problem, (15), can be written as

$$\max_{s_t^i, n_{1t}^i} (1 - s_t^i) \left(n_{1,t}^i\right)^\alpha p_H E_{t,\theta} \left[\beta \frac{U_{C_{t+1}}}{U_{C_t}} \theta_{t+1} F \left(\frac{1}{1 + \frac{\lambda_{t+1}}{U_{C_{t+1}}}} \frac{p_H s_t^i \hat{R}_{t+1}^i \left(\theta_{t+1}\right)}{w_{t+1}} \right) \right], \tag{$15'$}$$

which is solved subject to

$$s_t^i p_t = w_t n_{1,t}^i, \tag{3}$$

and $\left(16^L\right)$. Note that the entrepreneur takes into account the fact that his choices of project size, n_t^i, and outside equity, s_t^i, affect ρ_t^*, i.e., the likelihood of second-period financing of the liquidity need by the investor. In particular, a larger project (larger n_{1t}) tends to relax (16) such that an a larger maximum liquidity need (higher ρ_{t+1}^{i*}) will be financed. There are two potential variations on the solution to this problem depending on whether $\left(16^L\right)$ binds.

In what follows, we assume that the liquidity shock is uniformly distributed over $[0, \bar{\rho}]$ so that

$$F(\rho) = \frac{\rho}{\bar{\rho}}, \qquad 0 \leq \rho \leq \bar{\rho}. \tag{23}$$

The entrepreneur's problem is solved for this distribution of the liquidity shock. To this end, first maximize ($15'$) assuming that the entrepreneur's incentive-compatibility constraint does not bind. In that case, the solution to ($15'$) yields

$$s_t^i = \frac{1 + 2\alpha}{2(1 + \alpha)}, \tag{24}$$

and then (3) gives the value of $n_{1,t}^i$. Note that, while the value of s_t^i is independent of t in this case, $n_{1,t}^i$ and the project's output are nonetheless dependent on t.

Having solved for $n_{1,t}^i$, now check if $\left(16^L\right)$ holds. For this to be the case, $n_{1,t}^i$ must be greater than the threshold value

$$\tilde{n}_{1,t}^i = \left[\frac{(1+2\alpha)J}{\theta_{Lt}\left(p_H - p_L\right)}\right]^{\frac{1}{\alpha}}. \tag{25}$$

If $n_{1,t}^i < \tilde{n}_{1,t}^i$, the incentive-compatibility constraint binds and one needs to solve (3) and $\left(16^L\right)$ jointly for $n_{1,t}^i$ and s_t^i (with the latter holding with equality) and there is no further maximization involved; there is only one feasible choice.

As mentioned earlier, having solved for the optimal values of s_t^i and n_t^i, (17d) determines the equilibrium price of the shares of the projects, once equilibrium is imposed, to which we turn next.

3.3 Imposing the Equilibrium

In equilibrium, the only goods that are produced are from projects that received a sufficiently low liquidity shock, i.e., for the projects of each household $\rho_t^i \leq \rho_t^*$. As all projects are *ex ante* identical and as all goods enter symmetrically in the utility function, then for each household $\rho_t^i \leq \rho_t^*$, and equilibrium conditions become:

$$s_t^i = s_t \tag{26a}$$

$$y_t^i = y_t = \theta_t (n_{1,t-1})^\alpha \tag{26b}$$

$$q_t^i = q_t = Q_t = 1 \tag{26c}$$

$$\hat{R}_t^i = \hat{R}_t = y_t \tag{26d}$$

with labor market equilibrium given by

$$n_{1,t} + \bar{n}_{2,t}\left(\rho_t^*\right) F\left(\rho_t^*\right) = n_t \tag{27}$$

where

$$\bar{n}_{2,t}\left(\rho_t^*\right) = \int_0^{\rho_t^*} \rho \frac{f\left(\rho\right)}{F\left(\rho_t^*\right)} d\rho \tag{28}$$

is the average additional labor requirement, conditional on the liquidity need being financed. Furthermore, the household's time constraint must be satisfied

$$n_t + L_t = 1. \tag{29}$$

The clearing of the market for the aggregate good requires

$$C_t + M_{t+1} - M_t = y_t\left(\theta_t\right) p_H F\left(\rho_t^*\right) = \theta_t\left(n_{1,t-1}\right)^{\alpha} p_H F\left(\rho_t^*\right) \tag{30}$$

The equilibrium demand for liquidity cannot exceed the supply so that

$$\int_0^{\rho_t^*} s_{t-1} m_t(\rho) f\left(\rho\right) d\rho \leq M_t \tag{31}$$

