Entry and disclosure in group contests

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Abstract

We study information disclosure policies for contests among groups. Each player endogenously decides whether or not to participate in competition as a member of their group. Within-group aggregation of effort is best-shot, i.e., each group's performance is determined by the highest investment among its members. We consider a generalized all-pay auction setting, in which the group with the highest performance wins the contest with certainty. Players' values for winning are private information at the entry stage, but may be disclosed at the competition stage. We compare three disclosure policies: (i) no disclosure, when the number of entrants remains unknown and their values private; (ii) within-group disclosure, when this information is disclosed within each group but not across groups; and (iii) full disclosure, when the information about entrants is disclosed across groups. For the benchmark case of contests between individuals, information disclosure always reduces expected aggregate investment. However, this is no longer true in group contests: Within-group disclosure unambiguously raises aggregate investment, while the effect of full disclosure is ambiguous.

Keywords: group contest, best shot, endogenous entry, information disclosure
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1 Introduction

Across a wide range of economic, political, and social environments, competition takes place between groups of individuals who align themselves in pursuit of some common goals. For example, many interest groups engaged in lobbying activities consist of diverse collections of individuals with broadly aligned objectives. Within organizations, managers may solicit project submissions from multiple divisions or teams of employees and reward the team with the best proposal. Crowdsourcing R&D platforms such as the XPRIZE Foundation invite groups to compete for prizes solving complex innovation problems. Such competitive settings can be broadly categorized as *group contests*. In each group, individuals invest effort or other resources to increase their group's performance, which in turn improves the group's chances of winning a valuable prize.

In this paper, we study competition between groups in which the decision to enter competition as a member of a particular group is endogenous. In our setting, there are a fixed number of competing groups, each with a pool of *potential* participants. In an initial entry stage, each player decides whether or not to participate in competition as a member of their group. Participants must forgo some outside option or, equivalently, face a cost of entry. Then, in a second stage, participants make investment or effort decisions that determine the group's performance level. We analyze and compare three *disclosure policies* that dictate the information available to participants at the time they make their investments.

We consider activities in which group performance is determined solely by the *best-shot*, i.e., the maximum investment chosen by a group member (Baik and Shogren, 1998; Chowdhury, Lee and Sheremeta, 2013).[1] Such a setting may arise, for example, in lobbying environments where the official who is lobbied considers only the individual agents on each side of an issue who made the most compelling case; or within an organization where a team of employees pitches only the most promising project idea to the management. One can think also of a market setting where a client, such as a real-estate developer, solicits projects from multiple firms, and each firm conducts an internal selection process and presents its best proposal.

We model group competition as a generalized all-pay auction (Baye, Kovenock and De Vries, 1996; Siegel, 2009) in which the group with the highest performance (the highest

[1]Other popular approaches are to model the group performance level using perfect substitutes aggregation technology (Katz, Nitzan and Rosenberg, 1990; Baik, 1993, 2008; Baik, Kim and Na, 2001; Münster, 2009) or weak-link (perfect complements) technology (Lee, 2012). Kolmar and Rommeswinkel (2013) and Brookins, Lightle and Ryvkin (2015) consider varying degrees of complementarity with a CES technology.

best-shot) wins the contest with certainty.[2] Moreover, we assume that the prize associated with winning is a group-specific public good. Thus, the value of winning for each member of the winning group is equal to her individual private value, regardless of the size of the group.[3] Together, these features of the environment generate incentives for individuals within groups to free-ride on the investments made by their fellow group members. These free-riding incentives underscore one of the key differences between contests among groups and contests among individuals. As such, the environment we consider provides a rich and previously unexplored interplay between endogenous entry, free riding and information disclosure in contests.

The game proceeds in two stages. At the initial entry stage, players' values for the prize are private information, although it is common knowledge that they are drawn independently from the same commonly known distribution. Depending on the disclosure policy, information about the number of participating group members and their values may become known to participants at the beginning of the second stage, prior to their investment decisions.[4] In the second stage, participants simultaneously choose an irreversible, costly investment, and the outcome of the contest is determined.

We first consider, as a benchmark, the case of contests among individuals and show that expected aggregate investment is always lower when information about others' types is disclosed to those who enter. In this case, equilibrium entry is independent of disclosure. However, equilibrium aggregate investment corresponds to the (truncated) expectation of the second highest valuation without disclosure; and to the sum of the expected bids by the two highest valuation entrants in a complete information all-pay auction, with disclosure. The latter entails an efficiency loss because the lower valuation player can win, and a loss of revenue because the lower valuation player can bid zero, with a positive probability. There is also no scope for free riding or a coordination problem disclosure may alleviate; thus, nondisclosure leads to a larger investment.

We then turn to contests among groups and broaden our analysis to consider three different disclosure rules, each approximating prominent real-world information structures.

[2]This setting corresponds to a perfectly discriminating contest success function (CSF). Alternative environments, in which the contest is imperfectly discriminating, include those with a lottery CSF of Tullock (1980) and its generalizations. In these settings, the group with the highest performance has a higher probability of winning, but does not win with certainty.

[3]Note that *potential members* of the winning group who do not participate do not receive any benefit from the prize being awarded to the group they could have joined. This is the case, for example, for researchers deciding whether or not to join their colleagues on a grant application, politicians joining various caucuses or factions, or (potential) plaintiffs in group litigation.

[4]We assume this disclosure of information is exogenous. It may be the effect of explicit design decisions, for example, by a contest sponsor; or it may be a result of naturally occurring dissemination of information due to environmental factors, such as the spatial or network structure of agents' interactions.

In our "no disclosure" (ND) setting, all entry decisions and valuations remain private information throughout the investment stage. Thus, entrants face incomplete information regarding the number of entrants in their own group and in other groups, as well as about others' valuations for the prize. This situation occurs, for example, in large organizational settings that frequently re-assign workers to new project teams, or settings with little to no interaction between personnel (e.g., telecommuting workers or disjoint work schedules). In the "full disclosure" (FD) setting, entrants are informed, prior to making investment decisions, about the number and valuations of all entrants. This condition is applicable, for example, to online crowdsourcing competitions (e.g., TopCoder.com and Kaggle.com) whereby "leaderboards" continuously and publicly display team information, such as the number of team members and their skill levels, player bios and past accomplishments.[5] In the third setting, which we call "within-group disclosure" (WD), entrants are informed about the number and valuations of all entrants within their own group, but do not learn any information about the entrants in other groups (Brookins and Ryvkin, 2016). This information structure is found in many naturally occurring "blind" competitive settings that only announce competitor information (e.g., background, performance, experience and ability) at the end of the contest stage, such as competition for federal research grants.

We show that, compared to the ND setting, expected aggregate investment is unambiguously *higher* in the WD setting, where information disclosure is restricted to be within groups. The key intuition for this result is that entrants who learn about types within their group are able to solve the coordination problem that arises without disclosure, alleviating the impact of free riding within groups. Equilibrium entry does not differ between the ND and WD settings. However, the reduction in free riding is sufficiently strong so that the expected aggregate investment supplied by the highest types in each group in the WD setting exceeds not only the same but even the sum of all entrants' investments in the ND case.

Turning to full disclosure (FD), we show that the effect of disclosure on expected aggregate investment in contests among individuals is often *reversed* in contests between groups. While in individual contests FD unambiguously leads to a lower expected aggregate investment than ND (or WD, which is equivalent to ND in this case), in group contests aggregate investment may be higher or lower in the FD setting than under either of the other two disclosure rules. When entrants are informed about the number and types of entrants in all groups, the contest collapses to an individual all-pay auction of complete information among the groups' "leaders." In this setting, we first prove that

[5]See, e.g., the following leaderboard on Kaggle.com: https://www.kaggle.com/c/data-science-bowl-2018/leaderboard.

there is more entry under FD than under ND and WD. However, as in a typical all-pay auction equilibrium, at most two highest types among the group leaders actively invest with probability one. As a consequence, it is possible for expected aggregate investment to be lower under FD than under ND. Nevertheless, provided the distribution of types is sufficiently elastic and the (expected) group size is large enough, the distributions of the top two order statistics among the groups' best entrants are shifted towards the upper bound of the type space, which leads the two active entrants to invest more aggressively. As a result, when the distribution of types satisfies this (sufficient) elasticity condition, full disclosure also increases expected aggregate investment compared to the setting with no disclosure.

The rest of the paper proceeds as follows. In Section 2, we review related literature. Section 3 describes the model. The benchmark case of contests among individuals is considered in Section 4, and our main results and numerical illustrations for group contests are presented in Section 5. Section 6 concludes. All missing proofs are collected in Appendix A.

2 Related literature

A fundamental difference between group contests and individual contests is that players in the former typically face some incentive to free-ride on the investments made by other group members. In general, the impacts of such free-riding incentives depend on the size of the group, the type of within-group investment aggregation technology, and on the information available to the participants. Moreover, informational conditions may affect the decision to participate in the first place. Recent years have seen a revival of interest in contests with endogenous entry and in the effect of disclosing the number of entrants. However, virtually all of the existing research focuses on contests among individuals. As such, one of our goals in this paper is to explore the interaction between endogenous entry and free-riding incentives that is unique to group contests.

