Contests with Network Externalities: Theory & Evidence

Luke A. Boosey* Christopher L. Brown†

Abstract

This paper considers the impact of identity-dependent externalities on competitive behavior in all-pay contests. We introduce a model of *network contest games*, in which the prize generates externalities for players directly linked to the winner, and establish existence and sufficient conditions for uniqueness of Nash equilibria. Both the structure of the network and nature of the externalities have intuitive consequences for equilibrium investment. In general, positive externalities introduce free-riding incentives, whereas negative externalities intensify competition, especially among highly connected agents. Results from a laboratory experiment provide robust empirical support for the comparative static predictions of the model.

Keywords: contests, networks, identity-dependent externalities, network games, best-response potential, experiment

JEL: C72, C92, D72, D74, D85, Z13

1 Introduction

In virtually all areas of social and economic interaction, one can find examples of agents competing with each other in pursuit of some valuable prize. Individuals and organizations frequently expend significant resources on marketing, advertising, and lobbying in order to outperform their rivals or command a greater influence over market allocations or political outcomes. Research in industrial economics, public choice, and political economy has explored competitive behavior in rent-seeking environments, R&D competition, patent races, political campaigns,

*Department of Economics, Florida State University, 113 Collegiate Loop, Tallahassee, FL 32306-2180, USA; E-mail: `lboosey@fsu.edu`

†Krannert School of Management, Purdue University, 403 W State St, West Lafayette, IN 47907-2056, USA; E-mail: `cbrown.econ@gmail.com`.

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and promotion tournaments. Many of these settings are modeled as *contests*, in which agents exert costly effort or make irreversible investments and the winner takes all.

The standard assumption in contest models is that losing agents are indifferent to the identity of the winner. However, agents may have considerably more general preferences over the possible allocations of the prize. In particular, agents who do not win the contest may care a great deal about who does, especially if the allocation of the prize affects the nature of subsequent interactions between the contestants. In the related context of winner-pay auctions, Jehiel, Moldovanu and Stacchetti (1996) introduced the notion of *identity-dependent externalities* (or IDEs) as a way of capturing the consequences of the allocation for bidders in post-auction interactions. Such externalities may arise in relation to the assignment of exclusive licensing agreements (Brocas, 2003), the sale of a nuclear weapon or location of environmentally hazardous enterprises (Jehiel, Moldovanu and Stacchetti, 1996), competition for access to a cost-reducing process innovation, or the allocation of talent across teams (Das Varma, 2002).

There are relatively few studies that consider the implications of IDEs for *all-pay contests* (see, e.g., Linster, 1993; Esteban and Ray, 1999; Konrad, 2006; Klose and Kovenock, 2015); and yet, there remain many interesting questions to explore. For instance, in many settings the structure of IDEs is governed by an underlying network of connections. As such, there are naturally arising questions regarding the impact of network structure on competitive behavior which, to date, have not been addressed by the existing literature on IDEs in auctions and contests.

In this paper, we study the effects of network-based identity-dependent externalities on competitive behavior in all-pay contest environments. To do so, we develop and analyze a theoretical model of a *network contest game*. Our framework builds on recent developments to the understanding of strategic behavior in games played on networks (Bramoullé, Kranton and D'Amours, 2014). We concentrate on Tullock (1980) contests—one of the most commonly studied formulations of *imperfectly-discriminating* all-pay contests—wherein each player's probability of winning the contest is increasing in her own effort investment, relative to the investments of others. The primary innovation of our model is the introduction of a network that governs the flow of externalities from the winning player to her neighbors.

As a motivating example, consider a collection of community councils lobbying a city planning committee in charge of selecting the location for a new public facility. Each community's ideal outcome would be to have the facility located within their own neighborhood. However, if the facility generates positive externalities or is more easily accessible to neighborhoods that are sufficiently close to the eventual location, it is natural to expect that lobbying activity will depend on the geographical network connecting the communities. If the externalities are sufficiently strong, or the communities sufficiently well-connected, they may engage in less lobbying activity than if it is more difficult to access a facility located outside their own neighborhood.

Along similar lines, the investment decisions made by firms competing for an exclusive licensing agreement will typically depend on the rivalry structure in the firms' product market space. Firms who operate in close proximity to the winning firm may be significantly worse off than other unsuccessful firms.[1] How might the structure of product market rivalries affect rent-seeking behavior in this setting? The natural intuition in this case suggests that the negative externalities associated with the exclusive license will intensify competition among firms who are engaged in markets with more heated rivalry.

Our main contributions in this paper are theoretical. We start by establishing the existence of a Nash equilibrium for general network structures and externalities (Theorem 1). The main challenge to existence is the fact that payoff functions in the network contest game are (like the standard contest environment) discontinuous at zero. We rely on results from Reny (1999) and Bagh and Jofre (2006) to prove existence. In addition, we provide closed-form characterizations of equilibria for two broad classes of network structures: *regular* networks and (a subclass of) *core-periphery* networks, to highlight key characteristics of the relationship between externalities, network properties, and equilibrium behavior.

For regular networks, there exists a symmetric equilibrium in any network contest game. Moreover, comparative statics with respect to the size of the externality and the density of the network are consistent with the intuition highlighted by the motivating examples given above. For instance, positive externalities introduce incentives for players to free ride on their neighbors' investments, leading

[1] Another similar example can be made in the context of professional sporting organizations competing for the services of a talented free-agent athlete. For instance, in Major League Baseball, the Boston Red Sox (part of the American League East Division) might be much happier to see a top free agent player sign a deal with the San Diego Padres (who are in the National League West Division) than with the New York Yankees, who play in the same League and Division as Boston. There are, of course, several other considerations that influence the negotiations between sporting teams and free agent athletes, including salary demands, team budgets, contract length, synergies with existing team members, and the athlete's locational preferences. Nevertheless, the point is that competition in these kinds of settings, which may include both winner-pay and all-pay components, is likely influenced by the anticipated interest and activity of rival teams.

to lower equilibrium investment. Conversely, negative externalities drive up the effective value of winning the contest, intensifying competition and increasing equilibrium investment. Each of these effects is amplified as the network becomes more densely connected, as captured by an increase in the common *degree* for regular networks. Nevertheless, the symmetric equilibrium in regular networks is typically not unique. For instance, when externalities are positive and sufficiently strong, there may also exist a *specialized equilibrium*, in which some subset of the players choose to be inactive (invest nothing) in the contest.

Similarly, semi-symmetric equilibria in a subclass of core-periphery networks also take the form of a specialized equilibrium for sufficiently strong, positive externalities. In particular, highly connected core players, facing stronger free-riding incentives than peripheral players, invest nothing in equilibrium. In contrast, when the prize allocation generates strong negative externalities, the core players—who are more exposed by the structure of the network—increase their equilibrium investment substantially compared to the peripheral players.

We then provide sufficient conditions for there to be a *unique* Nash equilibrium (Theorem 2). Our characterization closely follows the seminal approach developed by Bramoullé, Kranton and D'Amours (2014) for network games with linear best replies. However, adapting their results to the network contest game is a non-trivial exercise. In particular, because best replies are non-linear in the network contest game (just as they are for standard Tullock (1980) contests), the main results derived by Bramoullé, Kranton and D'Amours (2014) cannot be directly applied.[2] Nevertheless, we demonstrate that the key insights provided by Bramoullé, Kranton and D'Amours (2014) can be suitably adapted to the network contest game.

One key condition of our uniqueness theorem relates the size of the externalities in the network contest game to the lowest eigenvalue of the network, which also plays a crucial role in Bramoullé, Kranton and D'Amours (2014).[3] While Bramoullé, Kranton and D'Amours (2014) exploit the theory of potential games (Monderer and Shapley, 1996) to derive their results, our formulation does not admit an exact potential function. Instead, we establish that the network contest game is a *best-response potential game* (Voorneveld, 2000), which allows us

[2]Moreover, approaches based on variational inequalities (VI) that have been applied to network games without linear best replies (see, e.g., Melo, 2018; Parise and Ozdaglar, 2019; Zenou and Zhou, 2020) also do not apply.

[3]As discussed by Bramoullé, Kranton and D'Amours (2014), the lowest eigenvalue captures the "two-sidedness" or "bipartiteness" of the graph. When the lowest eigenvalue (which is negative) is sufficiently large in magnitude, the amplification of agents' interactions increases the chances of multiple equilibria.

to take an analogous approach. Altogether, our theoretical framework establishes new results extending both the well-developed literature on contest theory and the growing body of work studying strategic behavior in network games.

Finally, we test the main predictions of the model in a controlled laboratory experiment. In our experiment, subjects are placed into groups of six and assigned to positions in one of four network configurations—the complete network, a circle network, a star network, and a core-periphery network with two core players. We implement three different conditions that vary the size and sign of the externality: a strong negative externality, a strong positive externality (of the same magnitude as in the negative condition), and a baseline control in which the network structure is retained, but the externalities are set equal to zero. Overall, our main experimental findings provide strong support for the theoretical predictions; at the aggregate level, the comparative static predictions across treatments are well supported by the observed patterns of mean investment.

The remainder of the paper is organized as follows. In Section 2, we introduce the theoretical model of a network contest game. Section 3 presents the equilibrium analysis, including our main results on existence and uniqueness of Nash equilibria in the network contest game. Specific results for the class of regular networks and a class of *core-to-periphery* structures are also provided, with several examples, in this section of the paper. Section 4 describes the design of our experiment and presents the main experimental findings. We discuss related prior literature in Section 5 and provide brief concluding remarks in Section 6.

2 The Network Contest Game

Consider an environment with a set of individuals $N = \{1, \ldots, n\}$ arranged in a network, described by the adjacency matrix $\mathbf{G}$, where $g_{ij} = g_{ji} = 1$ if distinct agents i and j are linked, and $g_{ij} = g_{ji} = 0$ otherwise. We follow the convention that $g_{ii} = 0$ for all $i \in N$. Each individual competes in a contest by choosing a level of investment (or effort) $x_i \geqslant 0$. All players have the same linear cost of effort function, $c(x_i) = x_i$. Let $\mathbf{x}_{-i}$ denote the vector of investments chosen by all individuals other than i and suppose the probability of player i winning the contest is given by the Tullock (1980) lottery contest success function. That is,

$$P_i(x_i, \mathbf{x}_{-i}) = \begin{cases} \frac{1}{n}, & \text{if } \sum_{h=1}^{n} x_h = 0, \\ \frac{x_i}{\sum_{h=1}^{n} x_h}, & \text{otherwise.} \end{cases} \qquad [1]$$

The winner of the contest receives a prize $V > 0$. We assume, without loss of generality, that the value of the prize is normalized to $V = 1$. In the standard contest setting, player i's payoff from winning is $V = 1$, while the payoff from losing is zero, regardless of who among the other players wins the contest. In such a setting, it is a well-known result (see, e.g., Szidarovszky and Okuguchi, 1997) that the unique equilibrium is symmetric, given by $x_i = \bar{x}$ for all $i = 1, \ldots, n$, where

$$\bar{x} = \frac{n-1}{n^2} \qquad [2]$$

The main innovation in our model is that there are identity-dependent externalities generated by the prize that, together with the network, lead to different possible payoffs for player i when she does not win the contest.

In particular, if a player does not win the contest, her payoff depends on whether or not she is linked to the winner. The allocation of the prize to a player i imposes an externality αV, with $\alpha \in [-1, 1)$, on each agent who is connected to i; i.e., each agent j with $g_{ij} = 1$. If $g_{ij} = 0$, no externality is imposed on player j.[4] Thus, the expected payoff to player i from a profile of investments $(x_i, \mathbf{x}_{-i})$ is given by

$$\pi_i(x_i, \mathbf{x}_{-i}; \mathbf{G}) = P_i(x_i, \mathbf{x}_{-i}) - x_i + \alpha \sum_{j=1}^{n} g_{ij} P_j(x_j, \mathbf{x}_{-j}). \qquad [3]$$

Throughout the paper, we refer to the game as a *network contest game*, represented in normal form as $\Gamma = (X_i, \pi_i)_{i=1}^{n}$ where $X_i = \mathbb{R}_+$ represents the strategy set for player i, and $\pi_i(\cdot)$ is the payoff function defined in [3].

3 Equilibrium Analysis

We start our analysis by noting that any strategy profile with only one active agent cannot be a Nash equilibrium. Indeed, for a strategy profile $\mathbf{x}$ with $x_j > 0$ and $\mathbf{x}_{-j} = \mathbf{0}$, player j's best response function is empty. Similarly, given $\alpha < 1$, it is also straightforward to show that $\mathbf{x} = \mathbf{0}$ is not an equilibrium. Thus, we can restrict attention to strategy profiles with at least two active agents.