Finally, in equilibrium net supply of bonds is zero, or

$$B_t = 0, \qquad \forall t. \tag{32}$$

The equations for $(17a - 17e)$, $(26b - 26d)$, (27), and $(29 - 32)$ contain the following endogenous variables: s_t, p_t, y_t, $n_{1,t}$, ρ_t^*, q_t, $\hat{R}_t$, w_t, L_t, n_t, C_t, M_{t+1}, R_t, λ_t, and B_{t+1}. So, we have 15 variables and 13 equations. Depending on which situation applies to the entrepreneur's optimization problem, either (3) and (24) or (3) and (16^L) will be added to the set of equations to have a complete system with 15 equations in 15 unknowns. In the case where (24) is used, it must be checked that the computed solution for n_1 is greater than $\tilde{n}_{1,t}$ in (25).

4 Calibrating the Model

We begin by specifying the functional forms and distributional assumptions followed by the calibration of the model to the data.

4.1 Functional Forms etc.

We posit the utility function of the following form:

$$U(C, L) = \ln C + \eta \ln L. \tag{33}$$

In addition, we assume the aggregate productivity shock follows an autoregressive process

$$\ln \theta_t = \psi_\theta \ln \theta_{t-1} + \varepsilon_t, \tag{34}$$

with serial correlation ρ_θ where the innovation to aggregate productivity, ε_t, is assumed to be normally distributed with mean zero and a standard deviation of σ, but truncated at some lower bound, $\varepsilon_t \geq \varepsilon_L$. Hence, in the non-stochastic steady state $\theta_{ss} = 1$. The truncation of ε is necessary under a continuous distribution in order to prevent shirking in any aggregate state. The specification enables us to define

$$\theta_{Lt} = \theta_t^{\psi_\theta} \exp(\varepsilon_L), \tag{35}$$

the lowest draw possible for next period's productivity, θ_{t+1}, given current productivity. With serially correlated θ, θ_{Lt} becomes time dependent.

Given the distribution of the liquidity shock in (23), from (28), (5), and (20) we have

$$\bar{n}_{2,ss}(\rho^*_{ss}) = \frac{\rho^*_{ss}}{2} \tag{28$'$}$$

$$\bar{m}(\rho^*_{ss}) = \frac{1}{2}\frac{w_{ss}\rho^*}{s_{ss}} = \frac{1}{2}m(\rho^*_{ss}) \tag{20$'$}$$

Using the functional form of the utility function, the optimality conditions $(17a - 17e)$ can be simplified as follows:

$$\frac{w_{ss}}{C_{ss}} = \frac{\eta}{L_{ss}} \tag{17a$'$}$$

$$1 = \beta R_{ss} \tag{17b$'$}$$

$$1 = \beta \frac{p_H \hat{R}_{ss}}{m(\rho^*_{ss})} = \beta \frac{s_{ss} p_H \hat{R}_{ss}}{\rho^*_{ss} w_{ss}} \tag{17c'}$$

$$p_{ss} = \beta p_H \hat{R}_{ss} F(\rho^*_{ss}) \left[1 - \frac{\bar{m}(\rho^*_{ss})}{m(\rho^*)}\right] = \frac{\beta}{2} p_H \hat{R}_{ss} F(\rho^*_{ss}) \tag{17d'}$$

$$\lambda_{ss} = \frac{1}{C_{ss}} \left[\frac{p_H \hat{R}_{ss}}{m(\rho^*)} - 1\right] \tag{17e'}$$

4.2 Calibration

Using $(17c' - 17d')$ and (3), one can solve for

$$n_{1,ss} = \frac{(\rho^*_{ss})^2}{2\bar{\rho}} = \frac{n_{ss}}{2}, \tag{33}$$

where the last equality follows from (27) and (28′).

We calibrate the model so that in the non-stochastic steady state $n_{ss} = .36$, which is in line with results from survey data discussed in Juster and Stafford (1991). For an annual calibration, we set β to the usual value of .96. The cost share of labor in production, α is set to 1/3, about half the value commonly used in the RBC literature. We note that on average, only half of total labor hours is devoted to new projects; the other half goes to finalize projects initiated in the previous period. Finally, the innovation in aggregate productivity σ is set at .02, and the lower bound for ε at $-.03$. The aggregate shock has serial correlation $\rho_\theta = .80$, a value widely assumed in annually calibrated RBC models, broadly equivalent to the quarterly value of 0.95 [see e.g Kydland and Prescott (1982)].