Our general setting is closely related to the literature on contests in which the prize is a group-specific public good (see, e.g., Katz, Nitzan and Rosenberg, 1990; Baik, 1993; Chowdhury, Lee and Sheremeta, 2013; Kolmar and Rommeswinkel, 2013; Barbieri and Malueg, 2016; Eliaz and Wu, 2018; Barbieri et al., 2019).[6] Only a handful of papers in this literature consider the setup with private information (Fu, Lu and Pan, 2015; Barbieri and Malueg, 2016; Brookins and Ryvkin, 2016; Eliaz and Wu, 2018; Barbieri et al., 2019).

[6] There is also a large literature on group contests where the prize is a private good that must be divided between the members of the winning group. For example, see Nitzan (1991); Lee (1995); Skaperdas (1998); Wärneryd (1998); Konrad and Leininger (2007); Münster (2007) and Nitzan and Ueda (2009, 2011).

The baseline features of our group contest environment are most similar to Barbieri and Malueg (2016). As in their model, we consider a setting where players' values for winning are private information, group performance is determined by the "best-shot" of its members, and the highest performing group wins the contest. The focus of the analysis in Barbieri and Malueg (2016) is on the comparative statics of individual, group, and aggregate investment, and of the equilibrium probability of winning, as the size of the groups and the number of groups are varied. In contrast, our goals in the current paper are to understand competition when the decision to join a group is endogenous, and to compare equilibrium behavior across different information disclosure policies. To this end, we incorporate into the model two features that have previously been studied only in contests and auctions among individuals. First, we allow for endogenous entry by potential group members prior to the investment stage. Second, we vary the information disclosure that takes place between the entry stage and the investment stage regarding group size and participants' types, which facilitates a comparison between the analyses for the complete information and incomplete information environments.

While our study is, to the best of our knowledge, the first to consider endogenous entry into competing *groups*, there is substantial previous work on *individual* contests and auctions with endogenous entry. In all-pay contest environments, key theoretical insights regarding entry are provided by Higgins, Shughart and Tollison (1985); Gradstein (1995); Fu and Lu (2010); Kaplan and Sela (2010) and Fu, Jiao and Lu (2015). Similarly, in the auction literature, endogenous entry has been modeled and analyzed by Levin and Smith (1994) and Pevnitskaya (2004).[7] In addition, much of the recent research on contest design has focused on the impact or optimality of different information disclosure policies. For instance, Lim and Matros (2009) and Fu, Jiao and Lu (2011) consider the effect of disclosing the number of actual participants in contests with stochastic entry. They show that the disclosure policy is irrelevant for expected total effort in Tullock contests. However, Fu, Jiao and Lu (2011) further show that in contests with a more general "ratio-form" CSF, the optimal disclosure rule depends on the shape of the CSF's impact function. Recent work by Ryvkin and Drugov (2020) generalizes these results by showing that the effect of disclosure in a general tournament model depends on the

[7]There is also a considerable amount of work that examines exogenous, or stochastic, entry into contests (see, e.g., Münster, 2006; Myerson and Wärneryd, 2006; Lim and Matros, 2009; Fu, Jiao and Lu, 2011; Kahana and Klunover, 2015, 2016; Ryvkin and Drugov, 2020) and into auctions (see, e.g., McAfee and McMillan, 1987; Harstad, Kagel and Levin, 1990; Levin and Ozdenoren, 2004). For experimental evidence related to both exogenous and endogenous entry, see Anderson and Stafford (2003); Eriksson, Teyssier and Villeval (2009); Morgan, Orzen and Sefton (2012); Morgan et al. (2016); Hammond et al. (2019); Boosey, Brookins and Ryvkin (2017, 2020); Aycinena and Rentschler (2019) in relation to contests, and Dyer, Kagel and Levin (1989); Ivanova-Stenzel and Salmon (2004); Isaac, Pevnitskaya and Schnier (2012); Palfrey and Pevnitskaya (2008); Aycinena and Rentschler (2018) in relation to auctions.

curvature of the cost function of effort.[8] For an all-pay auction environment, Chen, Jiang and Knyazev (2017) show that compared with full concealment, disclosing the number of actual participants in an all-pay auction with private values decreases the expected total investment if and only if the participants' cost functions are concave.

In all of the aforementioned studies, disclosure relates exclusively to the *number* of entrants in the contest. Yet, there are also several studies that explore the impact of disclosing participants' (initially private) valuations on expected total investment. For the standard single-item all-pay auction environment, Morath and Münster (2008) establish that expected total effort is lower under complete information (i.e., with disclosure) than under private information (i.e., without disclosure). Their result is generalized to a contest with multiple prizes by Fu, Jiao and Lu (2014).[9] Feng (2023) considers an all-pay auction with entry, but also restricts attention to one dimension of disclosure (valuations), while Zhang and Zhou (2016) employ a Bayesian persuasion approach to show that, in general, contest designers may benefit by adopting a policy of partial disclosure.

To conclude, the existing literature has something to say about the effects of endogenous entry and disclosure in individual contests, and the main contribution of this paper is our extension of the analysis of these phenomena to contests among groups. The only other study that we are aware of that considers disclosure of the number of entrants in group contests is by Boosey, Brookins and Ryvkin (2019), who examine Tullock contests among groups with stochastic group sizes and players with a common (and publicly known) prize valuation. In this paper, we consider the effects of disclosing both the number of entrants *and* their private valuations. The structure of competition among groups also allows us to examine within-group disclosure—a particular form of partial disclosure, in which individuals learn about the number and valuations of entrants in their own group, but not of those in other groups.

3 Model setup

There are $n \geqslant 2$ groups, indexed by $i = 1, \ldots, n$, with $m \geqslant 1$ players in each group, indexed by $ij = i1, \ldots, im$. The players are risk neutral expected payoff maximizers. Each player ij is endowed with prize valuation v_{ij}, which is initially the player's private information, drawn independently from a commonly known distribution with interval support $V = [\underline{v}, \overline{v}] \subseteq \mathbb{R}_+$, absolutely continuous cdf $F(\cdot)$ and continuous, positive a.e. pdf

[8] See also Fu, Lu and Zhang (2016), who study a generalized Tullock contest with two players who are asymmetric in terms of both their values and their stochastic entry probabilities.

[9] For two other studies that explore a slightly richer set of disclosure policies, see Lu, Ma and Wang (2018) and Serena (2022).

$f(\cdot)$.

The game consists of two stages. In the first stage, each player ij decides whether or not to enter the group contest as a member of group i. The players who decide to stay out receive an outside option payoff $\omega \in \mathbb{R}_+$. The entrants proceed to the second stage. Let $M_i \subseteq \{i1, \ldots, im\}$ denote the set of entrants in group i. In the second stage, depending on a disclosure condition, some information may be revealed to entrants, after which they choose their investment levels $x_{ij} \in \mathbb{R}_+$. Group i's aggregate investment is determined by the best-shot technology as $X_i = \max_{ij \in M_i} x_{ij}$. The group with the highest investment wins the contest, and all entrants in that group receive their valuations. Entrants in all other groups receive zero. Ties are broken randomly, but occur with probability zero in equilibrium. All entrants pay their investments.

Information disclosure We consider three information settings implementing different modes of information disclosure at the beginning of the second stage, before entrants make their investment decisions.

(i) No disclosure (ND): Entrant ij knows only v_{ij} (observed prior to entry).

(ii) Within-group disclosure (WD): Entrant ij observes v_{ik} for all $ik \in M_i$.

(iii) Full disclosure (FD): Entrant ij observes v_{lk} for all $lk \in \cup_{l=1}^{n} M_l$.

In the ND setting, no new information is revealed between the stages, and the game effectively collapses into one stage. In the case of within-group disclosure (WD), entrants observe others' valuations within their own groups. Finally, in the FD setting all entrants' valuations become public information. In cases (ii) and (iii) it is implied that entrants observe also the *number* of other entrants in their own groups and in all groups, respectively.

Analysis We look for a symmetric cutoff equilibrium in which there is a valuation $v^* \in V$ such that a player with valuation v enters the contest if and only if $v \in V^* = [v^*, \overline{v}]$.[10] We assume that $\omega < \overline{v}$ so that at least some entry occurs with positive probability. Let $q = 1 - F(v^*)$ denote the *ex ante* probability of entry. Further, let $\tilde{F}(v) = \frac{F(v) - F(v^*)}{q} \mathbb{1}_{v \geqslant v^*}$ and $\tilde{f}(v) = \frac{f(v)}{q} \mathbb{1}_{v \in V^*}$ denote the updated cdf and pdf of entrants' valuations.

Throughout our main analysis, we (implicitly) consider a principal whose objective is to maximize the *expected aggregate investment* of all groups, $\mathbb{E}(\sum_{i=1}^{n} X_i)$. This objective is standard in the literature and is suitable for an organizational setting where the manager's goal is to incentivize investments from all teams. In Section 5.4, we extend our analysis to consider two alternative objectives: *expected total investment*, $\mathbb{E}(\sum_{i=1}^{n} \sum_{ij \in M_i} x_{ij})$, and

[10] If $v^* > \underline{v}$ then, by continuity, the marginal player with $v = v^*$ is indifferent between entering and staying out. For concreteness, and without loss, we assume that such a player enters.

expected highest investment, $\mathbb{E}(\max_{i\in\{1,\dots,n\}} X_i)$. The former describes a setting where the manager cares about the effort of all employees, even those whose proposals are dominated by others within their teams. The latter, at the other end of the spectrum, is more relevant for innovation contests where the principal only cares about the highest quality proposal overall, which will eventually be implemented.