Consider player i and fix a profile $\mathbf{x}_{-i}$ with at least one strictly positive invest-

[4] Notice that the model incorporates a few stylized assumptions about the externality. In particular, the externality generated by allocating the prize to player i is the same for all of player i's neighbors, and does not spillover beyond the winner's immediate neighbors. We view these as natural starting points from which the model might be generalized. We also assume that the externality parameter does not depend on the winner's identity. As such, all of the heterogeneity that arises in the model is captured by an agent's position within the network.

ment. The expected payoff for player i in equation (3) can be rewritten as

$$\pi_i(x_i, \mathbf{x}_{-i}; \mathbf{G}) = \frac{x_i}{\sum_{h=1}^n x_h} - x_i + \alpha \sum_{j=1}^n g_{ij} \frac{x_j}{\sum_{h=1}^n x_h}$$

for all $x_i \geqslant 0$ and all $\mathbf{x}_{-i} \neq \mathbf{0}$. Note that $\partial^2 \pi_i / \partial x_i^2 < 0$ so that the payoff functions are strictly concave. Thus, player i's best response to $\mathbf{x}_{-i}$ is a well-defined, single-valued function given by

$$f_i(\mathbf{x}_{-i}; \alpha, G) = \max \left\{ 0, \left[\sum_{h \neq i} x_h (1 - \alpha g_{ih}) \right]^{0.5} - \sum_{h \neq i} x_h \right\}. \qquad [4]$$

As in the standard contest game, the best response functions are non-linear. As such, the main analysis of uniqueness and stability for network games developed in Bramoullé, Kranton and D'Amours (2014) cannot be directly applied. Moreover, the payoff functions do not satisfy the assumptions on the objective function required to apply the variational inequalities approach followed by Parise and Ozdaglar (2019) and Melo (2018) for network games with non-linear best replies.[5] When $\alpha = 0$, the best response functions are, as expected, the same as those for the standard contest game, for which existence and uniqueness are well established. For $\alpha \neq 0$, the issue is not quite as straightforward. We investigate the issue of uniqueness in section 3.3. To prove existence of a pure strategy Nash equilibrium, we rely on results from Reny (1999) and Bagh and Jofre (2006), to deal with the fact that payoff functions are discontinuous at $\mathbf{x} = 0$.

Theorem 1 (Existence)**.** *The network contest game possesses a pure strategy Nash equilibrium.*

Here, we highlight the main idea behind the proof of Theorem 1, which is detailed along with all of the other proofs in Appendix A. In particular, existence follows from Theorem 3.1 in Reny (1999). In order to apply Reny's theorem, we establish that the network contest game is compact, quasi-concave, and better-reply secure. For the last property, we show that the game is payoff secure and *weakly reciprocal upper semicontinuous* (wrusc), which is a condition introduced by Bagh and Jofre (2006) who prove that payoff security and wrusc imply better-reply security.

[5]They each consider games in which the objective function depends on x_i and a neighborhood aggregate, $\sum_h g_{ih} x_h$, but does not depend otherwise on x_j if $g_{ij} = 0$. In our setting, the payoff of an agent i depends on each x_j through the CSF, even if $g_{ij} = 0$.

Next, we provide a characterization of equilibrium profiles. For a given profile $\mathbf{x}$, we denote the set of active agents (those for whom $x_i > 0$) by A and the set of inactive agents by $N - A$. The following lemma provides a straightforward characterization of the set of Nash equilibria for the network contest game with externality α and network $\mathbf{G}$.

Lemma 1. *An investment profile* $\mathbf{x}$ *with active agents* A *is a Nash equilibrium if and only if* $|A| \geqslant 2$ *and*

(i) *for all* $i \in A$,

$$\sum_{j \in A}(1 - \alpha g_{ij})x_j - x_i = \left(\sum_{j \in A} x_j\right)^2 \qquad [5]$$

(ii) *for all* $i \in N - A$,

$$\sum_{j \in A}(1 - \alpha g_{ij})x_j \leqslant \left(\sum_{j \in A} x_j\right)^2 \qquad [6]$$

Consider first the special case of a *complete* network, in which each agent is linked to every other agent.

Proposition 1. *Consider the game in which* $\mathbf{G}$ *is the complete network,* $\mathbf{K}_n$*. For any* $\alpha \in [-1, 1)$*, there exists a unique Nash equilibrium, in which all players are active and choose the symmetric investment*

$$\bar{x}_K^\alpha = \frac{(n-1)(1-\alpha)}{n^2}.$$

Since the proof is straightforward, we instead highlight the underlying intuition. In the complete network, every non-winning agent is always impacted (symmetrically) by the winning agent, rendering the externalities identity-*independent*.[6] As a result, the game can be reformulated as a standard contest without externalities but with a modified prize value equal to the difference between the payoff from winning and the payoff from losing, which is $V - \alpha V = V(1 - \alpha)$.[7]

Although there is a unique equilibrium (which is symmetric) when the network is complete or when $\alpha = 0$ (a standard contest), there need not be a unique equilibrium for incomplete networks with non-zero externalities. In particular, for many networks, when α is positive and sufficiently large, there may exist both

[6]Note that this characterization relies on the modeling assumption that the size of the externality is homogenous across winning agents and their neighbors.

[7]Redefining $\widehat{V} = V(1-\alpha)$ (and setting $V = 1$) the result follows from the fact that the unique equilibrium in a standard contest with prize $\widehat{V}$ is $(n-1)\widehat{V}/n^2$.

a symmetric equilibrium (in which all agents actively invest) and a *specialized equilibrium*, defined next, in which some agents are inactive.

Definition 1. A *specialized equilibrium* is a Nash equilibrium $\mathbf{x}^*$ in which the set of active players A forms a maximal independent set. That is, for any two players $i, j \in A$, $g_{ij} = 0$, while for every $k \in N - A$, $\sum_{j \in A} g_{kj} \geqslant 1$.

For a given network $\mathbf{G}$ and a set of active agents A, let $d^i_A = \sum_{j \in A} g_{ij}$ denote the number of active agents linked to agent $i \in N$. Then, define $d_{N-A,A} = \min_{i \in N-A} d^i_A$. Finally, let $n_A = |A|$ denote the number of active agents in A.

Proposition 2. *Consider the game with network* $\mathbf{G}$ *and externality* $\alpha \in [-1, 1)$.

(i) *There exists a specialized equilibrium,* $\mathbf{x}^*$*, with active agents* A *and inactive agents* $N - A$*, if and only if* $\alpha \geqslant \frac{1}{d_{N-A,A}}$.

(ii) *In every specialized equilibrium,* $x^*_i = \bar{x}_A$ *for all* $i \in A$*, where* $\bar{x}_A = \frac{n_A - 1}{n_A^2}$.

Proposition 2 establishes that, in fact, in any specialized equilibrium, each inactive player must be linked to at least two active players.[8] Moreover, a specialized equilibrium is symmetric for players in A. That is, each active player chooses the same investment, corresponding to the equilibrium investment in a standard contest (without externalities) among only the n_A active agents.

Corollary 1. *Specialized equilibria do not exist for negative externalities* ($\alpha < 0$).

When α is sufficiently large, inactive players are content to exit the competition for the prize because they can free ride off their active neighbors and enjoy the positive externality that accrues if one of their neighbors wins. The greater the number of active neighbors, the lower the externality can be for the inactive player to opt out of the competition, but α must always be positive for a specialized equilibrium to exist.

3.1 Equilibria in Regular Networks

For the network graph $\mathbf{G}$, we let $d_i = \sum_j g_{ij}$ denote player i's degree. Then $\mathbf{G}$ is a regular network (or regular graph) of degree k if $d_i = k$ for all $i \in N$. The next result establishes existence of a *symmetric* equilibrium in any regular network $\mathbf{G}$ for any $\alpha \in [-1, 1)$.

[8] Bramoullé and Kranton (2007) describe a maximal independent set *of order* r as a maximal independent set, A, with each node $j \in N - A$ connected to at least r nodes in A. As they note, while a maximal independent set exists for any graph, maximal independent sets of order r with $r > 1$ need not exist.

Proposition 3. *Consider the game with network* $\mathbf{G}$ *and externality* $\alpha \in [-1, 1)$*. If* $\mathbf{G}$ *is a regular network of degree* $k \in \{0, \ldots, n-1\}$*, then there exists a symmetric, pure strategy Nash equilibrium,* $\mathbf{x}^* = (x^*, \ldots, x^*)$*, where*

$$x^* = \frac{n-1-\alpha k}{n^2}. \qquad [7]$$

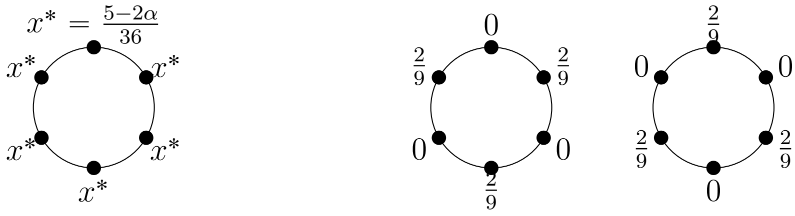
Note that, as should be expected, when $\alpha = 0$ or $k = 0$ (which is the case when $\mathbf{G}$ is the empty network), we obtain $x^* = \bar{x}$, which corresponds to the standard contest with no externalities. Furthermore, when $k = n-1$, $\mathbf{G}$ is the complete network $\mathbf{K}_n$, and we obtain $x^* = \bar{x}_K^\alpha$.

More importantly, comparative statics with respect to α and k have natural and intuitive interpretations. For positive externalities ($\alpha > 0$), free-riding incentives reduce the equilibrium investment compared to a standard contest without externalities. For negative externalities ($\alpha < 0$), the effective value of winning the contest increases so that competition intensifies, pushing equilibrium investment higher than in the standard contest. For both positive and negative externalities, these effects are amplified as k increases, which corresponds to an increase in network density.

Combining Proposition 2 with Proposition 3, it follows that for *regular networks*, there may exist multiple equilibria. Whenever the graph has a maximal independent set A with $\alpha \geqslant 1/d_{N-A,A}$, there is both a specialized equilibrium and the symmetric equilibrium with full participation.[9] In addition, in many cases, there may exist multiple specialized equilibria corresponding to different maximal independent sets of agents. To illustrate this multiplicity, we present two examples of regular networks and highlight the ranges of α for which there exist both specialized equilibria and a symmetric equilibrium with $A = N$.

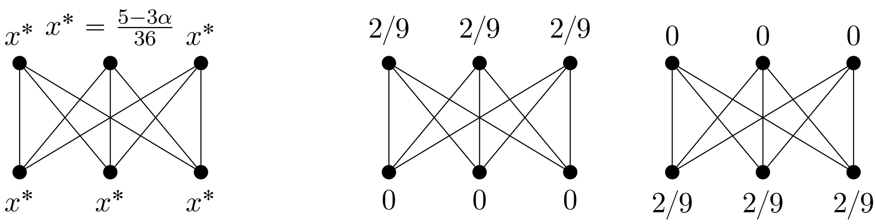
Example 1 (A circle network). In the circle network, the players are arranged around a circle and linked to the two agents on either side. Thus, the circle network is regular of degree $k = 2$. Hence, there exists a symmetric equilibrium for any $\alpha \in [-1, 1)$, in which all agents are active and each invests $x^* = \frac{5-2\alpha}{36}$; see panel (a) in Figure 1. Moreover, for $n = 6$, there are two maximal independent sets, as shown in Figure 1, panel (b). For each of these, $n_A = 3$, so that each active agent invests $\bar{x}_A = 2/9$. Furthermore, since every inactive player is linked to two active

[9] Note that in some cases, such a maximal independent set may not exist. For instance, consider the circle network with $n = 5$ agents. In this network, every maximal independent set is of order at most one, meaning that there is always at least one inactive agent who is connected to only one active agent, i.e., $d_{N-A,A} = 1$. In this case, a specialized equilibrium does not exist for any $\alpha < 1$.



(a) Symmetric equilibria, $\alpha \in [-1.1)$ **(b)** Specialized equilibria, $\alpha \geqslant 0.5$

Figure 1. Equilibria in the circle network with $n = 6$ agents. Panel (a): A symmetric equilibrium with all agents active exists for any $\alpha \in [-1, 1)$. Panel (b): When $\alpha \geqslant 0.5$, there are two specialized equilibria, each characterized by a maximal independent set of three agents, with each active agent investing $\bar{x}_A = 2/9$.



(a) Symmetric equilibria, $\alpha \in [-1, 1)$ **(b)** Specialized equilibria, $\alpha \geqslant 1/3$

Figure 2. Equilibria in the complete bipartite network with $n = 6$ agents. Panel (a): A symmetric equilibrium with all agents active exists for any $\alpha \in [-1, 1)$. Panel (b): When $\alpha \geqslant 1/3$, there are two specialized equilibria, each characterized by a maximal independent set of three agents, with each active agent investing $\bar{x}_A = 2/9$.

players, $d_{N-A,A} = 2$. Thus, the specialized equilibria exist if and only if $\alpha \geqslant 0.5$.

Example 2 (A bipartite network). $\mathbf{G}$ is a bipartite graph if the nodes (agents) can be partitioned into two disjoint sets A and B, with $g_{ij} = 0$ for all $i, j \in A$ and $g_{kl} = 0$ for all $k, l \in B$. Figure 2 illustrates a complete bipartite graph with $n = 6$ agents. This network is regular of degree $k = 3$. Hence, there exists a symmetric equilibrium for any $\alpha \in [-1, 1)$, in which all agents are active and each invests $x^* = \frac{5-3\alpha}{36}$; see panel (a) in Figure 2. Moreover, the three agents on the top and the three agents on the bottom represent the two maximal independent sets (as well as the two elements of the partition); see panel (b) in Figure 2. Given $n_A = 3$, each active agent invests $\bar{x}_A = 2/9$. Since the graph is a complete bipartite graph, each inactive agent in a specialized profile is linked to all of the active agents, so that $d_{N-A,A} = 3$. Thus, the specialized equilibria shown exist if and only if

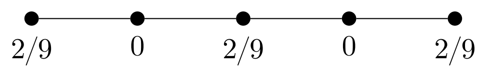


Figure 3. A specialized equilibrium for the line network with $n = 5$ agents exists if and only if $\alpha \geqslant 0.5$. The center agent and the agents at the endpoints of the line form a maximal independent set. Each active agent invests $\bar{x}_A = 2/9$.

$\alpha \geqslant 1/3$.