To ensure that it is never optimal to let the entrepreneur shirk, p_H is given a relatively high value of .9 and p_L is set to a low value of .4. The liquidity shock distribution parameter $\bar{\rho}$ is set to .5 so that second-period liquidity needs of approximately 85 per cent of the projects are financed.

With nonbinding IC, we have $s_{ss} = \frac{1+2\alpha}{2(1+\alpha)} = \frac{5}{8}$. The calibrated steady state is shown in Table 1. The steady state is independent of the value of J as long as J is less than the threshold value $J^* = .1660$ that solves (25) for $\tilde{n}_1 = n_{1,ss}$.

5 Results

The results from simulating three model versions are summarized in Table 2. The first specification is based on a lognormally, iid θ, the aggregate productivity shock. The other two assume an autoregressive θ, one with the incentive compatibility (IC) constraint never binding, the other with an occasionally binding IC constraint. The difference between the two lies in the values attached to J, in (11), the parameter capturing the gain from shirking. In the former case, J is set low enough for (11^L) never to bind; in the latter, it is set somewhat below its steady state 'threshold' value of 0.166, resulting in a tendency for the IC to bind during periods of low aggregate productivity. The reported standard deviations are all in percentage terms.[11]

5.1 IID Shocks

The statistics in the first two columns in Table 2 refer to the most basic model version, a model with an iid productivity shock and a non-binding IC constraint. In order to obtain a comparable volatility of θ across all model versions, σ and ε_L are adjusted accordingly in the basic version, to 0.033 and -0.1 respectively. Several observations are noteworthy here. First, consumption exhibits just about as much volatility as output, the two being near perfectly correlated. Hence, very limited consumption smoothing takes place in the model. The reason is the absense of any asset (such as capital) that would serve to smooth out consumption across time in a significant manner. The stock of liquidity, M, only amounts to around 30% of C in the nonstochastic steady state, limiting its stabilizing role. Second, total employment, n, as well as employment engaged in new projects, n_1, are both smoother than output, which is in line with U.S. data. However, while the latter is procyclical, the former is notably countercyclical, clearly at variance with data. With the limited smoothing possibility in consumption, the households will require a large enough increase in the

[11] The model was solved by parameterizing the expectations in the Euler equations (19b) - (19d), a method proposed by Marcet (1988), and extended to include occasionally binding constraints by Christiano and Fisher (1994).

real wage in order to encourage work effort in the face of a positive productivity shock. In this model specification, the increase in w simply falls short of providing a strong enough substitution effect. Third, the current cut-off value of the liquidity shock, ρ^*, is negatively correlated with current output, whereas next period's ρ^* shows a positive correlation. The negative contemporaneous correlation owes to the fact that a positive productivity shock raises the real wage, the unit cost of ongoing projects. With a given amount of liquidity, M, on hand, less funds are available for any single project. Fourth, we see a near perfectly negative correlation between the risk free gross real interest rate, R, and a positive correlation between share prices, p, and output. The former follows from the very limited smoothing opportunities in the model: about the only channel for households to absorb a higher output level is to increase consumption, which demands a falling interest rate.

5.2 Serially Correlated Shocks

The next pair of statistics in Table 2 is based on a serially correlated productivity shock, setting the autocorrelation coefficient at 0.8. The entrepreneurs never experience a binding IC constraint at any time. Time series plots for the endogenous prices and quantities are also shown in Figure 2. The innovation to the productivity shock is adjusted accordingly, keeping the standard deviation of θ intact across these two model versions. Autocorrelated aggregate productivity generally increases the volatility in the economy. Output, consumption, the real wage, and share prices all show increased volatility. Two factors are at work here. First, the negative correlation between ρ^* and current output has loosened somewhat, with the coefficient reduced from -0.76 to -0.38. From the goods market equilibrium in (30), this should increase output for any given θ. Second, employment in projects initiated in the last period, $n_{1,t-1}$ is now more closely correlated with current output, where the coefficient has risen from -0.01 to 0.14. In anticipation of θ remaining high after a positive productivity shock, the households now respond by setting aside more liquidity in order to meet increased demand for second period financing of projects beginning in the current period. The cut-off value, ρ^*, is still negatively correlated with current output, but the correlation is significantly reduced (in absolute value). The reason is that a high current productivity is now likely to go together with a relatively high level of beginning-of-period liquidity, neutralizing to some extent the countercyclicality of ρ^*. Again, consumption smoothing is minimal, and for the same reason: the lack of any asset that could serve as a means to absorb idiosyncratic or aggregate shocks. This also explains the prevalent, albeit reduced, countercyclicality in total employment, n. Increased volatility in the real wage has a mitigating effect, although not big enough to turn n procyclical. Perhaps counterintuitively, the correlation between ρ^*_{t+1} and current output is reduced, from 0.45 to 0.13. With autoregressive θ, increased current productivity should, other things equal, raise the expected next-period cut-off point. However, this is complicated by the increasingly volatile and highly procyclical real wage, w. From (21) it is clear that a higher w_{t+1} takes ρ^*_{t+1} in the other direction.