4 Contests among individuals ($m = 1$)

As an important benchmark setting, we first consider contests among individuals. Within-group disclosure always takes place in this case, by definition, and is equivalent to no disclosure; therefore, we only compare no disclosure and full disclosure. Observe that the cutoff valuation, v^*, is independent of disclosure. Indeed, without disclosure the marginal player will invest zero and can only win the contest if she is the only entrant. In all other cases, she will lose with probability one and earn zero payoff. Under (full) disclosure, the marginal player will also invest zero and win if she is the only entrant. If there are several entrants, the investment stage game is an all-pay auction of complete information, with equilibrium in mixed strategies (see, e.g., Baye, Kovenock and De Vries, 1996). However, the marginal player's valuation will be the lowest with probability one; therefore, in equilibrium she will earn zero in expectation.

Thus, the cutoff type, v^*, is determined by the indifference condition equating the payoff of the marginal player when she is the only entrant to the outside option:

$$v^* F(v^*)^{n-1} = \omega. \tag{1}$$

Under our assumption that $\omega < \overline{v}$, Eq. (1) has a unique solution.

No disclosure Let $b^{(1)}_{\mathrm{ND}}(v)$ denote the symmetric monotone bidding function for entrants (here and in what follows, subscript "ND" stands for nondisclosure; superscript "(1)" distinguishes contests between individuals). Using the standard approach, consider an entrant with valuation $v \in V^*$ bidding as if her valuation is $\hat{v} \in V^*$. The entrant's payoff is then given by $\Pi^{(1)}(v, \hat{v}; v^*) = v p^{(1)}_{\mathrm{ND}}(\hat{v}) - b_{\mathrm{ND}}(\hat{v})$, where the probability of winning is

$$p^{(1)}_{\mathrm{ND}}(\hat{v}) = (1-q)^{n-1} + \sum_{k=1}^{n-1} \binom{n-1}{k} q^k (1-q)^{n-1-k} \tilde{F}(\hat{v})^k. \tag{2}$$

The first term represents the situation when there are no other entrants, while the second term sums over all possible numbers of other entrants $k = 1, \dots, n-1$. The first-order condition $\Pi^{(1)}_{\hat{v}}(v,v;v^*) = 0$ produces a differential equation for the unknown bidding function $b^{(1)}_{\text{ND}}(v)$, which, together with the initial condition $b^{(1)}_{\text{ND}}(v^*) = 0$, gives

$$b^{(1)}_{\text{ND}}(v) = (n-1)\int_{v^*}^{v} tF(t)^{n-2}\mathrm{d}F(t). \tag{3}$$

Following the standard argument, we verify that $\Pi^{(1)}_{\hat{v}}(v,\hat{v};v^*)$ changes sign at $\hat{v} = v$ in a way that makes $b^{(1)}_{\text{ND}}(v)$ given by (3) the unique symmetric equilibrium bidding function for a given v^*. Equation (3) has exactly the same form as the equilibrium bidding function in independent private value all-pay auctions without entry (see, e.g., Krishna and Morgan, 1997), except it is shifted down by a constant and truncated at v^*. The effect of endogenous entry is contained entirely in v^*.

Expected aggregate investment under no disclosure, therefore, is

$$B^{(1)}_{\text{ND}} = nq\int_{v^*}^{\overline{v}} b^{(1)}_{\text{ND}}(v)\mathrm{d}\tilde{F}(v) = n(n-1)\int_{t_1\geqslant t_2\geqslant v^*} t_2F(t_2)^{n-2}\mathrm{d}F(t_1)\mathrm{d}F(t_2). \tag{4}$$

Full disclosure Suppose there are $k \geqslant 2$ entrants,[11] and let $v_{(1)} > v_{(2)} \ldots > v_{(k)}$ denote their ranked valuations (ties in valuations are probability zero events; therefore, we can generically assume the strict inequalities). In equilibrium, the entrants with valuations $v_{(1)}$ and $v_{(2)}$ bid according to mixed strategies with common support $[0, v_{(2)}]$ and cdfs $G_1(x_1) = \frac{x_1}{v_{(2)}}$ and $G_2(x_2) = 1 - \frac{v_{(2)}}{v_{(1)}} + \frac{x_2}{v_{(1)}}$, respectively, while all other entrants bid zero. The corresponding probabilities of winning are $p_1 = 1 - \frac{v_{(2)}}{2v_{(1)}}$ and $p_2 = \frac{v_{(2)}}{2v_{(1)}}$. Average bids are $b_1 = \frac{v_{(2)}}{2}$ and $b_2 = \frac{v^2_{(2)}}{2v_{(1)}}$. Finally, the expected payoffs are $\pi_1 = v_{(1)} - v_{(2)}$ and $\pi_2 = 0$, respectively (Baye, Kovenock and De Vries, 1996).

Expected aggregate investment under full disclosure, therefore, is

$$B^{(1)}_{\text{FD}} = \sum_{k=2}^{n}\binom{n}{k}q^k(1-q)^{n-k}\int_{t_1\geqslant t_2\geqslant v^*}\left(\frac{t_2}{2}+\frac{t_2^2}{2t_1}\right)\tilde{f}_{(1,2:k)}(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2,$$

where $\tilde{f}_{(1,2:k)}(t_1,t_2)$ is the joint pdf of the top two order statistics in a sample of size k from distribution $\tilde{F}(\cdot)$. This pdf is given by (David and Nagaraja, 2003)

$$\tilde{f}_{(1,2:k)}(t_1,t_2) = k(k-1)\tilde{F}(t_2)^{k-2}\tilde{f}(t_1)\tilde{f}(t_2)\mathbb{1}_{t_1\geqslant t_2},$$

[11] States with one or zero entrants contribute nothing to aggregate investment.

resulting in

$$B_{\mathrm{FD}}^{(1)} = n(n-1)\int_{t_1 \geqslant t_2 \geqslant v^*} \left(\frac{t_2}{2} + \frac{t_2^2}{2t_1}\right) F(t_2)^{n-2} \mathrm{d}F(t_1)\mathrm{d}F(t_2). \tag{5}$$

Comparing (4) and (5), we observe that $B_{\mathrm{FD}}^{(1)} < B_{\mathrm{ND}}^{(1)}$. This gives our first result.

Proposition 1 *In contests among individuals, expected aggregate investment under (full) disclosure is lower than under no disclosure:* $B_{\mathrm{FD}}^{(1)} < B_{\mathrm{ND}}^{(1)}$.

Proposition 1 is an important benchmark result that serves as a motivation for what follows. It shows that in contests among individuals the disclosure of types has an unambiguous negative effect on aggregate investment. It holds for any cutoff valuation v^*, which includes contests where the number of players is fixed ($v^* = \underline{v}$), generalizing the result of Morath and Münster (2008). It also holds when the number of players is stochastic following an exogenous distribution.

The mechanism behind Proposition 1 is as follows. Without disclosure, the allocation of the prize is efficient and aggregate investment is given by the expectation of the second highest valuation, $v_{(2)}$—the same as in other revenue-equivalent auctions, such as the first-price or second-price auction. The expectation is truncated due to endogenous entry, but since the cutoff type is independent of disclosure, this truncation is irrelevant. Under disclosure, the efficiency is lost; *and* the player with valuation $v_{(2)}$ bids zero with probability $1 - \frac{v_{(2)}}{v_{(1)}}$, and hence her contribution to aggregate investment is reduced ($\frac{v_{(2)}^2}{2v_{(1)}} < \frac{v_{(2)}}{2}$ with probability one).

5 Contests among groups ($m \geqslant 1$)

In this section, we consider contests among groups, starting with the no disclosure (ND) case in Section 5.1. In Section 5.2, we then consider within-group disclosure (WD) and compare it to ND. In Section 5.3, we characterize full disclosure (FD) and compare it to both ND and WD. Finally, in Section 5.4 we consider the impact of disclosure policies on two alternative objectives of the principal—expected total investment and expected highest investment.

5.1 No disclosure

Let $b_{\mathrm{ND}}(v)$ denote the symmetric monotone bidding function for entrants. Again following the standard approach, consider an entrant with valuation $v \in V^*$ bidding according to a valuation $\hat{v} \in V^*$. The entrant's payoff is $\Pi(v, \hat{v}; v^*) = v p_{\text{ND}}(\hat{v}) - b_{\text{ND}}(\hat{v})$, where the probability of winning is

$$p_{\text{ND}}(\hat{v}) = (1-q)^{nm-m} + \sum_{k_1=0}^{m-1} \sum_{k_2=1}^{nm-m} \binom{m-1}{k_1} \binom{nm-m}{k_2} q^{k_1+k_2} (1-q)^{nm-1-k_1-k_2} \times \\ \times \left[\tilde{F}(\hat{v})^{k_1+k_2} + (1 - \tilde{F}(\hat{v})^{k_1+k_2}) \frac{k_1}{k_1+k_2} \right]. \tag{6}$$

The first term represents the situation when there are no entrants in other groups. The second term sums over all possible configurations of the numbers of entrants in the player's own group (k_1) and other groups (k_2). The first term in square brackets is the probability that the player's valuation (and hence the bid) is the highest of them all, while the second term is the "free-riding component" where the player's valuation is not the highest but she nevertheless wins because someone else in her group has the highest valuation.