Although the prior examples illustrate specialized equilibria in the context of regular networks, specialized equilibria may arise in other classes of networks. To underscore this point, consider the example of a line network with $n = 5$ agents, which is not regular.[10]

EXAMPLE 3 (A line network). In the line network, whenever n is odd, there is a specialized equilibrium associated with the maximal independent set consisting of the endpoints of the line and every second node in between (see Figure 3). Every inactive agent is connected to two active agents, so that $d_{N-A,A} = 2$. Thus, the specialized equilibrium exists if and only if $\alpha \geqslant 0.5$.

Further examples of specialized equilibria arise in the context of another commonly studied class of networks; those that exhibit a core-periphery structure.

3.2 Equilibria in a Subclass of Core-Periphery Networks

The class of *core-periphery networks* is comprised of networks consisting of two types of agents—a set of highly connected *core* players, and a set of less connected *periphery* players. While this class of networks is very broadly defined, we restrict attention to a subset of the class that includes many of the most commonly studied core-periphery structures.

In particular, we define a subclass of core-periphery structures referred to as *core-to-periphery* networks. In a *core-to-periphery* network, there are $n_c \geqslant 1$ core players. All of the core players are connected to each other, creating a dense, or completely connected core. In addition, each core player is connected to $m \geqslant 1$ periphery players. We further assume that each periphery player is connected to a *single* core player and no other periphery players. Thus, there are $n = n_c(1 + m)$ total players, comprised of $n_c m$ periphery players, all with degree 1, and n_c core players, each with degree $k = (n_c - 1) + m$.

[10]Note that, for a line network with an even number of agents (n even), if the set of active agents form a maximal independent set, there is always at least one inactive agent who is linked to just one active agent. Thus, by Proposition 2, there does not exist a specialized equilibrium for the line if n is even.

The conditions laid out in the previous paragraph are satisfied by, for instance, the *star network*, which has a single core player ($n_c = 1$) connected to m periphery players. For all such *core-to-periphery* networks, we characterize the semi-symmetric equilibrium in which all players of the same type choose identical levels of investment. We denote the investment levels by x_c and x_p for core and periphery players, respectively.

Proposition 4. *Consider the game with network* $\mathbf{G}$ *and externality* $\alpha \in [-1, 1)$. *Suppose* $\mathbf{G}$ *is a core-to-periphery network with* n_c *core players, each connected to* m *peripheral players. Then there exists a semi-symmetric, pure strategy Nash equilibrium in which every core player chooses the same investment* x_c^*, *and every peripheral player chooses the same investment* x_p^*, *where*

(i) *if* $\alpha < \frac{1}{m}$, *then* $x_c^* = [1 - \alpha m]\Delta$ *and* $x_p^* = [1 + \alpha(n_c - 2)]\Delta$, *where*

$$\Delta = \frac{n_c[1 + m + \alpha m(n_c - 3)] - [1 + \alpha(n_c - 1 - \alpha m)]}{n_c^2[1 + m + \alpha m(n_c - 3)]^2} \geqslant 0.$$

(ii) *if* $\alpha \geqslant \frac{1}{m}$, *then* $x_c^* = 0$ *and* $x_p^* = \frac{n_c m - 1}{(n_c m)^2}$.

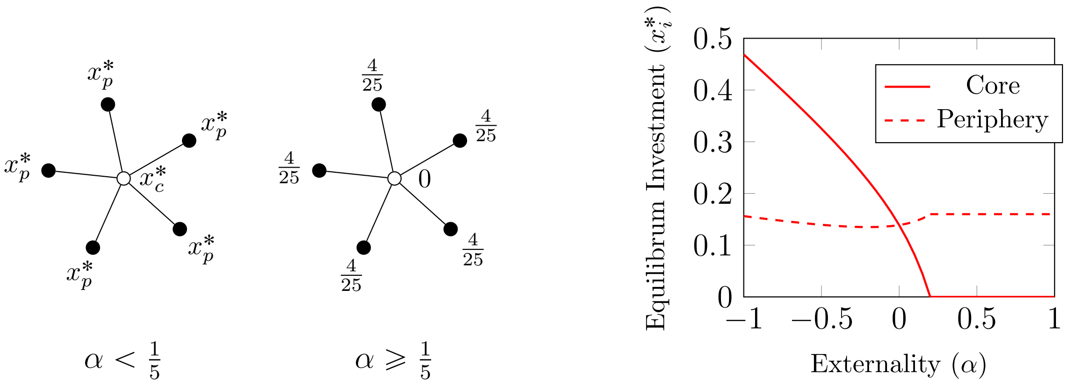
Note that when $\alpha = 0$, the equilibrium investments reduce to the standard contest equilibrium,

$$x_c^* = x_p^* = \frac{n_c(1 + m) - 1}{n_c^2(1 + m)^2} = \frac{n - 1}{n^2}.$$

For negative externalities and sufficiently small, positive externalities ($\alpha < 1/m$), the semi-symmetric equilibrium is interior; that is, both sets of agents are active. In addition, the semi-symmetric equilibrium investment for core players is decreasing in the externality (and strictly decreasing until they become inactive). In contrast, for periphery players, equilibrium investment is non-monotonic in α.

Moreover, for $\alpha < 0$, we have $x_c^* > x_p^*$. Intuitively, the core players are structurally more *exposed* to the negative externality than are the less connected periphery players (who are linked only to a single core agent, by assumption). Accordingly, for $\alpha > 0$, free-riding incentives are also stronger for core players than for periphery players, so that $x_c^* < x_p^*$ in the semi-symmetric equilibrium with positive externalities.

When the positive externality becomes sufficiently large ($\alpha \geqslant 1/m$), the semi-symmetric equilibrium is a *specialized equilibrium.* Free-riding incentives for the core players are sufficiently strong that they choose to be inactive in the contest.



(a) Star network, for $\alpha < \frac{1}{5}$ and $\alpha \geqslant \frac{1}{5}$ **(b)** Equilibrium investment, $\alpha \in [-1, 1)$

Figure 4. Semi-symmetric equilibria in the star network.

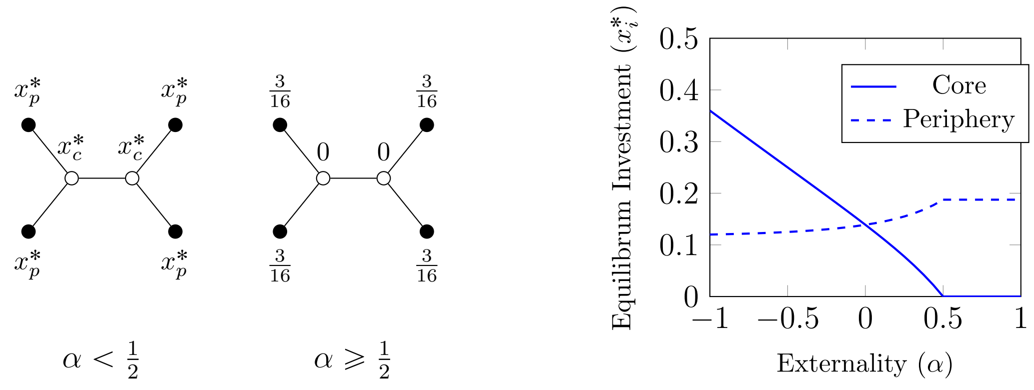
When this is the case, only the periphery players are active, and since they are not connected to each other, they form a maximal independent set and their equilibrium investment coincides with the equilibrium for a standard contest between $n_c m$ players (i.e., the total number of periphery players). Thus, for the subclass of core-to-periphery network structures, strong positive externalities lead to polarization of competition in the semi-symmetric equilibrium. The following examples serve to illustrate the semi-symmetric equilibria in two common core-to-periphery network structures.

Example 4 (A star network). In a star network, there is a single core-player, such that $n_c = 1$, and m peripheral players connected to the core (see Figure 4a where the core player is distinguished by the hollow node). For $m = 5$, the semi-symmetric equilibrium involves full participation when $\alpha < \frac{1}{5}$, with

$$x_c^* = \frac{5(1-5\alpha)(1-\alpha)^2}{4(3-5\alpha)^2} \qquad \text{and} \qquad x_p^* = \frac{5(1-\alpha)^3}{4(3-5\alpha)^2}.$$

When $\alpha \geqslant \frac{1}{5}$, the semi-symmetric equilibrium is a specialized equilibrium with A equal to the set of peripheral players, with $x_c^* = 0$ and $x_p^* = \frac{4}{25}$. Figure 4 shows the two cases on the network graph in panel (a) and in a graph that plots the equilibrium investment against α for both player types.

Example 5 (A core-periphery network with $n_c = 2$). In the CP2 network (see Figure 5a), there are $n_c = 2$ core players (distinguished by hollow nodes), each connected to 2 peripheral players. Thus, the semi-symmetric equilibrium involves



(a) CP2 network, for $\alpha < \frac{1}{2}$ and $\alpha \geqslant \frac{1}{2}$ **(b)** Equilibrium investment, $\alpha \in [-1, 1)$

Figure 5. Semi-symmetric equilibria in the CP2 network.

full participation when $\alpha < \frac{1}{2}$, with

$$x_c^* = \frac{(1-2\alpha)(5(1-\alpha)+2\alpha^2)}{4(3-2\alpha)^2} \qquad \text{and} \qquad x_p^* = \frac{5(1-\alpha)+2\alpha^2}{4(3-2\alpha)^2},$$

and is the specialized equilibrium with $x_c^* = 0$ and $x_p^* = \frac{3}{16}$ whenever $\alpha \geqslant \frac{1}{2}$. These equilibria are again illustrated on the network graph and plotted against α in panels (a) and (b) of Figure 5.

3.3 Uniqueness of Equilibria

In this section, we provide a more general treatment of uniqueness in the network contest game. Since the game does not admit linear best replies, we cannot directly apply the results from Bramoullé, Kranton and D'Amours (2014) to characterize a sufficient condition for uniqueness. However, using a similar approach, combined with direct argument, we are able to provide a related characterization of sufficient conditions under which the network contest game possesses a unique equilibrium.

To facilitate the exposition, we provide a general description of our approach. First, we show that while the contest game with network externalities is not an exact potential game, it is a *best-response (or best-reply) potential game* (Voorneveld, 2000). That is, there exists a function $\mathbf{P}$ (called a BR-potential) with the same best replies as the network contest game. Thus, the set of Nash equilibria in the game coincide with those strategy profiles that maximize the BR-potential, $\mathbf{P}$.

Second, we partition the domain $\mathbf{X}$ of the BR-potential $\mathbf{P}$ into two subsets: $\mathbf{X}^H$, consisting of strategy profiles $\mathbf{x}$ such that $\sum_h x_h \geqslant 0.5$, and $\mathbf{X}^L$, consisting of strategy profiles $\mathbf{x}$ such that $\sum_h x_h < 0.5$. For $\mathbf{X}^H$, the BR-potential $\mathbf{P}$ is strictly concave in $\mathbf{x}$ as long as $[\mathbf{I} + \alpha\mathbf{G}]$ is positive definite, which is true if and only if $\alpha < 1/|\lambda_{min}(\mathbf{G})|$, where $\lambda_{min}(\mathbf{G})$ is the lowest eigenvalue of $\mathbf{G}$. This is the familiar sufficient condition provided by Bramoullé, Kranton and D'Amours (2014) for uniqueness in network games with linear best replies.

For $\mathbf{X}^L$, the BR-potential $\mathbf{P}$ need not be strictly concave in $\mathbf{x}$, even if $\alpha < 1/|\lambda_{min}(\mathbf{G})|$. That is, the condition that $[\mathbf{I} + \alpha\mathbf{G}]$ is positive definite does not assure that $\mathbf{P}$ is strictly concave over $\mathbf{X}^L$. Nevertheless, we show directly that if there exists a Nash equilibrium in $\mathbf{X}^L$, we must have either $\alpha > 0.5$ (if the Nash equilibrium involves at least one inactive agent) or $\alpha > 0.5(n-2)/\Delta(\mathbf{G})$, where $\Delta(\mathbf{G}) \equiv \max_i d_i$ is the maximum degree in the graph (if the Nash equilibrium involves all agents being active).

Before stating the result, we first introduce the definition of a best-response potential game (Voorneveld, 2000) and the BR-potential function, $\mathbf{P}$.

Definition 2. A game $\Gamma = (X_i, \pi_i)_{i=1}^n$ with strategy space $\mathbf{X} = X_1 \times \ldots \times X_n$ and payoff functions $\pi_i : \mathbf{X} \to \mathbb{R}$ for players $i \in N = \{1, \ldots, n\}$ is called a *Best-Response potential game (BR-potential game)* if there exists a function $\mathbf{P} : X \to \mathbb{R}$ such that

$$\arg\max_{x_i \in X_i} \mathbf{P}(x_i, \mathbf{x}_{-i}) \quad = \quad \arg\max_{x_i \in X_i} \pi_i(x_i, \mathbf{x}_{-i}) \qquad [8]$$

for any $i \in N$ and any $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$. The function $\mathbf{P}$ is called a *BR-potential* for Γ.

Next, we construct a BR-potential for the network contest game. Note that, for any $\mathbf{x} \in \mathbf{X}$, we let $|A(\mathbf{x})|$ denote the number of nonzero entries in the vector $\mathbf{x}$ (i.e., the number of active agents under profile $\mathbf{x}$). In addition, let $X_{tot} = \sum_h x_h$ be the sum of investments for the profile $\mathbf{x}$.