Finally, the results from forcing the IC to bind occasionally for the entrepreneurs are reported in the last two columns of Table 2, together with time series plots in Figure 3. The productivity shock remains autoregressive. Overall, the occasionally binding IC further increases the volatility in output, consumption, real wages, real liquidity, and share prices. Notable is the vast increase in the standard deviation of n_1, from 0.29 to 0.53 percent. Further, the number of issued shares, s, is no longer constant, now being determined either by (24) (if the IC is slack), or by (16^L) (if it is binding.) The tightness of the liquidity constraint, as measured by the term λ_t/U_{Ct} in (17)e is highly volatile, ranging from close to zero to about 0.09 in the sample shown in Figure 2. It has a procyclical tendency, albeit a weak one, with a correlation coefficient of 0.33. Two counteracting factors are present here. First, in periods of high expected aggregate productivity, the demand for new projects increases, tightening the constraint. Second, the provision of liquidity increases, easing the constraint.

In cases where the IC binds during periods of adverse productivity shocks, the economy is further dragged down from what would otherwise be the case, as can be seen from the impulse response functions in Figure 3. Notice how an adverse current productivity shock lowers the amount of the liquidity (M) set aside for future second-period financing of projects. This is also reflected in the negative response of the expected cut-off value of ρ^*_{t+1}. As in the other specifications, very limited consumption smoothing is at work. However, the real wage is now sufficiently volatile to make total employment, n, procyclical, changing the correlation coefficient from -0.49 to 0.11, bringing it a good deal closer to U.S. data. In other words, the real wage volatility is now strong enough to generate a dominating substitution effect in the labor-leisure margin, (17a). Finally, the impulse response function for the tightness of the liquidity constraint, λ_t/U_{Ct}, displays a clear easing on impact during a severe downturn. However, the constraint quickly tightens as recovery is underway.

6 Conclusions

This paper examines the importance that occasionally binding incentive compatibility constraints in financial contracts have in affecting macroeconomic performance over the business cycle. The principal result is that sufficiently strong adverse aggregate (productivity) shocks cause these constraints to bind, in which case considerable volatility is added to the economy and the dynamic behavior of the economy changes. Of particular significance is the amplification of economic downturns as a result of a drying up of liquidity. Not only does the adverse aggregate shock reduce funding of new investment projects as new equity issues decline when the incentive constraint binds and credit is rationed, but also the shock reduces the willingness of shareholders in these firms to provide any additional future funding that may be needed for completion of ongoing projects, many of which would otherwise have received funding. Those projects are terminated due to lack of liquidity in the economy, further reducing employment and output from what otherwise would have been the case for the same aggregate shock had the incentive constraints not been binding and credit not been rationed.

This model is highly stylized in order to focus cleanly on a key feature of major economic downturns: the significant contraction in aggregate liquidity. For example, bank lending standards tighten and commercial paper issuance can all but dry up during recessions. For us, it is important to distinguish between savings channeled into new investments versus liquid low-yielding (in our model, non-interest bearing) funds. The former is used to exploit new investment opportunities, while the latter is used to continue ongoing investments, which would otherwise be terminated. An aggregate shortage of liquid funds so-defined is what we take to be the basis of liquidity crises, and normally occurs only during severe economic downturns. This phenomenon is what our model is intended to capture. Determining the quantitative significance of this channel requires incorporating this financial and production structure in a more elaborate model.

There are a number of extensions of this basic model that we believe would be useful. We list three: the introduction of financial intermediaries explicitly to examine such issues as the role of bank capital in cushioning liquidity crises; endogeneity of the "private benefit" on which the key incentive constraints in the contract is built and how protective covenants may affect the frequency with which the incentive constraint binds; and the role of the government provision of liquidity, and when, how much, and in what market should their intervention take place to mitigate the macroeconomic consequences of a severe economic downturn.