The first-order condition $\Pi_{\hat{v}}(v, v; v^*) = 0$ produces a differential equation for the unknown bidding function $b_{\text{ND}}(v)$, which, together with the initial condition $b_{\text{ND}}(v^*) = 0$, gives

$$b_{\text{ND}}(v) = m(n-1) \int_{v^*}^{v} t F(t)^{nm-2} \mathrm{d}F(t). \tag{7}$$

The details of the derivation are provided in Appendix A. We again verify that $\Pi_{\hat{v}}(v, \hat{v}; v^*)$ changes sign at $\hat{v} = v$ in a way that makes $b_{\text{ND}}(v)$ given by (7) the unique symmetric equilibrium bidding function for a given v^*. Similar to individual contests, Eq. (7) has exactly the same form as the equilibrium bidding function in a group contest with a fixed number of players (Barbieri and Malueg, 2016), except it is shifted down by a constant and truncated at v^*.

In order to identify v^*, suppose $v^* \in \text{int}(V)$ and hence the cutoff type is indifferent between entering and not entering. Her payoff from entry is $\Pi(v^*, v^*; v^*) = v^* p_{\text{ND}}(v^*)$, where

$$p_{\text{ND}}(v^*) = (1-q)^{nm-m} + \sum_{k_1=0}^{m-1} \sum_{k_2=1}^{nm-m} \binom{m-1}{k_1} \binom{nm-m}{k_2} q^{k_1+k_2} (1-q)^{nm-1-k_1-k_2} \frac{k_1}{k_1+k_2} \\ = \frac{m-1}{nm-1} + \frac{m(n-1)}{nm-1} F(v^*)^{nm-1}. \tag{8}$$

For details, see Appendix A. It follows that $v^* = \underline{v}$, i.e., there is full entry, if $\omega \leqslant \frac{m-1}{nm-1}\underline{v}$. Otherwise, the cutoff is given by the unique solution of the equation $v^* p_{\text{ND}}(v^*) = \omega$.[12]

[12] As seen from (8), $p_{\text{ND}}(v)$ is continuous and (weakly) increasing in V and hence $v p_{\text{ND}}(v)$ is continuous and strictly increasing.

Expected aggregate investment in each group is given by the expectation of the maximum bid among entrants, which, together with (7), gives expected aggregate investment in the contest:

$$B_{\mathrm{ND}} = n\sum_{k=0}^{m}\binom{m}{k}q^k(1-q)^{m-k}\int_{v^*}^{\overline{v}} b_{\mathrm{ND}}(v)\mathrm{d}\tilde{F}(v)^k = n\int_{v^*}^{\overline{v}} b_{\mathrm{ND}}(v)\mathrm{d}F(v)^m$$

$$= m^2 n(n-1)\int_{t_1\geqslant t_2\geqslant v^*} t_2 F(t_2)^{nm-2}F(t_1)^{m-1}\mathrm{d}F(t_1)\mathrm{d}F(t_2). \tag{9}$$

5.2 Within-group disclosure

In this setting, valuations of entrants are revealed within each group before the group members decide on their investments. While it is clear that only one of the entrants within each group will be active in any (pure strategy) equilibrium, multiple such equilibria are possible, with different entrants being active. We assume that groups coordinate so that the *leaders*—the entrants with the highest valuations—are the active bidders. This assumption is quite reasonable in the case of best-shot aggregation where the most capable group member is the natural leader. It is also the only equilibrium that is parallel to the one arising in the setting without disclosure where each group's bid is by construction determined by the highest-valuation entrant.[13]

We again look for a symmetric cutoff entry equilibrium with some marginal type v^*. Let $b_{\mathrm{WD}}(v)$ denote the monotone bidding function of leaders in each group (subscript "WD" stands for within-group disclosure). Following the same steps as in Section 5.1, consider a leader with valuation $v \in V^*$ that is bidding as if her valuation is $\hat{v} \in V^*$. The leader's payoff is $\Pi(v, \hat{v}; v^*) = vp_{\mathrm{WD}}(\hat{v}) - b_{\mathrm{WD}}(\hat{v})$, where

$$p_{\mathrm{WD}}(\hat{v}) = (1-q)^{nm-m} + \sum_{k_2=1}^{nm-m}\binom{nm-m}{k_2}q^{k_2}(1-q)^{nm-m-k_2}\tilde{F}(\hat{v})^{k_2}.$$

Indeed, this leader's group wins if there are no entrants in other groups (the first term) or her valuation exceeds that of all leaders (or, equivalently, of all entrants) in all other groups (the second term). Solving the first-order condition $\Pi_{\hat{v}}(v, v; v^*) = 0$, we obtain

[13] As an alternative justification, suppose each group has a non-bidding manager who derives some value from the group winning the contest and is able to select one active bidder among the entrants in her group (subject to the entrants' participation constraints). Then each manager would select the entrant in her group with the highest valuation to be the active bidder. Similarly, this outcome would emerge if all entrants could select the active bidder via a binding majority voting procedure (with the highest valuation entrant appointed in the case of a tie when there are two entrants).

the bidding function

$$b_{\mathrm{WD}}(v) = m(n-1)\int_{v^*}^{v} tF(t)^{nm-m-1}\mathrm{d}F(t). \tag{10}$$

The optimality of $b_{\mathrm{WD}}(v)$ follows similar to Section 5.1. All entrants whose valuations are not the highest bid zero.

Consider now the payoff of the marginal type v^* in the case of entry. This type always bids zero and can win if there are no entrants in other groups or the leader in her group is the winner. Thus, the payoff of the marginal entrant is exactly the same as in the no disclosure case, and hence the equilibrium v^* under within-group disclosure is the same as the v^* under no disclosure identified in Section 5.1.

From (10), the expected aggregate investment by entrants is

$$\begin{aligned} B_{\mathrm{WD}} &= n\sum_{k=0}^{m}\binom{m}{k}q^k(1-q)^{m-k}\int_{v^*}^{\overline{v}} b_{\mathrm{WD}}(v)\mathrm{d}\tilde{F}(v)^k = n\int_{v^*}^{\overline{v}} b_{\mathrm{WD}}(v)\mathrm{d}F(v)^m \\ &= m^2n(n-1)\int_{t_1\geqslant t_2\geqslant v^*} t_2F(t_2)^{nm-m-1}F(t_1)^{m-1}\mathrm{d}F(t_1)\mathrm{d}F(t_2). \end{aligned} \tag{11}$$

Using Eqs. (9) and (11), the difference in expected aggregate investment between the within-group disclosure and no disclosure settings is[14]

$$\begin{aligned} &B_{\mathrm{WD}} - B_{\mathrm{ND}} \\ &= m^2n(n-1)\int_{t_1\geqslant t_2\geqslant v^*} t_2F(t_2)^{nm-m-1}F(t_1)^{m-1}\left[1-F(t_2)^{m-1}\right]\mathrm{d}F(t_1)\mathrm{d}F(t_2) \geqslant 0. \end{aligned}$$

Note that the inequality above is strict whenever $m > 1$, i.e., when within-group disclosure can actually reveal new information. Thus, we arrive at the following result.

Proposition 2 *For group contests with $m > 1$, expected aggregate investment under within-group disclosure is greater than under no disclosure, $B_{\mathrm{WD}} > B_{\mathrm{ND}}$.*

The difference stems from the fact that the leaders' identities in the no disclosure case are unknown; every entrant can be a leader with some probability, and there is always some probability that an entrant's bid will be wasted. With disclosure, the entrants are able to solve the coordination problem within groups and bid more effectively.

[14] Alternatively, observe from Eqs. (7) and (10) that $b_{\mathrm{WD}}(v) \geqslant b_{\mathrm{ND}}(v)$ for each v, i.e., the two individual bidding functions are clearly ranked pointwise. Since aggregate group investment comes from the highest valuation group member in both cases, the comparison between B_{ND} and B_{WD} follows immediately. We rely on this approach when we discuss the ranking of the expected highest investment in Section 5.4, but here we compute B_{ND} and B_{WD} directly to facilitate the comparisons with full disclosure in Section 5.3.

5.3 Full disclosure

In this setting, valuations of all entrants are revealed across groups. Similar to Section 5.2, we assume that within each group players coordinate so that the groups' leader—the highest-valuation entrant—is its (potentially) active bidder. In this case, the second stage effectively turns into an all-pay auction of complete information among the leaders. If there are entrants in two or more groups, the unique equilibrium (with probability one) involves two active group leaders with the highest valuations bidding according to mixed strategies while all other groups drop out.