Lemma 2. *The following function, $\mathbf{P}$, is a BR-potential for the network contest game.*

$$\mathbf{P}(x_1, \ldots, x_n) \quad = \quad \begin{cases} \sum\limits_{j<k} (1 - \alpha g_{jk}) x_j x_k - \frac{1}{3}(X_{tot})^3 & \text{if } |A(\mathbf{x})| \geqslant 2, \\ -\frac{1}{3} x_j \left[\max\limits_{i \neq j} (1 - \alpha g_{ij}) \right]^2 & \text{if } |A(\mathbf{x})| = 1 \text{ and } x_j > 0, \\ -\frac{1}{3}\frac{n-1}{n} & \text{if } |A(\mathbf{x})| = 0. \end{cases} \qquad [9]$$

The proof involves showing that the best responses coincide with those of the game, and closely follows the approach used by Ewerhart (2017) for the standard contest game without externalities.[11] Then, by Proposition 2.2 of Voorneveld (2000), a strategy profile $\mathbf{x}$ is a Nash equilibrium of the game if and only if it maximizes the BR-potential, $\mathbf{P}$. Therefore, if there exists a unique maximizer for $\mathbf{P}$, it is also the unique Nash equilibrium of the network contest game. We can now state our uniqueness result.

Theorem 2 (Uniqueness). *Consider the game with network* $\mathbf{G}$ *and externality* $\alpha \in [-1, 1)$. *The following three conditions are, when jointly satisfied, sufficient for there to exist a unique Nash equilibrium;*

(i) $\alpha \leqslant 0.5$;

(ii) $\alpha \leqslant \frac{0.5(n-2)}{\Delta(\mathbf{G})}$; *and*

(iii) $\alpha < \frac{1}{|\lambda_{min}(\mathbf{G})|}$.

Furthermore, whenever these conditions are satisfied, the unique equilibrium involves total investment $\sum_h x_h \geqslant 0.5$.

It is worth noting that, depending on the network, one of the conditions in Theorem 2 will always imply the other two. For instance, if $\Delta(\mathbf{G}) \equiv \max_i d_i < n-1$ (i.e., if no player is directly linked to every other player), then condition (i) implies condition (ii). Otherwise, condition (ii) implies condition (i). Similarly, if in addition to $\Delta(\mathbf{G}) < n-1$ we have $|\lambda_{min}(\mathbf{G})| \geqslant 2$, then condition (iii) is sufficient on its own. In particular then, for many networks, the condition derived by Bramoullé, Kranton and D'Amours (2014) for network games with linear best replies (our condition (iii)) is also sufficient for the network contest game.[12]

4 Experimental Evidence

In this section, we describe the design, procedures, and results of the laboratory experiment we conducted to test the predictions of our theoretical framework. Additional details regarding the experimental design are provided in the Online Appendix.

Design.—The basic decision environment in our experiment is a network contest game with $n = 6$ players. Each individual is given a fixed endowment, $\omega = 800$ tokens, and asked to choose how much to invest in a project. Within each group,

[11] Moreover, setting $\alpha = 0$ yields the same BR-potential he constructs.

[12] It is also straightforward to see that these conditions are in general sufficient, but not necessary for uniqueness. Consider the complete network with $\alpha \in (0.5, 1)$. The lowest eigenvalue is -1, so that condition (iii) is always satisfied. However, conditions (i) and (ii) are (clearly) not satisfied. Nevertheless, as shown in Proposition 1, there exists a unique equilibrium for all values of $\alpha \in [-1, 1)$.

only one player's project can be successful, and the probability that player i's project is successful is given by the Tullock (1980) lottery contest success function in equation (1). We set the value of the prize to be $V = 500$ tokens and assume that the value of the externality, αV, is proportional to the prize. The resulting material payoffs to player i, accounting for the network structure $\mathbf{G}$ and the externality parameter α are $1300 - x_i$ if player i wins the contest, $800 + 500\alpha - x_i$ if player i does not win, but is directly linked to the winner by $\mathbf{G}$, and $800 - x_i$ if player i does not win and is not directly linked to the winner, where x_i is i's investment.

We introduce two sources of treatment variation. First, we examine the four network structures shown in Figure 6, varied across sessions (i.e., between subjects). The Complete and Circle networks are both *regular* networks (with degree $k = 5$ and $k = 2$, respectively). The Star and CP2 networks are both *core-to-periphery* networks. Second, we examine three values of the externality parameter, α, in every session (i.e., within subjects). The first value, $\alpha = 0$, represents the baseline environment with no externality. The other two values capture a (strong) negative externality ($\alpha = -0.8$) and a (strong) positive externality ($\alpha = 0.8$).[13] Altogether, this generates 12 treatment conditions, distinguished by the network and the externality parameter.

Procedures.—Each session consisted of four blocks, with multiple rounds in each block. In all sessions, Block 1 consisted of 10 rounds with $\alpha = 0$ (the Baseline condition). For the other three blocks, we implemented the Negative condition (15 rounds), the Positive condition (15 rounds), and another Baseline condition (10 rounds), varying the order of the three conditions across sessions. Table 1 summarizes the treatment design, number of sessions, and number of independent groups.

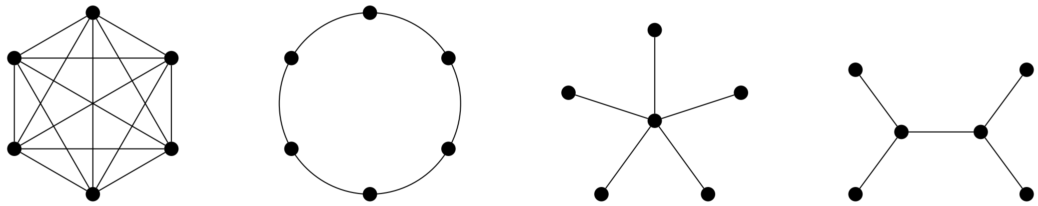
In total, we conducted 20 sessions at the XS/FS laboratory at Florida State University (FSU). Subjects could only participate in one session. The experiment was implemented using z-Tree (Fischbacher, 2007), with a total of 330 subjects, randomly recruited via ORSEE (Greiner, 2015) from a sub-population of FSU students who had all pre-registered to receive announcements about participation in experiments.

At the beginning of each session, subjects were randomly divided into groups

[13]For notational convenience, we occasionally describe the different treatment conditions by attaching a B (for Baseline) to indicate $\alpha = 0$, N (for Negative) to indicate $\alpha = -0.8$, or P (for Positive) to indicate $\alpha = 0.8$, at the end of the network name. For example, Complete-B refers to the Complete network with baseline $\alpha = 0$, while Star-P refers to the Star network with positive externality, $\alpha = 0.8$.

Table 1. Summary of experimental treatments.

Network	Treatment Order (Blocks 2–4)	Sessions	Groups	Subjects
COMPLETE	NPB, PNB, BNP, BPN	4	10	60
CIRCLE	NPB, PNB, BNP, BPN	4	11	66
STAR	NPB (2), PNB (2), BNP, BPN	6	18	108
CP2	NPB (2), PNB (2), BNP, BPN	6	16	96



(a) COMPLETE **(b)** CIRCLE **(c)** STAR **(d)** CP2

Figure 6. The set of networks

of six. Groups were fixed across all rounds and all blocks in every session. Participants were seated randomly at private computer terminals and given a set of written instructions. The experimenter then read the instructions aloud to facilitate common understanding.[14] Participants completed a short set of control questions to ensure they understood the instructions. The instructions were framed in terms of a general externality, X. Then, before each block, the experimenter announced the value of X $(= \alpha V)$ and reminded participants of the way payoffs are calculated. Participants were not informed about the number of blocks or the details of any future blocks until after the previous block was completed.

Before the four blocks that constituted the main part of the experiment, we also elicited subjects' attitudes towards risk, ambiguity, and losses, using a list-style procedure similar to the methods used by Holt and Laury (2002) and Sutter et al. (2013). We also included a decision task *after* the four blocks were completed, designed to provide a measure of each subject's *joy of winning*, following an approach introduced by Sheremeta (2010).[15] At the end of the experiment, subjects were paid for one randomly chosen period from each block, for the single decision round in the joy of winning elicitation task, and for one (randomly selected) of the risk, loss, or ambiguity aversion elicitation tasks. Tokens were

[14] A copy of the experimental instructions (for the Circle network) are provided in the Online Appendix.

[15] We do not address joy of winning or over-investment in the current paper, although it plays an important role in the experimental analysis in our companion paper Boosey and Brown (2021).

Table 2. Equilibrium predictions by treatment condition.

Network (position)	**Externality parameter** Negative	Baseline	Positive
COMPLETE	125.00	69.44	13.89
CIRCLE – *Symmetric*	91.67	69.44	47.22
CIRCLE – *Specialized*			
(active)	–	–	111.11
(inactive)	–	–	0.00
STAR (core)	206.63	69.44	0.00
STAR (peripheral)	74.39	69.44	80.00
CP2 (core)	157.89	69.44	0.00
CP2 (peripheral)	60.73	69.44	93.75

converted to US dollars according to the exchange rate 400 tokens = $1. Average earnings (including $7 show-up fee) amounted to $17.46.

Predictions.—Table 2 summarizes the equilibrium predictions for each treatment condition. First, for all networks, when $\alpha = 0$ (Baseline), the unique equilibrium investment is symmetric across positions and corresponds to the standard equilibrium investment for a contest with 6 players and a prize of $V = 500$. Furthermore, in the COMPLETE network, in light of Proposition 1, the unique equilibrium is the symmetric one corresponding to a standard contest with prize value equal to $V(1-\alpha)$. Accordingly, the equilibrium investment declines sharply as the externality increases from $\alpha = -0.8$ (Negative), to $\alpha = 0$ (Baseline), to $\alpha = 0.8$ (Positive).

In the CIRCLE network, there is a unique equilibrium for the Negative condition and the Baseline condition. However, for the Positive externality condition, there exists both a symmetric equilibrium with $x^* = 47.22$ and a pair of specialized equilibria, each consisting of three active players who invest $\bar{x}_A = 111.11$ and three inactive players.[16] This poses a potential coordination problem for the players in CIRCLE-P, since players may hold different beliefs about whether or not they are playing the symmetric equilibrium, the specialized equilibrium in which they are active, or the specialized equilibrium in which they are inactive.[17] Compared with

[16]The two specialized equilibria can be obtained by switching the sets of active and inactive players, since both are maximal independent sets.

[17]In the present paper, we focus on the comparative static predictions, which are similar whether we use the symmetric equilibrium or the specialized equilibrium. In our separate companion paper (Boosey and Brown, 2021), we examine the possibility that players are able to coordinate over time, and investigate whether play within groups is consistent with the maximal independent set characteristic of a specialized equilibrium.

the Complete network, the predicted investment in the symmetric equilibrium exhibits a much flatter decline as the externality increases. Thus, in line with the comparative static results discussed in Section 3.1, the effects of the externality (positive or negative) are increasing in the degree k for regular networks.[18]

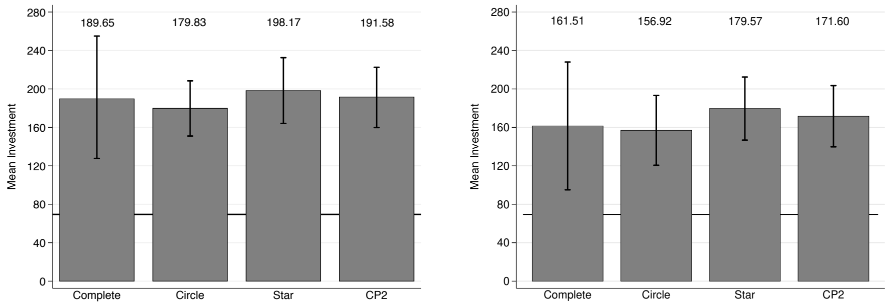
For the Star and CP2 networks, we examine the core player(s) and peripheral players separately. In both networks, the equilibrium is unique for all three externality conditions, is interior when $\alpha = -0.8$ and $\alpha = 0$, and specialized (with the core players inactive) when $\alpha = 0.8$. Thus, the equilibrium investment for the core player(s) is very high in the Negative condition, but equals zero in the Positive condition, reflecting their incentive to free ride in the specialized equilibrium. In contrast, for the peripheral players, equilibrium investment is fairly similar across all three values of α.

4.1 Main Results of the Experiment

In this section, we present the main results of the experiment as they relate to the predictions of our theoretical framework. To that end, we concentrate solely on the aggregate results concerning mean investment levels across networks and externality conditions. We focus on the comparative static predictions in order to highlight the main treatment effects. Throughout the analysis, we rely on non-parametric tests for treatment comparisons and on the wild cluster bootstrap method (Cameron, Gelbach and Miller, 2008) for post-estimation hypothesis tests on regression coefficients. When the relevant test is not indicated, the reported p-values correspond to a Wald test (with wild cluster bootstrap) comparing the estimated constant in a linear regression to the NE prediction. Furthermore, in all figures, error bars indicate 95% wild cluster bootstrap confidence intervals.

Baseline investment.—We first compare the mean investment level across networks in Block 1, where $\alpha = 0$. In this case, the network is payoff irrelevant and thus there should be no systematic differences across networks. Figure 7 shows that the mean Block 1 investment is 189.65 in Complete, 179.83 in Circle, 198.17 in Star, and 191.58 in CP2. Consistent with the prediction, we find no significant differences between networks (Kruskal-Wallis test, $p = 0.89$; also, for all pairwise comparisons between networks using the Wilcoxon ranksum test, $p > 0.401$). However, there is substantial over-investment, on average, relative to the Nash Equilibrium prediction (69.44) in all networks, which is consistent with the experimental literature on standard contests.