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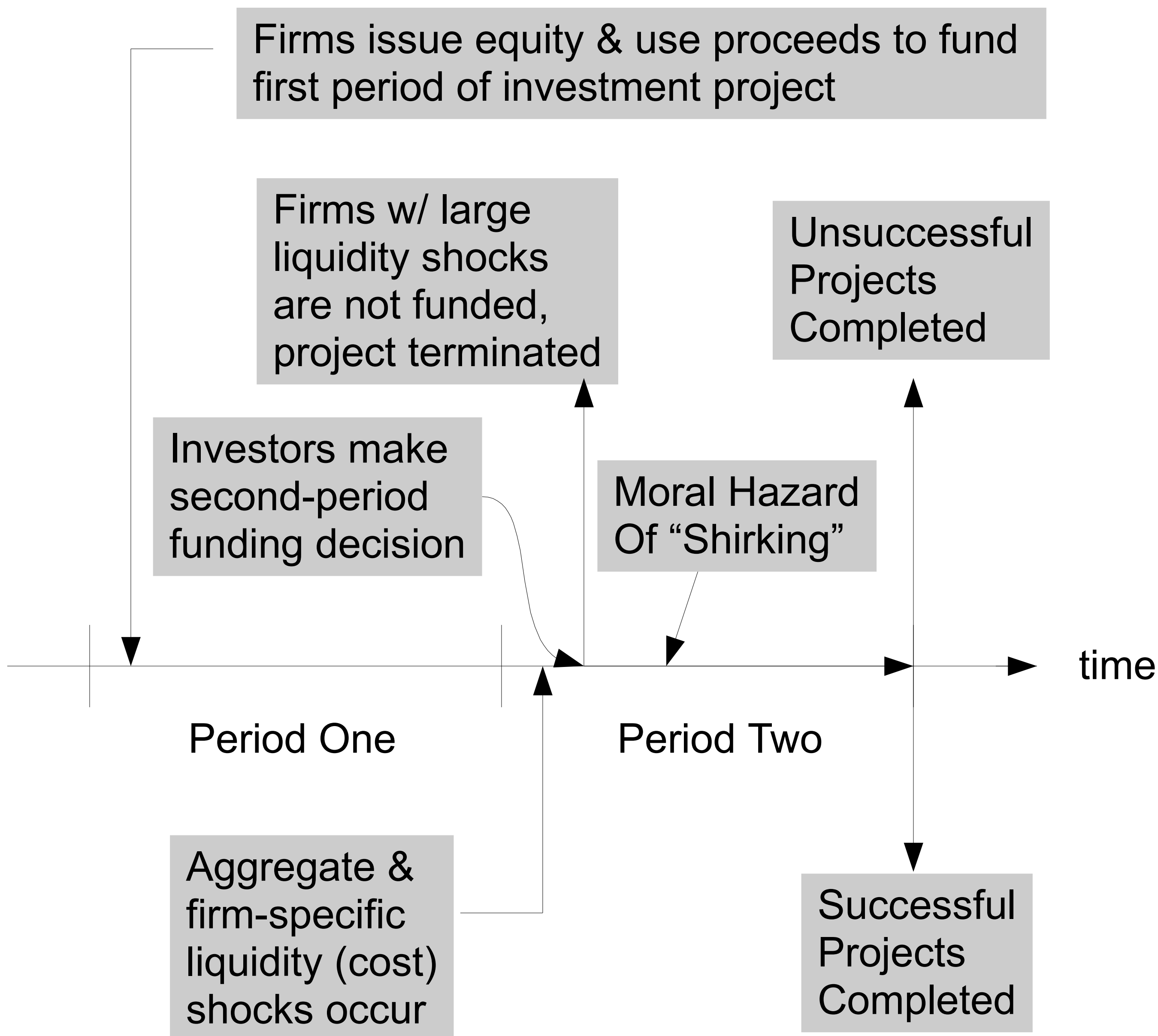