We again look for a cutoff entry equilibrium with some marginal type v^*. The payoff of the marginal type from entry is $v^* p_{\mathrm{FD}}(v^*)$ (subscript "FD" stands for full disclosure), where

$$p_{\mathrm{FD}}(v^*) = (1-q)^{nm-m} + (n-1)\sum_{k_1=0}^{m-1}\sum_{k_2=1}^{m}\sum_{k_3=0}^{nm-2m}\binom{m-1}{k_1}\binom{m}{k_2}\binom{nm-2m}{k_3}\times$$
$$\times\, q^{k_1+k_2+k_3}(1-q)^{nm-1-k_1-k_2-k_3}\left[\int_{t_1\geqslant t_2\geqslant t_3\geqslant v^*}\left(1-\frac{t_2}{2t_1}\right)\mathrm{d}\tilde{F}(t_1)^{k_1}\mathrm{d}\tilde{F}(t_2)^{k_2}\mathrm{d}\tilde{F}(t_3)^{k_3}+\right.$$
$$\left.+\int_{t_2\geqslant t_1\geqslant t_3\geqslant v^*}\frac{t_1}{2t_2}\mathrm{d}\tilde{F}(t_1)^{k_1}\mathrm{d}\tilde{F}(t_2)^{k_2}\mathrm{d}\tilde{F}(t_3)^{k_3}\right]. \tag{12}$$

As before, the first term describes the case when there are no entrants in other groups. The triple summation goes over the possible numbers of other entrants in the marginal entrant's own group (k_1), entrants in a second group (k_2), and entrants in all other groups (k_3). There are $n-1$ possible second groups in this context, hence the multiplier $(n-1)$. Winning occurs with positive probability when the marginal entrant's own group has a leader who is among the top two leaders; hence, integration is restricted to the domain with $\min\{t_1, t_2\} \geqslant t_3$. The equilibrium probabilities of winning (cf. Section 4) are used for the cases with $t_1 \geqslant t_2$ and $t_2 \geqslant t_1$ in the two integrals. Integration is over the highest order statistics in all cases.[15]

Simplifying (12), we obtain $p_{\mathrm{FD}}(v^*) = p_{\mathrm{ND}}(v^*) + A(v^*)$, where $p_{\mathrm{ND}}(v^*)$ is the marginal type's probability of winning in the case of nondisclosure, Eq. (6), and

$$A(v^*) = m(m-1)(n-1)\int_{t_1\geqslant t_2\geqslant v^*}\frac{t_2}{2t_1}F(t_1)^{m-2}F(t_2)^{nm-m-2}\times$$
$$\times\,[F(t_1)-F(t_2)]\mathrm{d}F(t_1)\mathrm{d}F(t_2) \geqslant 0. \tag{13}$$

[15]Note that, strictly speaking, p_{FD} is not the *probability* of winning. The marginal entrant can also win with positive probability if she is her group's leader (i.e., the only entrant) and there is only one other group with entrants. However, in this case the marginal entrant's expected payoff is zero, cf. Section 4.

For details, see Appendix A. Thus, the marginal type v^*_{FD} such that $v^*_{\text{FD}} p_{\text{FD}}(v^*_{\text{FD}}) = \omega$ is lower than in the case of no disclosure, $v^*_{\text{FD}} \leqslant v^*$, and the inequality is strict for $m > 1$ and $v^* > \underline{v}$. A larger mass of players enters the contest under full disclosure.

Proposition 3 *Suppose $m > 1$, and there is less than full entry under no disclosure, $v^* > \underline{v}$. Then there is more entry under full disclosure than under no disclosure: $v^*_{\text{FD}} < v^*$.*

Given the marginal type v^*_{FD}, it can be shown that the expected payoff from entering is less than ω for all types $v < v^*_{\text{FD}}$, and greater than ω for all types $v > v^*_{\text{FD}}$. First, for $v < v^*_{\text{FD}}$, type v either earns v or zero, and her probability of earning zero is the same as for the marginal type. Thus, the expected payoff of type $v < v^*_{\text{FD}}$ from entering must be lower than the expected payoff of the marginal type, which is, by definition, equal to ω. Second, for $v \geqslant v^*_{\text{FD}}$, we show in Appendix A that the expected payoff from entering, $\Pi(v, v; v^*_{\text{FD}})$, is increasing in v.

Recall our assumption that $\omega < \overline{v}$. The existence of a marginal type (i.e., an interior solution to $v^*_{\text{FD}} p_{\text{FD}}(v^*_{\text{FD}}) = \omega$) is then guaranteed by the following sufficient condition:

$$\frac{\omega}{\underline{v}} \geqslant \frac{m-1}{nm-1}\left(1 + \frac{m(n-1)}{2(nm-1)(nm-m-1)}\right). \tag{14}$$

The following example illustrates the comparison of equilibrium cutoffs for the full disclosure and no disclosure settings (Proposition 3).

Example 1 *Suppose $n = 2$, $m = 3$, and that valuations are drawn from a distribution of the form $F(v) = v^\alpha$, $\alpha > 0$, with interval support $V = [0, 1]$. In Figure 1, we plot the equilibrium cutoff valuation (type) as a function of the outside option, ω, for both the ND and FD cases with $\alpha = 1$ (uniform distribution) and $\alpha = 3$.*

From the left panel of Figure 1, which plots the cutoff valuations, it can be difficult to see that $v^*_{\text{FD}} < v^*$ for all (interior) values of ω. Thus, in the right panel, we also plot the difference in cutoff valuations, $v^* - v^*_{\text{FD}}$, as a function of ω, for the two different values of α. This better illustrates that the cutoff type is higher under ND than under FD, but also serves to illustrate that the difference is non-monotone and single-peaked. The nonmonotonicity of the difference $v^* - v^*_{\text{FD}}$ is expected because the two cutoffs have to be the same under full entry and no entry.

Expected aggregate investment under full disclosure is

$$B_{\text{FD}} = n(n-1)\sum_{k_1=0}^{m}\sum_{k_2=0}^{m}\sum_{k_3=0}^{nm-2m}\binom{m}{k_1}\binom{m}{k_2}\binom{nm-2m}{k_3}q^{k_1+k_2+k_3}(1-q)^{nm-k_1-k_2-k_3}\times$$

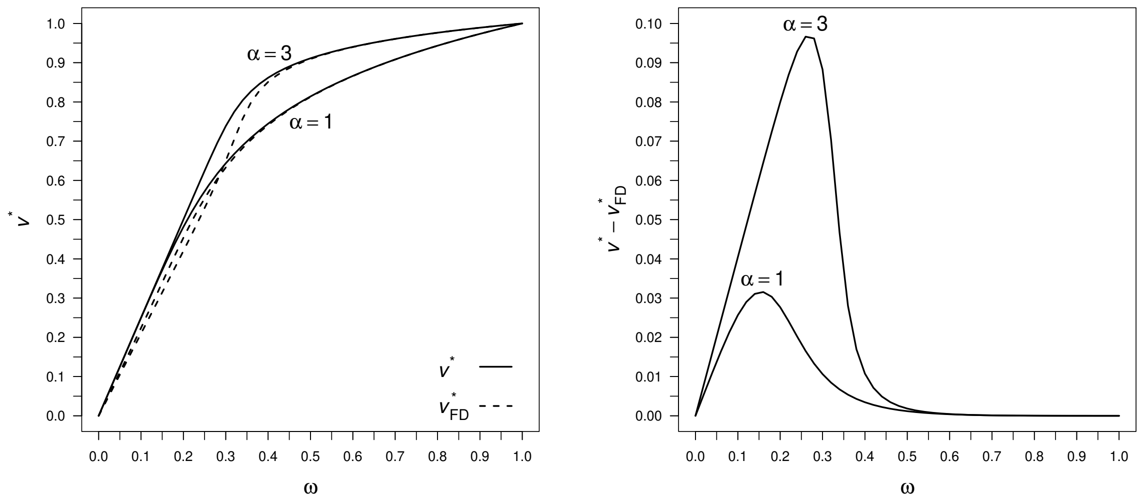

Figure 1: Cutoff valuations for ND and FD as a function of ω (left), and the difference in cutoff valuations, $v^* - v^*_{\mathrm{FD}}$, as a function of ω (right), in Example 1.

$$\times \int_{t_1 \geqslant t_2 \geqslant t_3 \geqslant v^*_{\mathrm{FD}}} \left(\frac{t_2}{2} + \frac{t_2^2}{2t_1} \right) \mathrm{d}\tilde{F}(t_1)^{k_1} \mathrm{d}\tilde{F}(t_2)^{k_2} \mathrm{d}\tilde{F}(t_3)^{k_3}. \tag{15}$$

Simplifying, obtain

$$B_{\mathrm{FD}} = m^2 n(n-1) \int_{t_1 \geqslant t_2 \geqslant v^*_{\mathrm{FD}}} \left(\frac{t_2}{2} + \frac{t_2^2}{2t_1} \right) F(t_1)^{m-1} F(t_2)^{nm-m-1} \mathrm{d}F(t_1) \mathrm{d}F(t_2). \tag{16}$$

For details, see Appendix A. The following proposition (proved in Appendix A) is our second major result.

Proposition 4 *Suppose $m > 1$, and the elasticity of the distribution of types, $\xi(t) = \frac{tf(t)}{F(t)}$, satisfies $\xi(t) \geqslant \frac{1}{m-1}$. Then expected aggregate investment under full disclosure is greater than under no disclosure, $B_{\mathrm{FD}} > B_{\mathrm{ND}}$.*

The proof of Proposition 4 is based on comparing B_{FD}, Eq. (16), to B_{ND}, Eq. (9). Recall that $v^*_{\mathrm{FD}} \leqslant v^*$; thus, the domain of integration for B_{FD} is larger. The lower bound on the elasticity of $F(\cdot)$ ensures that the integrand in (16) is also larger.

The next example illustrates the comparison of aggregate investment between FD and ND when the sufficient condition on the elasticity of the distribution of types provided by Proposition 4 is not satisfied. In particular, it demonstrates that the ranking of aggregate investment may be (but need not be) reversed.