[18] If subjects perfectly implement one of the specialized equilibria in Circle-P, the predicted average equilibrium investment is 55.55, which is higher than in the symmetric equilibrium.



(a) All rounds **(b)** Final six rounds

Figure 7. Mean investment levels in the Baseline condition ($\alpha = 0$) from Block 1, by network. The solid reference line indicates the NE point prediction (69.44). Error bars indicate 95% wild cluster bootstrap confidence intervals.

Result 1. *Mean investment in the Baseline condition (Block 1) does not differ across networks.*

Nevertheless, in all networks, mean investment is trending down towards the Nash Equilibrium point prediction over the course of Block 1, which is consistent with some learning by the subjects as they gain experience with the strategic environment.[19] Thus, in order to account for experience, we also replicate the analysis using only the final six rounds of the block (see Figure 7b).[20] The corresponding mean investments are 161.51 in Complete, 156.92 in Circle, 179.57 in Star, and 171.60 in CP2, which are also not significantly different from each other (Kruskal-Wallis test, $p = 0.772$; for all pairwise comparisons using the Wilcoxon ranksum test, $p > 0.322$). Although these investment levels are lower than when we use all 10 rounds, over-investment relative to the NE prediction persists.

4.1.1 Treatment comparisons

Next, we compare mean investment across networks and externality conditions using the data collected in Blocks 2–4. Table 3 reports the mean investment for each network and each externality condition, alongside the corresponding Nash equilibrium (NE) point predictions. As is typical in contest experiments, and consistent with behavior in Block 1, we observe considerable over-investment (over-dissipation) relative to the NE in all conditions. However, we focus in this paper on the treatment comparisons as they relate to the comparative static predictions of the model. We begin by considering the two regular networks, Complete and Circle, before turning our attention to the core-periphery structures, Star and

[19] We provide visual support for this downward trend in the Online Appendix.

[20] All of our results are qualitatively similar using only the last five rounds, or only the last ten rounds.

Table 3. Summary statistics for mean investment in Blocks 2-4 by treatment condition

	Externality					
	Negative ($\alpha = -0.8$)		Baseline ($\alpha = 0$)		Positive ($\alpha = 0.8$)	
Network	Observed	[NE]	Observed	[NE]	Observed	[NE]
Complete	180.28	[125.00]	123.36	[69.44]	53.85	[13.89]
Circle	168.90	[91.67]	114.72	[69.44]	103.06	[47.22]
						[55.55][†]
Star						
core	256.06	[206.63]	161.43	[69.44]	71.79	[0.00]
peripheral	179.16	[74.39]	143.15	[69.44]	126.14	[80.00]
CP2						
core	213.03	[157.89]	125.33	[69.44]	102.28	[0.00]
peripheral	168.37	[60.73]	137.36	[69.44]	140.09	[93.75]

Notes: [†] denotes the average equilibrium investment in the specialized equilibria for Circle-P.

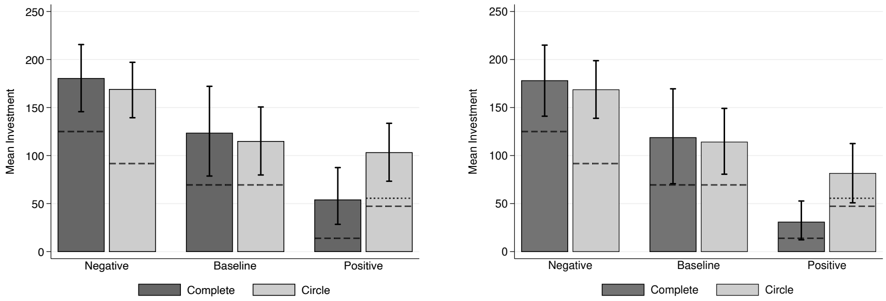
CP2.

Regular networks.—Figure 8a illustrates the mean investment reported in Table 3 (using all rounds) for each externality condition in the two regular networks, Complete and Circle. Comparisons within network (between-externality) are generally consistent with the comparative static predictions. In both networks, investment is highest for the negative externality (180.28 in Complete, 168.90 in Circle) and lowest for the positive externality (53.85 in Complete, 103.06 in Circle), as predicted by the theory. Using the mean investment across all rounds of a block within each independent group as a single observation, we observe significant differences for all pairwise comparisons between externality conditions in the Complete network (Wilcoxon Signed-Rank tests, $p = 0.047$ for N vs. B, $p = 0.005$ for N vs. P, $p = 0.007$ for P vs. B).

Similarly, in the Circle network, we observe significantly higher mean investment in the negative condition compared with the other two conditions (Wilcoxon Signed-Rank tests, $p = 0.016$ for N vs. B, $p = 0.008$ for N vs. P). However, mean investment in the Circle network is not significantly different between the baseline (zero externality) and positive externality conditions (Wilcoxon Signed-Rank test, $p = 0.374$), consistent with Figure 8a. We summarize our findings in the following two results.

Result 2. *For the* Complete *network, mean investment is strictly decreasing in the externality level, in line with the comparative static predictions.*

Result 3. *For the* Circle *network, mean investment is significantly higher with*



(a) All rounds **(b)** Final six rounds

Figure 8. Mean investment levels by externality in the Regular networks. Dashed lines indicate symmetric NE point predictions, while the short-dashed line for CIRCLE-P indicates specialized NE point prediction. Error bars indicate 95% wild cluster bootstrap confidence intervals.

the negative externality than with either of the other externality conditions. However, mean investment is no different with the positive externality than with zero externality.

The second part of the result for CIRCLE may be driven in part by the multiplicity of equilibria in the positive externality condition. In particular, while the symmetric equilibrium predicts lower investment in CIRCLE-P than in CIRCLE-B, the difference is quite small.[21] Furthermore, the predicted difference is even smaller if we posit that subjects play a specialized equilibrium (69.44 vs. 55.55). In addition, the likelihood of some coordination failure, especially in the early rounds, could explain why average investment remains at a similar level when α increases from 0 to 0.8.

We also examine investment using only the final six rounds of each block. Figure 8b shows that for the Negative and Baseline externality conditions, the mean investment levels using the final six rounds are no different than those reported in Table 3 and Figure 8a (which use all rounds). For both the COMPLETE and CIRCLE networks, over-investment relative to the NE prediction remains statistically significant. However, for the Positive externality condition, the mean investment over the final six rounds is considerably lower than it is using all rounds.[22] More importantly, when using only the final six rounds of each block, the comparative

[21] Note, however, that the difference is no smaller than the difference between CIRCLE-N and CIRCLE-B.

[22] In fact, over the final six rounds, over-investment in the COMPLETE-P condition is only marginally significant ($p = 0.0925$). Similarly, while the difference between mean investment and the symmetric equilibrium in CIRCLE-P over the final six rounds is still significant ($p = 0.032$), the difference relative to the average predicted investment in a specialized equilibrium is only marginally significant ($p = 0.0954$).

static prediction that investment in Circle-B is higher than in Circle-P is now supported (Wilcoxon Signed-Rank test, $p = 0.0076$). Thus, if we allow for learning (or experience) to take place in each block, we can remove the qualified support for the comparative static predictions in the Circle network altogether.

Next, we hold fixed the externality condition and compare investment levels *between* the two regular networks. As expected, in the Baseline condition, we find no significant differences, while in the Positive condition, investment is significantly higher in Circle than in Complete, which is consistent with both the symmetric and specialized equilibria for Circle-P.[23] However, contrary to the theoretical prediction, mean investment in the Negative condition is not significantly different between Complete and Circle (Wilcoxon Ranksum test using group-level means, $p = 1.000$ using all rounds, $p = 0.622$ using the final six rounds).

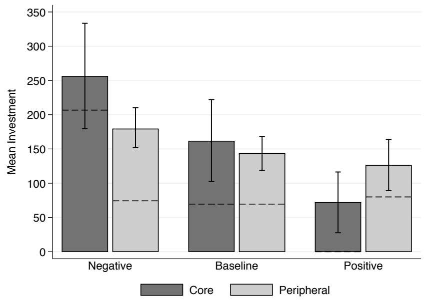
Result 4. *Average investment is significantly lower in* Complete-P *than in* Circle-P*, and is not significantly different between* Complete-B *and* Circle-B*, consistent with the predictions. In contrast, and contrary to the theoretical prediction, average investments in* Complete-N *and* Circle-N *are not significantly different from each other.*

Core-Periphery networks.—Figure 9a shows the mean investment across externality conditions for the core player and the peripheral players in the Star network. Core players invest significantly more than the NE point predictions in the Baseline and Positive conditions, but not in the Negative condition. Nevertheless, the comparisons between Negative, Baseline, and Positive for core players are all in line with the comparative static predictions. Specifically, investment in Negative is higher than in Baseline (Wilcoxon Signed-Rank test, $p = 0.018$) and Positive ($p < 0.001$), and investment in Baseline is higher than in Positive ($p = 0.004$).

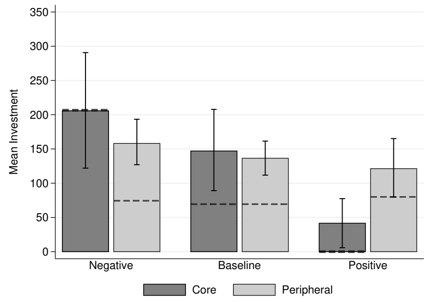
For peripheral players, the NE investment levels are very similar across the three externality conditions (cf. Table 2). However, Figure 9a shows that the mean investment is, in fact, slightly higher in Negative than in Baseline and Positive, which do not differ from one another.[24] The comparative static results for core players are all robust to using only the final six rounds of each block (see Figure 9b). However, the differences between mean investment of peripheral players for the different externality conditions are no longer statistically significant when

[23] For the Wilcoxon Ranksum test, using mean investment over all rounds for a single group as one observation, we have $p = 1.000$ for Baseline and $p = 0.029$ for Positive. Nothing substantive changes if we use the final six rounds, with $p = 0.994$ for Baseline and $p = 0.006$ for Positive.

[24] Wilcoxon Signed-Rank tests, $p = 0.043$ for N vs. B, $p = 0.003$ for N vs. P, $p = 0.223$ for P vs. B.



(a) All rounds



(b) Final six rounds

Figure 9. Mean investment levels by externality in the Star network. Dashed lines indicate NE point predictions. Error bars indicate 95% wild cluster bootstrap confidence intervals.

we restrict attention to the final six rounds.[25] Our next result summarizes these findings for the Star network.

Result 5. *For the* Star *network,*

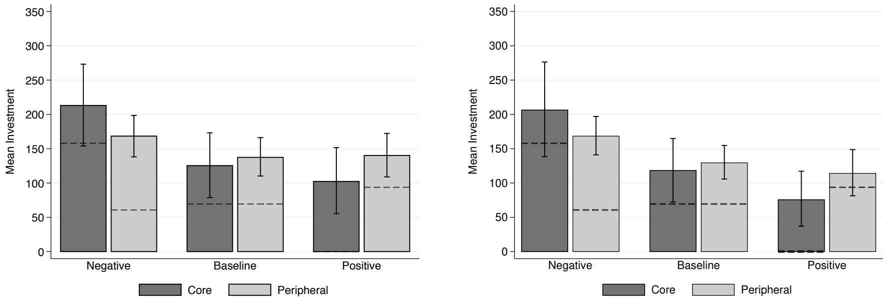
(i) mean investment by the core players is strictly decreasing in the externality level, in line with the comparative static predictions;

(ii) mean investment by the peripheral players is significantly higher with the negative externality than with the other two externality conditions when using all rounds, but does not differ between the three externality conditions when using the final six rounds of each block.

Figure 10a shows the mean investment in the CP2 network. As in the Star network, the core players invest significantly more than the NE point predictions in the Baseline and Positive conditions, but not the Negative condition. Nevertheless, the comparisons between externality conditions are all consistent with the comparative static predictions for the core players. Mean investment in Negative is higher than in both Baseline (Wilcoxon Signed-Rank test, $p = 0.007$) and Positive ($p = 0.006$), while investment in Baseline is higher than in Positive ($p = 0.030$). Figure 10b shows that each of these comparisons is also robust to using only the final six rounds of each block ($p < 0.01$ for each pairwise comparison).

The pattern of behavior for peripheral players in CP2 is also very similar to what we observe in the Star network. Mean investment is higher in Negative than in Baseline ($p = 0.010$) but only marginally higher than in Positive ($p = 0.079$), while Baseline and Positive are not significantly different ($p = 0.796$). Moreover, using only the final six rounds actually widens the difference between investment

[25] Wilcoxon Signed-Rank tests, $p = 0.396$ for N vs. B, $p = 0.085$ for N vs. P, $p = 0.122$ for P vs. B.



(a) All rounds

(b) Final six rounds

Figure 10. Mean investment levels by externality in the CP2 network. Dashed lines indicate NE point predictions. Error bars indicate 95% wild cluster bootstrap confidence intervals.

in the Negative condition and investment in the other two externality conditions by the peripheral players (Wilcoxon Signed-Rank tests, $p < 0.001$ for N vs. B; $p = 0.008$ for N vs. P; $p = 0.234$ for P vs. B).

Result 6. *For the* CP2 *network,*

(i) mean investment by the core players is strictly decreasing in the externality level, in line with the comparative static predictions;

(ii) mean investment by the peripheral players is significantly higher with the negative externality than with the other two externality conditions, and does not differ between the baseline and positive externality.