Figure 1: Timing of Projects

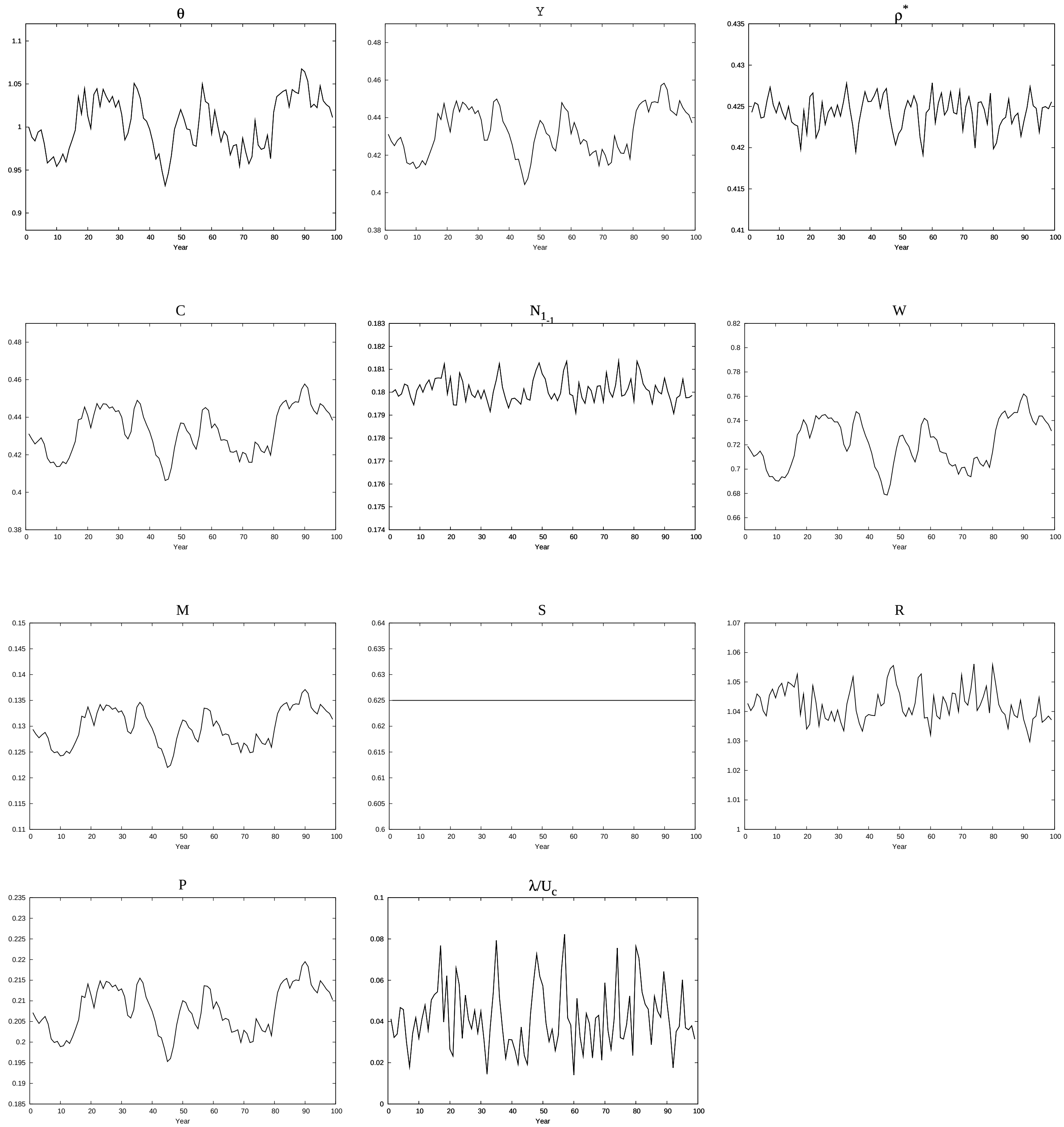

Figure 2: Autoregressive θ, nonbinding IC.