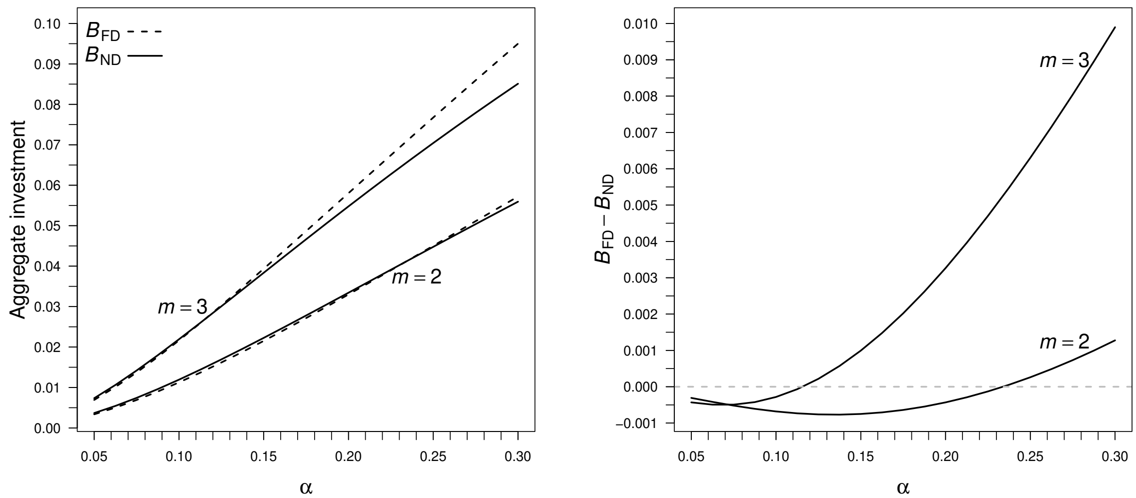

Figure 2: Aggregate investment for ND and FD as a function of α, for $m = 2$ and $m = 3$ (left), and the difference, $B_{\text{FD}} - B_{\text{ND}}$, as a function of α, for $m = 2$ and $m = 3$ (right), Example 2.

Example 2 *As in Example 1, suppose* $n = 2$*,* $V = [0, 1]$*, and* $F(v) = v^{\alpha}$*. Fix* $\omega = 0.4$ *and consider two cases corresponding to* $m = 2$ *and* $m = 3$*. In Figure 2, we plot the aggregate investment levels for values of* α *(the elasticity of the distribution of types) between 0.05 and 0.3.*

Proposition 4 implies that, for $m = 2$ (respectively, $m = 3$), $\alpha \geqslant 1$ (respectively, $\alpha \geqslant 0.5$) is sufficient for $B_{\text{FD}} > B_{\text{ND}}$. Thus, the sufficient condition is not satisfied for either m in Figure 2.

The left panel establishes that it is possible for aggregate investment to be lower under FD than under ND. For both values of m, B_{FD} (the dashed lines) is below B_{ND} (the solid lines) for at least some values of α. In order to better highlight the comparison, we also plot the difference, $B_{\text{FD}} - B_{\text{ND}}$, as a function of α, in the right panel.

When $m = 3$, $B_{\text{FD}} < B_{\text{ND}}$ for very small values of α. However, as the elasticity of the distribution of types increases, aggregate investment under FD grows much faster than under ND such that, even though the condition of Proposition 4 is not satisfied, we still obtain $B_{\text{FD}} > B_{\text{ND}}$. Similarly, when $m = 2$, if α is low, aggregate investment is higher under ND than under FD. Still, once α becomes large enough, full disclosure leads to higher aggregate investment than no disclosure.

The intuition for this relationship between the elasticity of the distribution of types and the effect of full disclosure is as follows. The effect of full disclosure on aggregate investment depends on three competing effects, two of which lead to an increase and one—to a decrease in aggregate investment as compared to ND. First, as is the case for the WD setting, full disclosure allows entrants to solve the coordination problem within their own group, which has a positive effect on expected group-level investment, as it reduces the effect of free riding on the highest valuation entrant's investment.

Second, full disclosure also reduces the number of active bidders among entrants to just two (the two highest valuation leaders of their respective groups) who play according to the standard mixed-strategy equilibrium in an all-pay auction of complete information. In this equilibrium, the second-highest valuation player places a mass on zero investment that reduces the expected aggregate investment. Importantly, this mass is increasing in the difference between the highest and second-highest valuations. As the elasticity of $F(\cdot)$ increases, draws from the distribution of valuations shift closer to the upper bound of the support, such that, in expectation, the difference between the highest and second-highest valuations becomes smaller. Thus, with an increase in the elasticity of the distribution, the negative effect of full disclosure becomes less important, allowing for FD to increase aggregate investment above the level in the ND setting.

Finally, due to Proposition 3 there is more entry under FD, which also increases aggregate investment. This explains why the sufficient condition in Proposition 4 is not very tight, cf. Example 2.

Note that a similar comparison cannot be made between B_{FD} and B_{WD}, Eq. (11). While the domain of integration is larger in B_{FD}, we observe that the integrand is always larger in B_{WD}. These competing effects—a larger mass of players entering but bidding lower under full disclosure—make the comparison ambiguous. As such, there is no systematic condition, independent of the cutoff, that suffices to establish an unambiguous ranking of aggregate investment between WD and FD. Our third example demonstrates this ambiguity, and shows that even when the condition in Proposition 4 is satisfied, it is possible for B_{FD} to be higher or lower than B_{WD}.

Example 3 *Suppose $n = 2$, $m = 3$, $\omega = 0.4$, and $F(v) = v^{\alpha}$. In Figure 3, we plot aggregate investment under FD (dashed lines) and under WD (solid lines) for values of $\alpha \in [1, 2]$. Note that for $m = 3$, the elasticity of the distribution of types satisfies $\alpha > 1/(m-1)$, so that the condition in Proposition 4 is satisfied. Figure 3 shows that B_{WD} may nevertheless be higher or lower than B_{FD}.*

Yet, the above discussion implies that an unambiguous comparison between WD and FD can be made in the important special case of full entry (or, in other words, when the number of players in the contest is fixed). Since there is, generically, more entry under

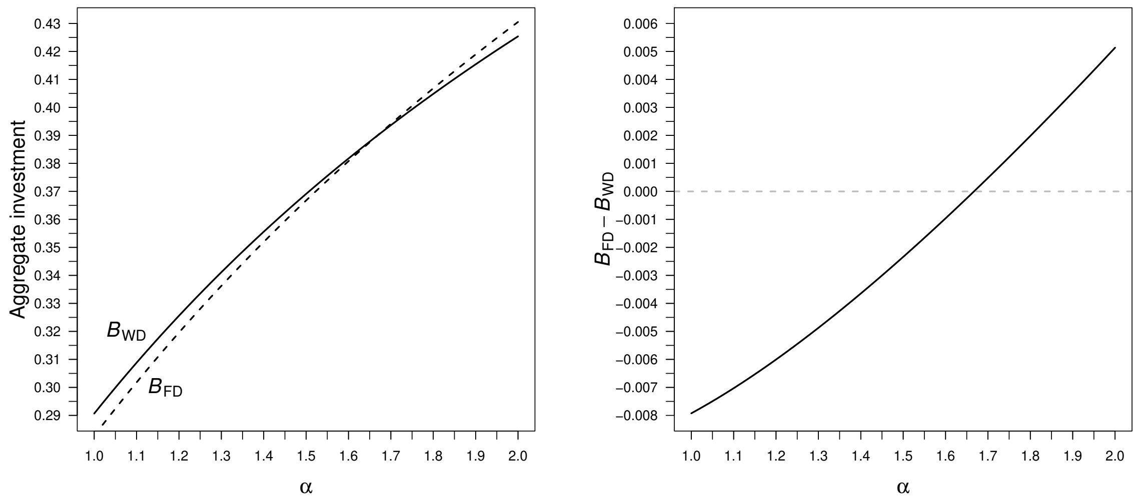

Figure 3: Aggregate investment for WD and FD as a function of α (left), and the difference, $B_{\text{FD}} - B_{\text{WD}}$, as a function of α (right).

FD, a necessary and sufficient condition for full entry under all three disclosure rules is that $v^* = \underline{v}$, i.e., $\omega = 0$. Recall also that, from Proposition 2, WD dominates ND for any cutoff, including the full entry case. We, therefore, have the following result.

Corollary 1 *Suppose $m > 1$ and there is full entry under all three disclosure rules (i.e., $\omega = 0$ or the number of players in the contest is fixed). Then expected aggregate investment under within-group disclosure is greater than under both no disclosure and full disclosure.*

5.4 Alternative objectives

5.4.1 Total investment

The "best-shot" production technology within groups implies that the investments of group members that are below the maximum bid are essentially wasted. However, from the contest designer's or policy perspective it may be of interest to consider *total investment*, defined as the sum of all individual investments, as a relevant criterion. While the wasted portion of total investment does not contribute directly to output (e.g., it does not improve the quality of the resulting innovation produced by the group), it may have spillovers the designer cares about. From (7), the expected total investment by entrants under no disclosure is

$$B_{\mathrm{ND}}^{\mathrm{tot}} = nmq \int_{v^*}^{\overline{v}} b_{\mathrm{ND}}(v)\mathrm{d}\tilde{F}(v) = m^2 n(n-1) \int_{t_1 \geqslant t_2 \geqslant v^*} t_2 F(t_2)^{nm-2} \mathrm{d}F(t_1)\mathrm{d}F(t_2). \tag{17}$$

Notice that under both within-group disclosure and full disclosure, expected total investment coincides with expected aggregate investment, Eqs. (11) and (16), respectively, because only leaders are (potentially) active in each group. Comparing (17) and (11), we obtain

$$\begin{aligned} & B_{\mathrm{WD}} - B_{\mathrm{ND}}^{\mathrm{tot}} \\ & = m^2 n(n-1) \int_{t_1 \geqslant t_2 \geqslant v^*} t_2 F(t_2)^{nm-m-1} F(t_1)^{m-1} \left[F(t_1)^{m-1} - F(t_2)^{m-1}\right] \mathrm{d}F(t_1)\mathrm{d}F(t_2) \geqslant 0. \end{aligned}$$

Thus, aggregate (equivalently, total) investment under within-group disclosure exceeds total investment without disclosure. The inequality is strict for $m > 1$.[16] With disclosure, the entrants are able to solve the coordination problem within groups and bid more effectively. The impact of within-group disclosure is so strong that the resulting expected aggregate investment exceeds even the sum of individual investments under no disclosure.