Finally, we compare behavior of the core players and peripheral players in the Star and CP2 networks, holding the externality level fixed. Core players' mean investment is less in CP2 than in Star when the externality is negative and in the baseline condition, and higher in CP2 than in Star when the externality is positive, but none of the differences is statistically significant (Wilcoxon Ranksum test, $p = 0.370$ for Negative, $p = 0.309$ for Baseline, and $p = 0.124$ for Positive). Similarly, mean investment by the peripheral players is not different between Star and CP2 for any of the externality conditions (Wilcoxon Ranksum test, $p = 0.581$ for Negative, $p = 0.605$ for Baseline, and $p = 0.448$ for Positive).

Altogether, the aggregate findings from our experiment provide strong support for the comparative static predictions of the model. Since our focus in the present paper is on the theoretical framework, we omit a more extensive analysis of the experimental data. Nevertheless, we examine the experimental data in considerably greater detail in a companion paper (Boosey and Brown, 2021).

5 Relation to Prior Literature

Prior literature on IDEs, following Jehiel, Moldovanu and Stacchetti (1996) has mostly considered optimal selling procedures in the presence of identity-dependent externalities.[26] Other related work has focused on strategic non-participation in auctions, especially with negative externalities (see, e.g., Jehiel and Moldovanu, 1996; Brocas, 2003) and explored the notion of *type-dependent* externalities (Brocas, 2013*a*, 2014), according to which the externality flows are correlated with the players' private valuations and not just their identities. In all-pay contest environments, there are a handful of related studies, including Konrad (2006) and Klose and Kovenock (2015), both of which characterize equilibria in the context of (perfectly-discriminating) all-pay auctions. There are, however, relatively few studies that consider externalities in the context of *imperfectly-discriminating* all-pay contests.

One exception is Linster (1993), who analyzes the equilibrium of a generalized Tullock contest in which the players care about who wins the prize if they do not. Another exception is Esteban and Ray (1999), who explore the relationship between equilibrium conflict and the distribution of preferences over outcomes in a lottery contest between interest groups.[27] While both of these studies incorporate the notion of identity-dependent externalities into a Tullock-style contest, neither draws a formal connection between these externalities and the underlying network structure that governs them. In contrast, a key contribution of our study is to bring together the literature on identity-dependent externalities and the relatively more recent developments in the theory of network games.

Typically, the network games literature examines games with linear best replies (see, e.g., the linear-quadratic utility functions in Ballester, Calvó-Armengol and Zenou, 2006; Bramoullé and Kranton, 2007; Bramoullé, Kranton and D'Amours, 2014). Among those that consider games with non-linear best replies, Allouch (2015) studies the private provision of local (network-based) public goods, and Melo (2018), Parise and Ozdaglar (2019), and Zenou and Zhou (2020) apply tech-

[26]For instance, Jehiel, Moldovanu and Stacchetti (1996, 1999) characterize the revenue-maximizing auctions for alternative information structures (including the case where externality flows are private information), Jehiel and Moldovanu (2000) study efficient auction design with externalities, while Das Varma (2002) characterizes the revenue and efficiency rankings of the standard sealed-bid and open ascending bid auction formats. See Jehiel and Moldovanu (2006) for a summary of the literature on standard, winner-pay auctions with identity-dependent externalities. In addition, Lu (2006) and Brocas (2013*b*) extend the analysis of the optimal auction to include the possibility of externalities between the seller and the bidders, whereas Aseff and Chade (2008) derive the optimal mechanism for a seller with multiple identical units.

[27]A crucial aspect of their model is the introduction of a "metric" over the different groups, which allows for spatial preferences over the preferred outcomes of other interest groups.

niques based on variational inequalities (VI) to establish existence and uniqueness.[28] To the best of our knowledge, the only other study to forge the connection between network games and externalities in a lottery contest game is König et al. (2017). They develop a stylized model of conflict to capture the impact of informal networks of alliances and enmities on conflict expenditures and outcomes, then apply their model to study empirically the Second Congo War.[29] In their model, agents (or groups) compete for a divisible prize in which any group's share of the prize depends on the group's relative *operational performance*, which takes the form of a generalized Tullock CSF. However, in contrast with our model, there are no allocation-based spillovers in their setting.[30]

Finally, experimental research on network games has, for the most part, focused either on coordination problems and games with strategic complementarities (see, e.g., Keser, Ehrhart and Berninghaus, 1998; Berninghaus, Ehrhart and Keser, 2002; Cassar, 2007; Gallo and Yan, 2015) or on public goods games where actions are strategic substitutes (Rosenkranz and Weitzel, 2012; van Leeuwen et al., 2019). Charness et al. (2014) examine both games of strategic complements and strategic substitutes, varying whether subjects in the experiment have complete or incomplete information about the network, in order to test the predictions of Galeotti et al. (2010) for network games. We provide the first experimental study of a contest game played on a network. As such, our findings both expand the experimental literature on network games and provide a novel extension on the rich body of work on contest experiments.

6 Conclusion

In this paper, we introduce and analyze a model of contests with identity-dependent externalities that are governed by a network. Our theoretical results simultaneously broaden the scope of traditional contest theory and extend the network games literature to a setting in which players have non-linear best replies. The

[28] Our model also entails non-linear best replies. However, as noted above, the VI approaches adopted by Melo (2018) and Parise and Ozdaglar (2019) rely on an assumption that the objective function for each agent depends only on own action and a neighborhood aggregate, which is not satisfied in our contest game due to the dependence of the contest success function on all players' actions.

[29] There is also a related, though distinct literature on *conflict networks* (see, e.g., Goyal and Vigier, 2014; Franke and Öztürk, 2015; Matros and Rietzke, 2018; Kovenock and Roberson, 2018; Xu, Zenou and Zhou, 2019) and the formation of conflict networks (Hiller, 2017; Jackson and Nei, 2015). In contrast with both our model and the model in König et al. (2017), these studies typically focus on environments where the network is used to describe the structure of conflict between agents who participate in *multiple battles*.

[30] Instead, the effort investments of other groups in König et al. (2017) feed directly into each group's operational performance through the underlying network of alliances and enmities.

model allows for positive and negative externalities, stemming from the allocation of the prize, that impact the payoffs of all players directly connected to the winner of the contest. We establish the existence of Nash equilibria and characterize sufficient conditions—related to the structure of the network—for uniqueness. For two broad classes of networks (regular and core-to-periphery), we provide closed-form results and show that the comparative statics align with the intuition from our motivating examples. Our framework can serve as a basis for studying a wide range of competitive situations, whether between firms or other organizations, individuals connected in a social network, or lobbyists with preferences over a multi-dimensional policy space.

In order to test the main predictions of the model, we also conducted a laboratory experiment in which we systematically varied both the network and the externalities. The experimental findings lend considerable support to the comparative static predictions of the model.

There are, of course, several directions in which our research may be extended. Our theoretical framework is relatively stylized—for instance, we limit attention to contests in which the externality flows are all of the same size and sign and the identity-dependence is driven entirely by the structure of the network. In future work, it may be interesting to generalize the model to allow for both positive and negative externalities within the same network, or to allow for link-specific externality flows. A related extension might be to allow for externalities that travel beyond the winner's immediate neighborhood, but with diluted impact proportional to the distance traveled.

From an empirical perspective, there is considerably more that can be done to understand behavior in network contest games. For instance, in our experiment we observe mean over-investment in most treatment conditions, consistent with the existing experimental literature on contests. Nevertheless, the particular patterns of over-investment appear to depend on the network, the externality condition and, in the core-periphery structures, the player's position within the network. In a companion paper (Boosey and Brown, 2021), we examine these interactions more carefully and provide supporting evidence for the influence of two behavioral phenomena—joy of winning, and social efficiency concerns—that appear to play an important role in the network contest game. Beyond our current experiment, it would also be useful to explore the impact of externalities in other, potentially larger, network structures, and to generalize the experimental setting alongside the extensions to our theoretical framework.

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A Proofs

Proof of Theorem 1. We prove existence by applying Theorem 3.1 in Reny (1999). For completeness, we restate the theorem using our own notation.

Theorem (Theorem 3.1, Reny (1999))**.** *If $\Gamma = (X_i, \pi_i)_{i=1}^n$ is compact, quasiconcave, and better-reply secure, then it possesses a pure strategy Nash equilibrium.*

Let $\Gamma = (X_i, \pi_i)_{i=1}^n$ denote the normal-form of the network contest game. Note that while $X_i = \mathbb{R}_+$ for each $i \in N$, we can, without loss of generality, restrict the agents' strategies to compact subsets of $\mathbb{R}_+$. To see why, note that since $\alpha \geqslant -1$, $P_i \leqslant 1$, and $d_i = \sum_h g_{ih} \leqslant n-1$, all strategies $x_i > 1 + (n-1) = n$ are strictly dominated by $x_i = 0$. Thus, we can restrict the strategy sets to $\widehat{X}_i = [0, n]$, which is compact. Next, we note that each agent i's payoff function is concave, and thus also quasiconcave, in x_i. It remains to show that Γ is *better reply secure*. To do so, we first introduce some relevant definitions and another result by Bagh and Jofre (2006) that extends on Reny (1999).

Definition 3. In the game $\Gamma = (X_i, \pi_i)_{i=1}^n$, player i can *secure* a payoff of $\alpha \in \mathbb{R}$ at $x \in X$ if there exists $y_i \in X_i$ such that $\pi_i(y_i, x'_{-i}) \geqslant \alpha$ for all x'_{-i} in some open neighborhood of x_{-i}.

Definition 4. A game $\Gamma = (X_i, \pi_i)_{i=1}^n$ is *payoff secure* if for every $x \in X$ and every $\varepsilon > 0$, each player i can secure a payoff of $\pi_i(x) - \varepsilon$ at x.

Let $\Lambda = \{(\mathbf{x}, \pi) \in \mathbf{X} \times \mathbb{R}^n \,|\, \pi_i(\mathbf{x}) = \pi_i, \forall\, i\}$ denote the graph of the vector of payoff functions for the game and let $\overline{\Lambda}$ denote the closure of Λ in $\mathbf{X} \times \mathbb{R}^n$. Finally, define the frontier of Λ to be the set of points in $\overline{\Lambda}$ but not in Λ, denoted by $\mathrm{Fr}\Lambda = \overline{\Lambda} \backslash \Lambda$. The following definition is from Bagh and Jofre (2006).

Definition 5. A game $\Gamma = (X_i, \pi_i)_{i=1}^n$ is *weakly reciprocally upper semicontinuous* (wrusc) if, for any $(\mathbf{x}, \pi) \in \mathrm{Fr}\Lambda$, there is a player i and $\widehat{x}_i \in X_i$ such that $\pi_i(\widehat{x}_i, x_{-i}) > \pi_i$.

Having defined payoff security and wrusc, we then appeal to the following result from Bagh and Jofre (2006).

Proposition 5 (Proposition 1, Bagh and Jofre (2006)). *If the game $\Gamma = (X_i, \pi_i)_{i=1}^n$ is payoff secure and wrusc, then it is better reply secure.*

To prove that Γ is payoff secure and wrusc, we follow a similar approach to Bagh and Jofre (2006) in their Example 3, which considers (a generalized form of) the standard contest game with Tullock (1980) contest success function.

(i) First, we show that the game is payoff secure. Note that payoffs are continuous except at $\mathbf{x} = \mathbf{0}$, where they are given by

$$\pi_i(\mathbf{0}) = \frac{1 + \alpha d_i}{n}$$

where d_i is player i's degree in the network. Then note that for $\tilde{x}_i > 0$, we have $\pi_i(\tilde{x}_i, \mathbf{0}) = 1 - \tilde{x}_i$, which is higher than $\pi_i(\mathbf{0})$ if $\tilde{x}_i < (n - 1 - \alpha d_i)/n$. Since $d_i \leqslant n-1$ and $\alpha < 1$, the right hand side is strictly positive, so that such a $\tilde{x}_i > 0$ can be found. Then, since $\pi_i(\cdot)$ is continuous at $(\tilde{x}_i, \mathbf{0})$, there is a neighborhood V of $\mathbf{x}_{-i} = \mathbf{0}$ such that $\pi_i(\tilde{x}_i, \mathbf{x}'_{-i}) > \pi_i(0, \mathbf{0})$ for all $\mathbf{x}'_{-i} \in V$. Thus, the game is payoff secure at the point $\mathbf{x} = \mathbf{0}$. Payoff security at all other $\mathbf{x}$ is straightforward.

(ii) Second, we show that the game is wrusc. In this game (as in the standard contest game), the only points in FrΛ must be points of the form $(\mathbf{0}, \pi)$ where $\pi_i = \lim\limits_{\mathbf{x}^k \to \mathbf{0}} \pi_i(\mathbf{x}^k)$ for all i. Note that

$$\begin{aligned}
\sum_{i=1}^n \pi_i(\mathbf{x}^k) &= \sum_{i=1}^n P_i(\mathbf{x}^k) - \sum_{i=1}^n x_i + \alpha \sum_{i=1}^n \sum_{j=1}^n g_{ij} P_j(\mathbf{x}^k) \\
&= 1 - \sum_{i=1}^n x_i + \alpha \sum_{i=1}^n d_i P_i(\mathbf{x}^k) \\
&\leqslant 1 - \sum_{i=1}^n x_i + \alpha(n-1)
\end{aligned}$$

where the inequality follows from the fact that $d_i \leqslant n-1$ for all i and $\sum_{i=1}^n P_i(\mathbf{x}^k) = 1$. As such, $\lim\limits_{\mathbf{x}^k \to \mathbf{0}} \sum_{i=1}^n \pi_i(\mathbf{x}^k) \leqslant 1 + \alpha(n-1)$ and thus, there exists some i for whom

$$\pi_i \leqslant \frac{1 + \alpha(n-1)}{n}.$$

Notice that $\lim\limits_{x_i \to 0} \pi_i(x_i, \mathbf{0}) = 1$. Thus, there exists $\hat{x}_i > 0$ such that $\pi_i(\hat{x}_i, \mathbf{0}) > \pi_i$, because $\alpha < 1$ ensures that $(1 + \alpha(n-1))/n < 1$. It follows that the game is wrusc.