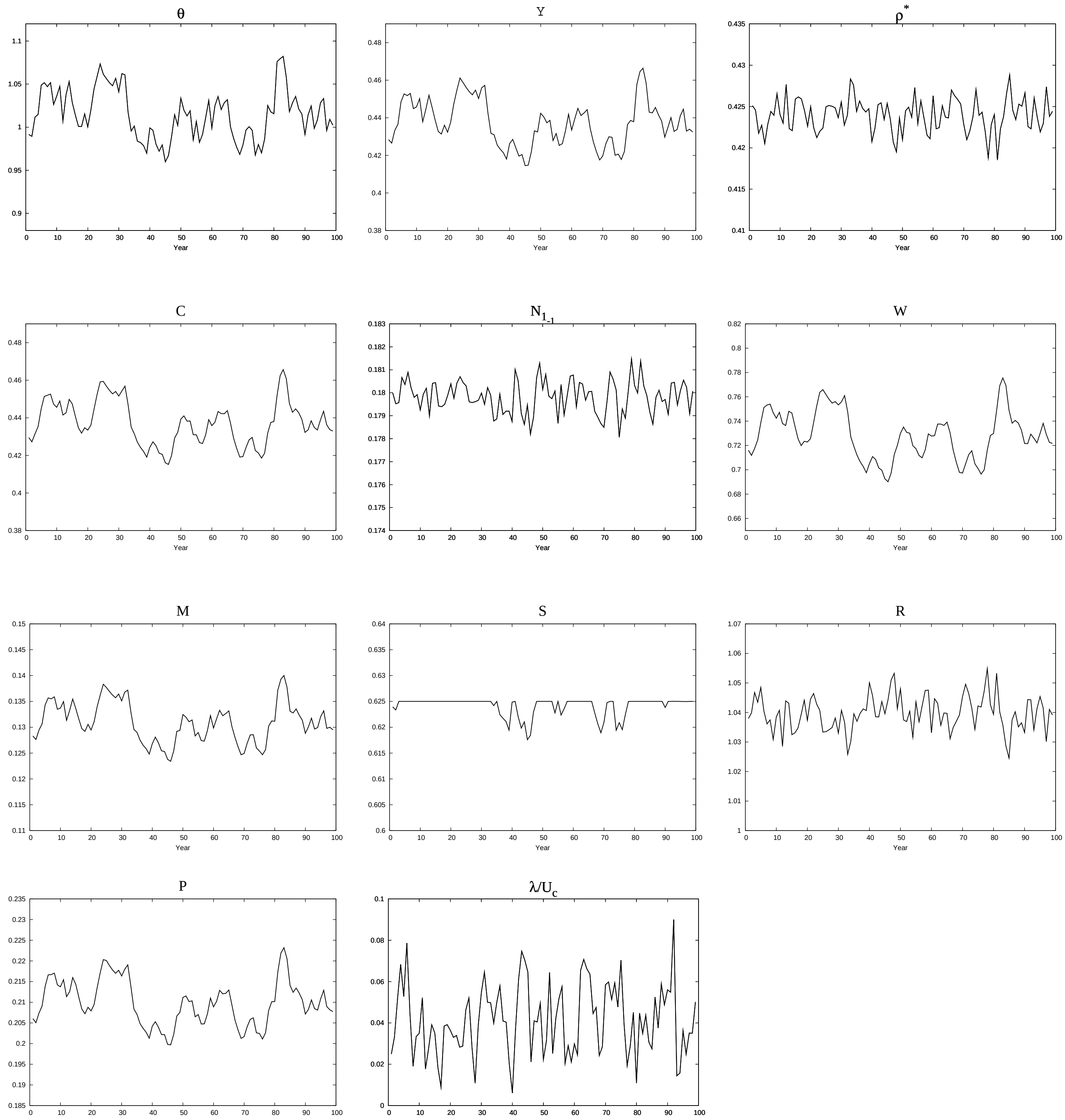

Figure 3: Autoregressive θ, occasionally binding IC.