This result also implies that the comparison of total investments between ND and FD is ambiguous. Specifically, we have $B_{\mathrm{WD}} > B_{\mathrm{ND}}^{\mathrm{tot}} > B_{\mathrm{ND}}$, while both B_{WD} and B_{ND} can be either above or below B_{FD}.

5.4.2 Expected highest investment

In an innovation or R&D competition setting, the principal may be interested in maximizing the expected highest investment across all participating groups. For example, if each group submits a solution to a challenge such as the Netflix Prize[17] or the XPRIZE Carbon Removal initiative,[18] only the best solution (if any) will eventually be implemented on a large scale. In this section we compare *expected highest investment* across the disclosure policies.

Under no disclosure, Eq. (7) gives the expected highest investment

$$\begin{aligned} B_{\mathrm{ND}}^{\max} &= \sum_{k=1}^{nm} \binom{nm}{k} q^k (1-q)^{nm-k} \int_{v^*}^{\overline{v}} b_{\mathrm{ND}}(v)\mathrm{d}\tilde{F}(v)^k = \int_{v^*}^{\overline{v}} b_{\mathrm{ND}}(v)\mathrm{d}F(v)^{nm} \\ &= m^2 n(n-1) \int_{t_1 \geqslant t_2 \geqslant v^*} t_2 F(t_2)^{nm-2} F(t_1)^{nm-1} \mathrm{d}F(t_1)\mathrm{d}F(t_2). \end{aligned} \tag{18}$$

[16] Recall that $m = 1$ implies WD and ND collapse to the same information environment.
[17] See https://www.thrillist.com/entertainment/nation/the-netflix-prize.
[18] See https://www.xprize.org/prizes/carbonremoval.

Under full disclosure, suppose there are at least two active groups, and $v_{(1)} > v_{(2)}$ are the two highest valuations. In equilibrium, the two entrants will bid according to the mixed strategies G_1 and G_2 described in Section 4. Expected highest investment conditional on $v_{(1)}$ and $v_{(2)}$ then is

$$\begin{aligned}&\int_{x_1>x_2} x_1 \mathrm{d}G_1(x_1)\mathrm{d}G_2(x_2) + \int_{x_1<x_2} x_2 \mathrm{d}G_1(x_1)\mathrm{d}G_2(x_2)\\ &= \int x_1 G_2(x_1)\mathrm{d}G_1(x_1) + \int x_2 G_1(x_2)\mathrm{d}G_2(x_2)\\ &= \int_0^{v_{(2)}} x_1 \left(1 - \frac{v_{(2)}}{v_{(1)}} + \frac{x_1}{v_{(1)}}\right)\frac{\mathrm{d}x_1}{v_{(2)}} + \int_0^{v_{(2)}} \frac{x_2^2}{v_{(2)}}\frac{\mathrm{d}x_2}{v_{(1)}} = \frac{v_{(2)}}{2} + \frac{v_{(2)}^2}{6v_{(1)}}.\end{aligned}$$

Using the joint distribution of $(v_{(1)}, v_{(2)})$ from (16), we obtain the unconditional expected highest investment:

$$B_{\mathrm{FD}}^{\max} = m^2 n(n-1)\int_{t_1 \geqslant t_2 \geqslant v_{\mathrm{FD}}^*} \left(\frac{t_2}{2} + \frac{t_2^2}{6t_1}\right) F(t_2)^{nm-m-1}F(t_1)^{m-1}\mathrm{d}F(t_1)\mathrm{d}F(t_2). \quad (19)$$

The comparison between (18) and (19) is ambiguous in general, but we establish the following result.

Proposition 5 *Suppose there is full entry (i.e., $v^* = v_{\mathrm{FD}}^* = \underline{v}$) and $\overline{v}$ is finite. Then $B_{\mathrm{FD}}^{\max} > B_{\mathrm{ND}}^{\max}$ for n or m sufficiently large.*

The reason why full disclosure dominates for sufficiently large n or m is that, as n or m increases, bids are shaded more under no disclosure because the probability for a player's bid to be the winning bid for a given prize valuation declines. This effect is absent under full disclosure where the two highest valuation players know that they are the only potential bidders. As the number of players increases, bid shading eventually undermines the expected highest investment in ND and, to a smaller extent, in WD, as compared to FD. Of course, under partial entry there is additional ambiguity due to the difference between the entry cutoffs. The assumption of bounded support is critical for the above argument, however, because the top order statistics converge to $\overline{v}$ at different speeds under ND and FD. As seen from (18) and (19), the speed of convergence is generally higher under ND. This implies that if the support of F is not bounded and the distribution has a sufficiently heavy tail, $B_{\mathrm{ND}}^{\max}$ can dominate $B_{\mathrm{FD}}^{\max}$.

Example 4 *Suppose $\omega = 0$, i.e., there is full entry under ND and FD. To illustrate Proposition 5, we consider Pareto distribution, $F(v) = 1 - \frac{1}{(v+1)^p}$, with support $[0, \infty)$, and its truncated version, $F_{\mathrm{t}}(v) = \frac{F(v)}{F(v_{\max})}$, supported in $[0, v_{\max}]$. Figure 4 shows the* *dependence of the difference $B^{\max}_{\mathrm{FD}} - B^{\max}_{\mathrm{ND}}$ on m, for n fixed, and on n, for m fixed. The support of F_{t} is bounded, and $B^{\max}_{\mathrm{FD}}$ eventually dominates $B^{\max}_{\mathrm{ND}}$ for m or n large enough. In contrast, $B^{\max}_{\mathrm{ND}}$ dominates $B^{\max}_{\mathrm{FD}}$ for n large (and m fixed) when the support is not bounded. Note that $B^{\max}_{\mathrm{FD}}$ still dominates $B^{\max}_{\mathrm{ND}}$ for m large (and n fixed), illustrating that the bounded support condition is sufficient but not necessary.*

A clear comparison of expected highest investment can be made between ND and WD. Indeed, as seen from Eqs. (7) and (10), the equilibrium bidding functions $b_{\mathrm{ND}}(v)$ and $b_{\mathrm{WD}}(v)$ are ranked pointwise such that $b_{\mathrm{WD}}(v) \geqslant b_{\mathrm{ND}}(v)$ for all $v \in V^*$, with strict inequality for $m > 1$ and $v > v^*$. Since the distribution of the highest valuation is the same in both cases, the highest investment in WD exceeds the highest investment in ND in the sense of first-order stochastic dominance. It is, therefore, immediate that expected highest investment is higher under WD. The effect becomes stronger as m increases because players benefit more from solving the coordination problem with increasing group size.

6 Concluding remarks

This paper is the first to study endogenous entry in contests between groups. Our focus is on the effects of information disclosure, which can be a designer's choice or institutionally predetermined. In settings where winning the contest provides a group-specific public good, group contests generate an interesting interplay of the standard contest incentives across groups with social dilemma-type free-riding incentives within groups. Endogenous entry and information disclosure can affect those incentives in several ways, leading to higher or lower aggregate equilibrium investment as compared to the no disclosure benchmark. The direction of the effect depends on how information is disclosed, the group size, and on the properties of the distribution of types.

Without disclosure, the competition between groups is efficient, in that the prize is allocated to the group containing the player with the highest value. However, significant investment within groups is wasted due to lack of coordination. Within-group disclosure, under which players only find out about the number and types of other members of their own group, helps players within groups coordinate on more socially efficient investment strategies at the group level. Competition across groups remains efficient, and the marginal type does not change, producing a higher aggregate investment overall.