Together, payoff security and wrusc imply better reply security, and applying Theorem 3.1 from Reny (1999), there exists a pure strategy Nash equilibrium. $\square$

Proof of Proposition 2. Both parts of the proposition follow directly. From condition (i) in Lemma 1, it follows from the fact that $g_{ij} = 0$ for all $i, j \in A$ in a specialized equilibrium, that $x_i = \sum_{j \in A} x_j - (\sum_{j \in A} x_j)^2$ for all $i \in A$, which implies that all active players must be choosing the same investment $\bar{x}_A = \frac{n_A - 1}{n_A^2}$. Therefore, total investment is given by $X_A = \sum_{j \in A} x_j = (n_A - 1)/n_A$. Then, for the second condition in Proposition 1 to be satisfied, it must be the case that for all $i \in N - A$,

$$\begin{aligned} & \frac{n_A - 1}{n_A^2}(n_A - \alpha d_A^i) \leqslant \frac{(n_A - 1)^2}{n_A^2} \\ \iff \quad & \alpha \geqslant \frac{1}{d_A^i} \end{aligned}$$

Taking $d_{N-A,A}$ to be the minimum of d_A^i over all $i \in N - A$ ensures that the inequality is satisfied for all inactive players. $\square$

Proof of Proposition 3. Suppose that $A = N$ (that is, all agents are active). From Lemma 1, only condition (i) needs to be satisfied. Summing equation [5] for all n active players and rearranging gives

$$(n-1)\sum_{i=1}^{n} x_i - \alpha \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij} x_j \quad = \quad n\left(\sum_{i=1}^{n} x_i\right)^2$$

and positing $x_i = x$ for all i yields

$$\begin{aligned} (n-1)nx - \alpha n k x &= n(nx)^2 \\ (n-1) - \alpha k &= n^2 x \end{aligned}$$

from which x^* follows. $\square$

Proof of Proposition 4. Suppose both types are active and consider condition (i) from Proposition 1. For each peripheral player, equation [5] reduces to

$$(n_c - \alpha)x_c + (n_c m - 1)x_p = (n_c x_c + n_c m x_p)^2$$

while for each core player, it simplifies to

$$(n_c - 1)(1 - \alpha)x_c + (n_c m - \alpha m)x_p = (n_c x_c + n_c m x_p)^2.$$

From this, we obtain $x_c(1+\alpha(n_c-2)) = (1-\alpha m)x_p$.

Substituting into the condition for the core players and solving yields the solution $x_c = (1-\alpha m)\Delta$ and $x_p = (1+\alpha(n_c-2))\Delta$, where

$$\Delta \quad = \quad \frac{n_c\left[1+m+\alpha m(n_c-3)\right] - \left[1+\alpha(n_c-1-\alpha m)\right]}{n_c^2\left[1+m+\alpha m(n_c-3)\right]^2} \geqslant 0.$$

For x_c to be strictly positive, we must have $\alpha < \frac{1}{m}$. Thus, a semi-symmetric equilibrium with full participation exists only when α is not too large. Once $\alpha \geqslant \frac{1}{m}$, there is a semi-symmetric equilibrium which is also a *specialized equilibrium* in which the core players are all inactive, while the peripheral players, who form a maximal independent set, invest the standard equilibrium investment for a contest between $n_c m$ individuals. □

Proof of Lemma 2. We proceed by cases. Fix a player i.

Case 1. Suppose $\mathbf{x}_{-i}$ has at least two strictly positive components. Then, for any x_i, $A(x_i, \mathbf{x}_{-i}) \geqslant 2$. It follows from [9] that

$$\begin{aligned}\frac{\partial \mathbf{P}}{\partial x_i} &= \sum_{h\neq i}(1-\alpha g_{ih})x_h - X_{tot}^2 \\ \frac{\partial^2 \mathbf{P}}{\partial x_i^2} &= -2X_{tot} < 0.\end{aligned}$$

It follows that $x_i \in \arg\max \mathbf{P}(x_i, \mathbf{x}_{-i})$ if and only if

$$x_i\Big(\sum_{h\neq i}(1-\alpha g_{ih})x_h - X_{tot}^2\Big) = 0$$

which implies

$$x_i = \max\left\{0, \sqrt{\sum_{h\neq i}(1-\alpha g_{ih})x_h} - \sum_{h\neq i} x_h\right\},$$

which is exactly the best response function $f_i(\mathbf{x}_{-i}, \alpha, \mathbf{G})$ derived in [4].

Case 2. Next, suppose $x_j > 0$ is the only positive component of $\mathbf{x}_{-i}$. From [9],

$$x_i > 0 \;\Rightarrow\; \mathbf{P}(x_i, \mathbf{x}_{-i}) = x_i x_j(1-\alpha g_{ij}) - \frac{1}{3}(x_i+x_j)^3$$

whereas

$$x_i = 0 \;\Rightarrow\; \mathbf{P}(x_i, \mathbf{x}_{-i}) = -\frac{1}{3}x_j\big[\max_{h\neq j}(1-\alpha g_{hj})\big]^2.$$

Taking the limit as x_i approaches zero from above, we have $\lim_{x_i\to 0}\mathbf{P}(x_i, \mathbf{x}_{-i}) =$

$-\frac{1}{3}x_j^3$, which is strictly greater than $\mathbf{P}(0, \mathbf{x}_{-i})$ if and only if

$$x_j < \max\big(1 - \alpha g_{ij}\big).$$

Multiplying through by x_j, player i's best response is interior at some $x_i > 0$ if and only if

$$x_j^2 < \max\big(1 - \alpha g_{ij}\big)x_j,$$

and is $x_i = 0$ otherwise, which again coincides with the best response function in [4].

Case 3. Finally, suppose $\mathbf{x}_{-i} = \mathbf{0}$. If $x_i > 0$, then

$$\mathbf{P}(x_i, \mathbf{0}) = -\frac{1}{3}x_i\big[\max_{h \neq i}(1 - \alpha g_{ih})\big]^2,$$

which approaches zero (from below) as x_i approaches zero from above. In contrast, $x_i = 0$ implies $\mathbf{P}(\mathbf{0}) = -\frac{1}{3}\frac{(n-1)}{n} < 0$. As such, a maximizer does not exist for $\mathbf{P}$, just as the best response function for π_i is empty when $\mathbf{x}_{-i} = \mathbf{0}$.

By means of the three cases, we have verified that for an arbitrary player i, the set of maximizers for $\mathbf{P}$ given any $\mathbf{x}_{-i}$ coincide with the best responses according to the payoff functions π_i. Thus, $\mathbf{P}$ is a BR-potential for Γ. □

Proof of Theorem 2. The network contest game is a best-response potential game (Voorneveld, 2000). Lemma 2 provides a BR-potential for the game, $\mathbf{P}$. Then, by Proposition 2.2 of Voorneveld (2000), the profile $\mathbf{x}$ is a Nash equilibrium of the network contest game if and only if it maximizes the BR-potential, $\mathbf{P}$. The remainder of the proof establishes conditions under which there is a unique maximizer for $\mathbf{P}$.

Recall that $\mathbf{P}$ is strictly concave if $\nabla^2\mathbf{P}$ is negative definite. Before deriving the Hessian for $\mathbf{P}$, note that we can restrict the search for maxima to investment profiles $\mathbf{x}$ with $|A(\mathbf{x})| \geqslant 2$, since we have already established that there are no Nash equilibria in which fewer than 2 players are active. Thus, for any such $\mathbf{x}$, the diagonal elements of the Hessian $\nabla^2\mathbf{P}$ are given by

$$\frac{\partial^2\mathbf{P}}{\partial x_i^2} = -2\sum_{h=1}^{n} x_h$$

while the cross-partial terms are symmetric and given by

$$\frac{\partial^2\mathbf{P}}{\partial x_i \partial x_j} = \frac{\partial^2\mathbf{P}}{\partial x_j \partial x_i} = (1 - \alpha g_{ij}) - 2\sum_{h=1}^{n} x_h.$$

Rewriting in matrix form and using $X_{tot} = \sum_h x_h$ gives

$$\nabla^2 \mathbf{P} \quad = \quad \big(1 - 2X_{tot}\big)\mathbf{J} - [\mathbf{I} + \alpha\mathbf{G}],$$

where $\mathbf{J}$ denotes the $n \times n$ matrix of ones. Note that even if $\mathbf{I} + \alpha\mathbf{G}$ is positive definite, if $X_{tot} < 0.5$ and is small enough, the Hessian need not be negative definite. Our approach to get around this problem is to partition the domain into two subsets, $\mathbf{X}^H$ and $\mathbf{X}^L$.

(i) If we restrict the domain of $\mathbf{P}$ to the set $\mathbf{X}^H$ of vectors $\mathbf{x}$ such that $X_{tot} \geqslant 0.5$, it is readily verified that $\mathbf{P}$ is strictly concave on the restricted domain if $\mathbf{I} + \alpha\mathbf{G}$ is positive definite, which is equivalent to the condition that $\alpha < 1/|\lambda_{min}(\mathbf{G})|$.

(ii) Nevertheless, this condition is not sufficient to establish strict concavity on the subdomain $\mathbf{X}^L$, which is composed of strategy profiles $\mathbf{x}$ with $X_{tot} < 0.5$. Instead, we proceed by direct argument. Suppose that $\mathbf{x}$ is a Nash equilibrium with $X_{tot} < 0.5$.

(a) Then, if there is any inactive player, k, we must have

$$X_{tot} - \alpha \sum_{h=1}^{n} g_{kh} x_h \quad \leqslant \quad \big(X_{tot}\big)^2.$$

Rearranging, we obtain

$$\alpha \quad \geqslant \quad \frac{X_{tot}\big(1 - X_{tot}\big)}{\sum_{h=1}^{n} g_{ih} x_h},$$

and since $g_{ih} \leqslant 1$ for all i, h, it follows that $\alpha \geqslant 1 - X_{tot} > 0.5$. Thus, if there is an equilibrium with an inactive player, such that $X_{tot} = \sum_h x_h < 0.5$, it must be the case that $\alpha > 0.5$.

(b) Then, suppose there is no inactive player for $\mathbf{x}$ with $X_{tot} < 0.5$. Then, for all n players, we must have

$$x_i + \alpha \sum_{h=1}^{n} g_{ih} x_h \quad = \quad X_{tot}(1 - X_{tot}).$$

Summing over all i, obtain

$$\begin{aligned}
& & \alpha \sum_{i=1}^{n} \sum_{h=1}^{n} g_{ih} x_h &= X_{tot}(n(1 - X_{tot}) - 1) \\
&\Rightarrow & \alpha \sum_{h=1}^{n} x_h \sum_{i=1}^{n} g_{ih} &> X_{tot} \left(\frac{n-2}{2} \right) \\
&\Rightarrow & \alpha \sum_{h=1}^{n} d_h x_h &> X_{tot} \left(\frac{n-2}{2} \right) \\
&\Rightarrow & \alpha \Delta(\mathbf{G}) X_{tot} &> X_{tot} \left(\frac{n-2}{2} \right) \\
&\Rightarrow & \alpha &> \frac{n-2}{2\Delta(\mathbf{G})},
\end{aligned}$$

where the second line follows from $1 - X_{tot} > 0.5$, and the fourth line from the fact that $\Delta(\mathbf{G})$ is the maximum degree of $\mathbf{G}$.

It follows that if $\alpha \leqslant 0.5$ and $\alpha \leqslant 0.5(n-2)/\Delta(\mathbf{G})$, there cannot be a Nash equilibrium in $\mathbf{X}^L$. By existence of an equilibrium, there must exist at least one equilibrium in $\mathbf{X}^H$. If we also have that $\alpha < 1/|\lambda_{min}(\mathbf{G})|$, then $[\mathbf{I} + \alpha\mathbf{G}]$ is positive definite, $\mathbf{P}$ is strictly concave on $\mathbf{X}^H$, and there exists a unique Nash equilibrium, $\mathbf{x} \in \mathbf{X}^H$, such that $X_{tot} \geqslant 0.5$. □

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B Block 1 (Baseline) Investment over time

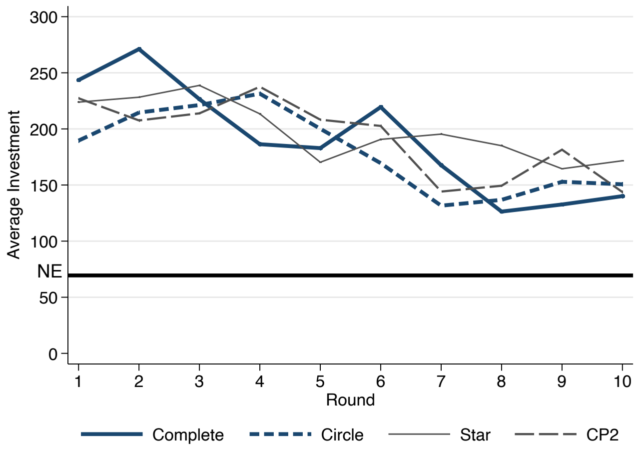

Figure B.1. Mean investment levels in the Baseline condition ($\alpha = 0$) from Block 1, by network and across rounds. The solid reference line indicates the NE point prediction (69.44).