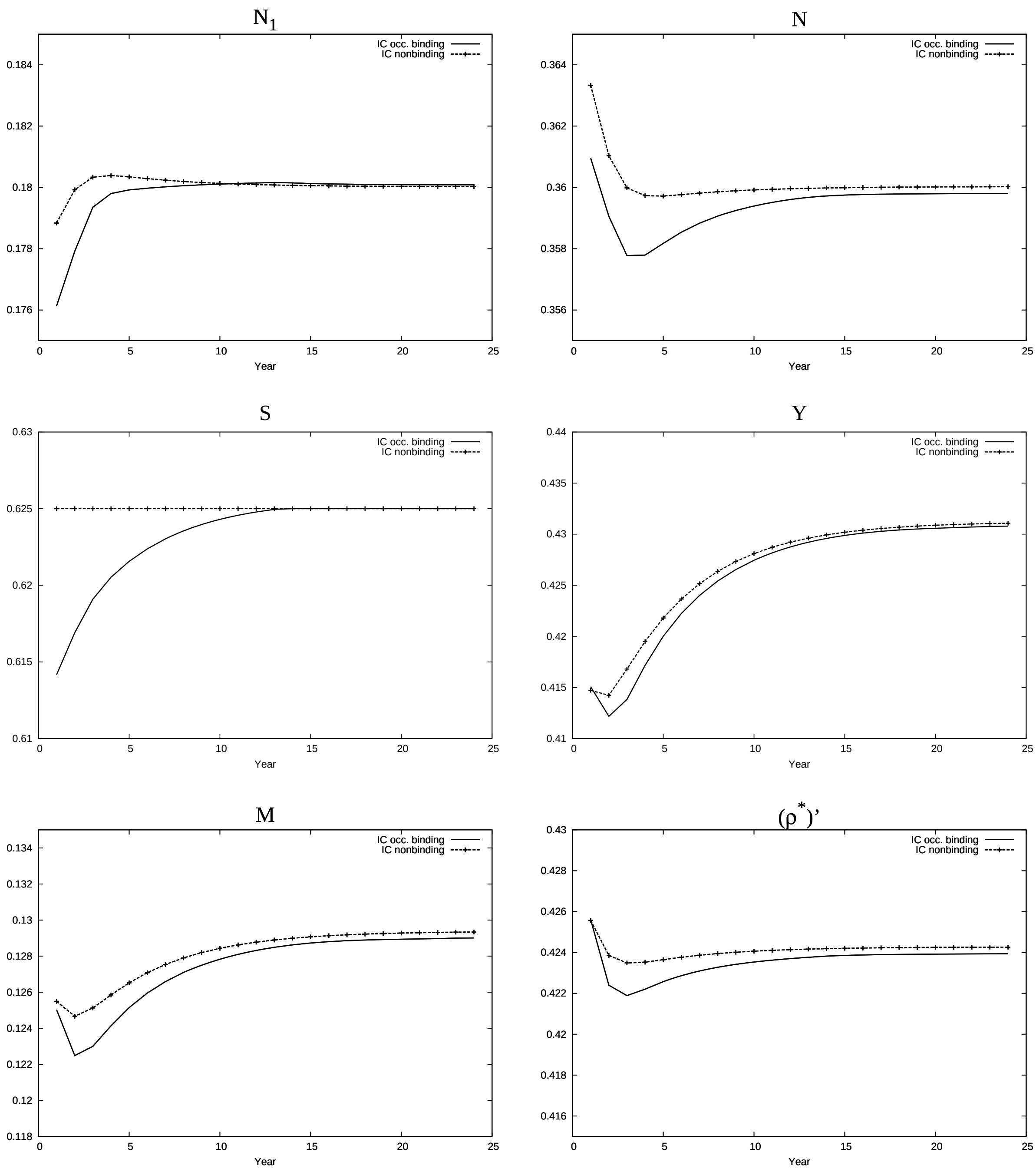

Figure 4: Impulse response functions. An ε_L shock in period 1 given $\theta_1 = 0.98$.

Table 1

Parameter values and steady state for the calibrated model with IC not binding

Preference Parameters

$\beta = 0.96, \;\; \eta = 1.0667$

Production Parameters

$\alpha = 1/3$

$\psi_\theta = 0.8, \;\; \sigma = 0.02, \;\; \varepsilon_L = -0.03$

$p_H = 0.9, \;\; p_L = 0.4$

$\bar{\rho} = 0.50$

Calibrated Steady State

$n = 0.36, \;\; n_1 = 0.18, \;\; n_1/n = 0.50$

$\rho^* = 0.4243, \;\; \rho^*/\bar{\rho} = 0.8485$

$C = 0.4312, \;\; M = 0.1294, \;\; M/C = 0.30$

$y = 0.5646; \;\; Y = 0.4312$

$p = 0.2070, \;\; s = 0.6250$

$w = 0.7186, \;\; R = 1.0417$

Table 2

Summary of Second Moments

	IID θ		Autoregressive θ			
	.		Nonbinding IC		Occasionally Binding IC	
Variable	stdev	corr w/y	stdev	corr w/y	stdev	corr w/y
y	2.49	1.00	2.91	1.00	3.08	1.00
c	2.12	0.98	2.81	0.99	2.98	0.99
θ	3.20	0.98	3.12	0.99	3.12	0.98
ρ^*	0.86	-0.76	0.49	-0.38	0.54	-0.17
ρ^*_{+1}	0.86	0.45	0.49	0.13	0.54	0.33
n_1	0.29	0.15	0.29	0.08	0.53	0.50
$n_{1,-1}$	0.29	-0.01	0.29	0.14	0.53	0.54
n	0.76	-0.84	0.36	-0.49	0.35	0.11
w	1.84	0.94	2.74	0.98	3.03	0.98
m	1.80	0.98	2.73	0.99	3.24	0.99
s	0.00	-	0.00	-	0.49	0.79
R	1.26	-0.98	0.50	-0.41	0.71	-0.26
p	1.80	0.98	2.73	0.99	2.86	0.99