C Additional Details of the Experimental Design

Main task procedures. Each player in a group was randomly assigned a letter ID from A to F. The letter ID and the position in the network were fixed across the entire experiment. In each round, players were shown the network, with their own ID and position highlighted. In addition, their direct neighbors in the network were highlighted in yellow, while those members of the group with whom they were not connected were shown in black. They were also reminded about the externality at the top of the screen, and prompted to enter the number of tokens they would like to invest in their project. After all players had made their decisions, an interim summary screen displayed a table showing all players' investments, the total investment, and the corresponding probability of winning.[31] After a few moments, the same screen was updated to also show the player the letter ID of the winner, whether or not they were affected by the externality (if they were not the winner), and the calculation of their payoffs for the round.

Joy of winning elicitation task. After the four blocks were completed, subjects were rematched into new groups of 6 subjects for a single decision round. They were given the same endowment of 800 tokens and asked to choose a project

[31] In the experiment, winning was not explicitly mentioned. Rather, we referred to the player's own project being the successful one.

investment, just as in the four main blocks. In contrast with the other four blocks, there were no network connections (and thus no externalities) and the winner received a prize of 0 tokens. This part of the experiment was designed to provide a measure of each subject's *joy of winning*, following the approach introduced by Sheremeta (2010).

D Experimental Instructions

Thank you for participating in today's experiment. I will read through the script so that everyone receives the same information. Please remain quiet and do not communicate with other participants during the experiment. Raise your hand if you have any questions and an experimenter will come to you to answer the question privately.

For your participation in today's experiment, you will receive the show-up fee of $7. In addition, during the experiment, you will have the opportunity to earn more money. Your additional earnings will depend on the decisions you make and on the decisions made by other participants. At the end of the experiment, you will be paid anonymously by check. No other participant will be informed about your payment.

The experiment consists of multiple parts. The instructions for subsequent parts will be given only after each previous part is completed. Below you will find the instructions for Part 1.

Part 1 Instructions

In this part, you will be asked to make three decisions. **One** of these three decisions will be randomly chosen at the end of the experiment and that decision will be used to calculate your actual earnings for Part 1.

The basic setups for the three decisions are similar. In each case, you will see a list of 20 choices between lotteries and sure amounts of money. Lotteries will always be on the left, and sure amounts of money on the right. The lists will be ordered such that you will prefer the lottery to the sure amount of money in the choice at the top of the list. As you go down the list, you will tend to like the lotteries less and less as compared to the sure amounts. At some point, you will be willing to switch from preferring a lottery to preferring the corresponding sure amount of money. At the point where you are willing to switch, please click on the SWITCH HERE button.

When you click on a SWITCH HERE button, lotteries will be your choice everywhere above that line, and sure amounts of money will be your choice everywhere below that line. All of the 20 choices that you generate will be highlighted. If you want to change your decision, simply click on another SWITCH HERE button. When you are ready to finalize your decision, click SUBMIT.

After you have made your decision, one of the 20 choices will be selected randomly. If your decision for that choice is a sure amount of money, you will earn that amount of money. If your decision for that choice is the lottery, then the outcome of the lottery will be determined according to the listed probabilities and your earnings will be equal to that outcome.

You will not be informed about your earnings from this part of the experiment until the very end of the session today, after you have completed all parts of the experiment.

Are there any questions before you begin making your decisions?

Part 2 Instructions

All amounts in this part of the experiment are expressed in **tokens**. The exchange rate is 400 tokens = $1.

This part of the experiment consists of a sequence of 10 decision rounds. At the beginning of round 1, you will be randomly assigned to a group consisting of 6 participants, including you. You will remain in this group for the duration of this part. That is, you will interact with the same 5 other participants in all 10 rounds.

Your group

Before round 1, you and the 5 other participants in your group will be randomly assigned to positions in the network graph shown in Figure D.2 below. One person will be assigned to each position. Each position is labeled with a letter ID, from A to F. Positions, and therefore also the letter IDs, will remain fixed for the duration of this part. In the network graph, a straight line between two positions indicates that players at those positions are "connected".

During the decision rounds, the network graph will be shown on the screen. Your own position will be highlighted in red. The players you are connected to will be highlighted in yellow, while those you are not connected to (if there are any) will be shown in black.

For example, Figure D.2 shows the network graph from player A's perspective. Thus, player A's position will be displayed in red, while the positions for player B and player C will be displayed in yellow. All of the other players' positions will be displayed in black, since only player B and player C are connected by an edge to player A in this network graph.

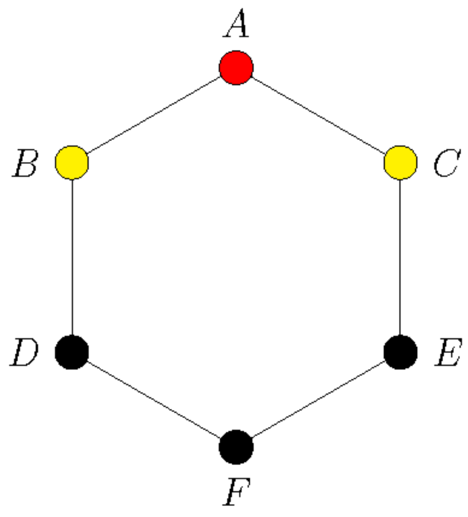

Figure D.2. The network graph - as viewed by player A

Your decision

In each round, you will be given an endowment of 800 tokens. You may use these tokens to make decisions in the round. Specifically, during the round, you can invest any integer number of tokens, from 0 to 800, into a project. Other participants in your group will face the same decision, with the same endowment of 800 tokens. After everyone has chosen a project investment, one participant in the group will be declared **the winner**, based on the following procedure. The probability that you are the winner is given by:

$$\frac{\text{Number of tokens you invested in your project}}{\text{Sum of the tokens invested in projects by all participants in your group}}$$

The computer program will determine the winner according to the probabilities calculated in this way.
Consider the following two examples.

Example 1: Suppose you invested 100 tokens in your project, while the other five participants in your group invested 150 tokens, 80 tokens, 100 tokens, 120 tokens, and 250 tokens, respectively. Then, the sum of the tokens invested in projects by all participants in your group will be $(100+150+80+100+120+250) = 800$ tokens. The probability you are the winner is then

$$\frac{100}{800} = \frac{1}{8} = 0.125 = 12.50\%$$

Example 2: For this example, suppose you invested 300 tokens in your project, while the other five participants in your group invested 20 tokens, 30 tokens, 0 tokens, 200 tokens, and 50 tokens, respectively. Then, the sum of the tokens invested in projects by all participants in your group will be $(300 + 20 + 30 + 0 + 200 + 50) = 600$ tokens. The probability you are the winner is then

$$\frac{300}{600} = \frac{1}{2} = 0.5 = 50.00\%$$

Your earnings

In each decision round, the winner will receive a prize of **500 tokens**. All participants (including the winner) must pay their project investments.

In addition, the earnings for each participant who is **connected to the winner** will be changed by X tokens. In general, X can be positive, negative, or zero.

Thus, your earnings in a given round are determined as follows:

If you are the winner:

+800 (endowment)
+500 (prize)
− (tokens you invested)

1300 − **(tokens you invested)**

If you are not the winner:

but are connected to the winner:

+800 (endowment)
+0 (no prize)
$+X$ (change in earnings)
− (tokens you invested)

800 + X − **(tokens you invested)**

and are not connected to the winner:

+800 (endowment)
+0 (no prize)
+0 (no change in earnings)
− (tokens you invested)

800 − **(tokens you invested)**

Example 3: Suppose you are the winner and your project investment was 100 tokens. Then your earnings for the round will be $1300 - 100 = 1200$ tokens.

Alternatively, suppose you are not the winner, and you **ARE NOT** connected to the winner. If your project investment was 100 tokens, then your earnings for the round will be $800 - 100 = 700$ tokens.

Finally, suppose you are not the winner, but that you **ARE** connected to the winner. Moreover, suppose $X = +200$. That is, the earnings of each player connected to the winner are **<u>increased</u>** by 200 tokens. If your project investment was 100 tokens, then your earnings for the round will be $800 + 200 - 100 = 900$ tokens.

If, instead, $X = -200$, the earnings of each player connected to the winner are **<u>decreased</u>** by 200 tokens. Thus, if your project investment was 100 tokens, your earnings for the round will be $800 - 200 - 100 = 500$ tokens.

Control Questions

In a moment, you will be asked to complete some control questions shown on the screen. These questions are only to help you understand the instructions - they will not affect your earnings. After several minutes, we will walk through the answers together, then move on to the next set of questions. After these are completed, we will continue with the instructions.

Feedback

After all participants have made their decisions, you will be shown the individual project investments for each participant in your group, the sum of all tokens invested in projects by participants in your group, and your probability of winning. Then, after the program determines the winner, the screen will display the position of the winner, whether or not you are connected to the winner, and a calculation of your earnings for the round.

Summary

Part 2 will consist of 10 decision rounds. In each round, you and the other participants in your group will choose project investments. The probability that your project wins depends on the share of your own project investment out of the total number of tokens invested by all participants in your group. Only one participant can be the winner in a given round. All participants must pay their project investments out of the endowment (800 tokens). The winner will receive a prize of 500 tokens. For any participant who does not win, but is connected to the winner, earnings will be changed by X tokens.

As a reminder, in the network graph shown on the screen, your position will be shown in red. The positions of the players with whom you are connected will be shown in yellow (in addition to being linked with your position by an edge). The positions of players who you are not connected to (if there are any) will be shown in black.

In Part 2, $X = 0$ for all 10 decision rounds. That is, the earnings for a participant who does not win, but is connected to the winner will not

be adjusted.
To make this clear, your earnings in any decision round will be given by:

1300 – (tokens you invested)	if you are the winner,
800 – (tokens you invested)	if you are not the winner, but are connected to the winner
800 – (tokens you invested)	if you are not the winner, and are not connected to the winner

At the end of the experiment, you will be paid for **one** randomly chosen decision round from Part 2. Each of the 10 decision rounds in this part is equally likely to be selected.

Part 3 Instructions

The instructions for Part 3 are almost identical to the instructions for Part 2. However, Part 3 will consist of a sequence of 15 decision rounds. Your group, the network graph, and your position will be the same as in Part 2.

In Part 3, $X = -400$ **for all 15 decision rounds. That is, the earnings for a participant who does not win, but is connected to the winner will be decreased by 400 tokens.**
To make this clear, your earnings in any decision round will be given by:

1300 – (tokens you invested)	if you are the winner,
400 – (tokens you invested)	if you are not the winner, but are connected to the winner
800 – (tokens you invested)	if you are not the winner, and are not connected to the winner

At the end of the experiment, you will be paid for **one** randomly chosen decision round from Part 3. Each of the 15 decision rounds in this part is equally likely to be selected.

Part 4 Instructions

The instructions for Part 4 are almost identical to the instructions for Part 3. Part 4 will also consist of a sequence of 15 decision rounds. Your group, the network graph, and your position will be the same as in Parts 2 and 3.

In Part 4, $X = +400$ **for all 15 decision rounds. That is, the earnings for a participant who does not win, but is connected to the winner will be increased by 400 tokens.**
To make this clear, your earnings in any decision round will be given by:

1300 – (tokens you invested)	if you are the winner,
1200 – (tokens you invested)	if you are not the winner, but are connected to the winner
800 – (tokens you invested)	if you are not the winner, and are not connected to the winner

At the end of the experiment, you will be paid for **one** randomly chosen decision round from Part 4. Each of the 15 decision rounds in this part is equally likely to be selected.

Part 5 Instructions

The instructions for Part 5 are **exactly** identical to the instructions for Part 2. Thus, it will consist of a sequence of 10 decision rounds. Your group, the network graph, and your position will be the same as in Parts 2, 3, and 4.

In Part 5, as in Part 2, $X = 0$ for all 10 decision rounds. That is, the earnings for a participant who does not win, but is connected to the winner will not be adjusted.

To make this clear, your earnings in any decision round will be given by:

1300 – (tokens you invested)	if you are the winner,
800 – (tokens you invested)	if you are not the winner, but are connected to the winner
800 – (tokens you invested)	if you are not the winner, and are not connected to the winner

At the end of the experiment, you will be paid for **one** randomly chosen decision round from Part 5. Each of the 10 decision rounds in this part is equally likely to be selected.

Part 6 Instructions

This part of the experiment consists of a single decision round. The basic setup is similar to the setup for Parts 2, 3, 4, and 5.

Before the round begins, you will be randomly rematched into a new group of 6 participants. In addition, there is no network graph connecting the participants for this part. However, you will still be randomly assigned a letter ID from A to F.

You and the other participants in your group will be given an endowment of 800 tokens each and asked to choose project investments. As in previous parts, the probability that your project wins depends on the share of your own project investment out of the total number of tokens invested by all participants in your group. All participants must pay their project investments out of the endowment.

There are two main differences from previous parts. The first is that **in this part, the winner will receive a prize of <u>0 tokens</u>.** The second is that, since there is no network graph connecting participants, **there is no adjustment** X to be made to the earnings of participants who are connected to the winner.

To make this clear, your earnings for this part (1 decision round only) will be given by:

800 – (tokens you invested)	if you are the winner,
800 – (tokens you invested)	if you are not the winner

After all participants have made their decisions, you will be shown the individual project investments for each participant in your group, the sum of all tokens invested in projects by participants in your group, and your probability of winning. Then, after the program determines the winner, the screen will display the letter ID of the winner, whether or not that is you, and a calculation of your earnings.