Full disclosure, under which the information about all contest participants is disclosed across groups, helps within-group coordination as well, but produces two additional (and possibly competing) effects. First, the payoff of the marginal type, and hence the mass of players entering the contest, increases. This is because under no disclosure (or under

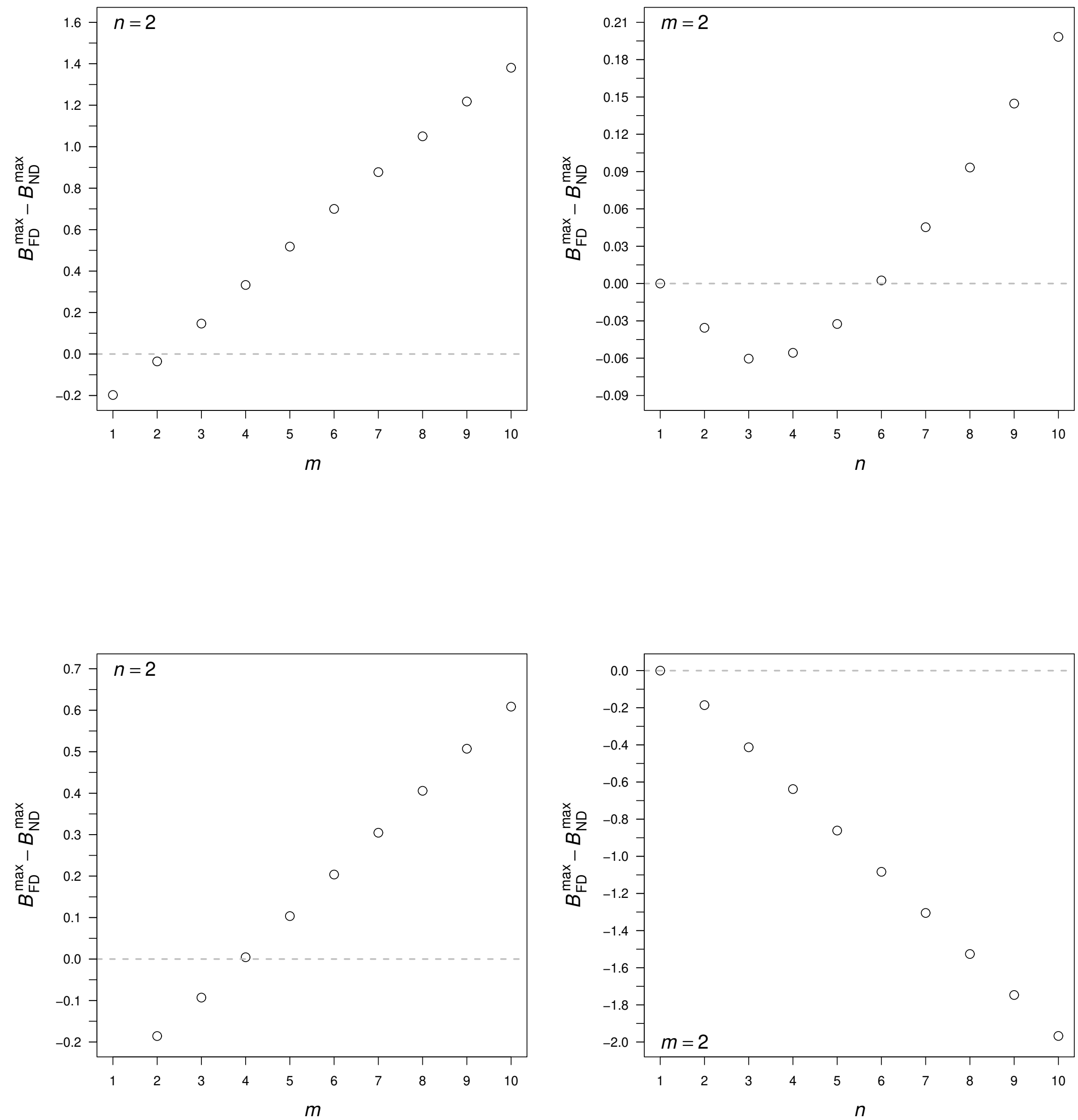

Figure 4: The difference in expected highest investment between FD and ND, $B^{\max}_{\mathrm{FD}} - B^{\max}_{\mathrm{ND}}$, as a function of m for $n = 2$ (the left panels), and as a function of n for for $m = 2$ (the right panels). The distribution of values is truncated Pareto (the top two panels) and Pareto (the bottom two panels), with $p = 1$ and $v_{\max} = 30$.

within-group disclosure) the marginal type can only win the contest with a positive probability if she is the only entrant or the highest value entrant in her group has the highest value overall. In contrast, under full disclosure the marginal type can also win with a positive probability when the highest value entrant in her group has the second highest value overall. This additional probability of winning is linked to the inefficiency of the mixed equilibrium in all-pay auctions under complete information.

Second, depending on the properties of the distribution of types, bidding conditional on type may increase or decrease under full disclosure as compared to no disclosure. We show that full disclosure produces a higher aggregate investment when the distribution of types is sufficiently elastic. The condition becomes weaker as the group size increases. Intuitively, when the density of types increases at the upper bound of its support, the top order statistics of types are sufficiently close, and hence the reduction in bidding due to the second highest type dropping out is less severe, more so the larger the (expected) group size.[19]

One clear implication of our results is that within-group disclosure is beneficial and should be facilitated by the contest designer when possible. This is true not only in cases where the designer's goal is to maximize expected aggregate investment, but also for a designer whose goal is to maximize expected total investment or expected highest investment.

At the same time, disclosure in the public domain, such as the various "leaderboard" practices in crowdsourcing, or sunshine laws in government practices, should be exercised with caution, more so the smaller the group size. In particular, for individual contests, full disclosure unambiguously leads to a lower investment and should be avoided.[20] However, full disclosure becomes optimal as the group size increases, especially if the distribution of types is sufficiently elastic. Our results, therefore, suggest that different disclosure policies can be optimal depending on whether a competitive task is performed by individuals, small groups or large groups, even if other features of the environment are similar.

Our results are, of course, restricted to (*ex ante*) symmetric group contests with a specific best-shot aggregation technology and an all-pay auction competition structure. Natural extensions of this work include alternative specifications of the model, such as a different or more flexible aggregation rule (e.g., perfect substitutes, perfect complements, or CES), imperfectly discriminating contest rules and asymmetric agents and/or groups.

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[20]For a fixed number of players, this goes back to Morath and Münster (2008).

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A Missing derivations and proofs

A.1 Equation (7)

The equation $\Pi_{\hat{v}}(v, v; v^*) = 0$ has the form $b'(v) = vp'(v)$, where, from (6),

$$
\begin{aligned}
p'(v) &= \sum_{k_1=0}^{m-1} \sum_{k_2=1}^{nm-m} \binom{m-1}{k_1} \binom{nm-m}{k_2} q^{k_1+k_2}(1-q)^{nm-1-k_1-k_2} \tilde{F}(v)^{k_1+k_2-1} \tilde{f}(v) k_2 \\
&= q\tilde{f}(v) \sum_{k_1=0}^{m-1} \binom{m-1}{k_1} (q\tilde{F}(v))^{k_1}(1-q)^{m-1-k_1} \sum_{k_2=1}^{nm-m} \binom{nm-m}{k_2} k_2 (q\tilde{F}(v))^{k_2-1}(1-q)^{nm-m-k_2} \\
&= f(v)[q\tilde{F}(v) + 1 - q]^{m-1} \frac{\partial(z+1-q)^{nm-m}}{\partial z}\bigg|_{z=q\tilde{F}(v)} = m(n-1)F(v)^{nm-2}f(v).
\end{aligned}
$$

This produces the equation $b'(v) = m(n-1)vF(v)^{nm-2}f(v)$, whose unique solution with initial condition $b(v^*) = 0$ is given by (7).

A.2 Equation (8)

We write the sum in (8) as

$$
\begin{aligned}
&\int_0^q \sum_{k_1=0}^{m-1} \sum_{k_2=1}^{nm-m} \binom{m-1}{k_1} \binom{nm-m}{k_2} z^{k_1+k_2-1}(1-q)^{nm-1-k_1-k_2} k_1 dz \\
&= \int_0^q \sum_{k_1=0}^{m-1} \binom{m-1}{k_1} k_1 z^{k_1-1}(1-q)^{m-1-k_1} \sum_{k_2=1}^{nm-m} \binom{nm-m}{k_2} z^{k_2}(1-q)^{nm-m-k_2} dz \\
&= \int_0^q \frac{\partial(z+1-q)^{m-1}}{\partial z} \left[(z+1-q)^{nm-m} - (1-q)^{nm-m}\right] dz \\
&= (m-1)\int_0^q (z+1-q)^{nm-2} dz - (1-q)^{nm-m}[1-(1-q)^{m-1}] \\
&= \frac{m-1}{nm-1}[1-(1-q)^{nm-1}] - (1-q)^{nm-m} + (1-q)^{nm-1} \\
&= \frac{m-1}{nm-1} + \frac{m(n-1)}{nm-1}(1-q)^{nm-1} - (1-q)^{nm-m}.
\end{aligned}
$$

Combining with the first term in (8), obtain the result.

A.3 Expected payoff for $v \geqslant v^*_{\text{FD}}$ increasing in v

Fix the marginal type to be v^*_{FD} and let $q = 1 - F(v^*_{\text{FD}})$. For type $v \geqslant v^*_{\text{FD}}$, the expected payoff from entering is given by

$$\Pi(v, v; v^*_{\text{FD}}) = v(1-q)^{nm-m} + (n-1)\sum_{k_1=0}^{m-1}\sum_{k_2=1}^{m}\sum_{k_3=0}^{nm-2m}\binom{m-1}{k_1}\binom{m}{k_2}\binom{nm-2m}{k_3}\times$$
$$\times\, q^{k_1+k_2+k_3}(1-q)^{nm-1-k_1-k_2-k_3}(Y_1+Y_2+Y_3+Y_4),$$

where

$$Y_1 = v\int\limits_{\overline{v}\geqslant t_1>v}\int_{t_1\geqslant t_2\geqslant t_3\geqslant v^*_{\text{FD}}}\left(1-\frac{t_2}{2t_1}\right)\mathrm{d}\tilde{F}(t_1)^{k_1}\mathrm{d}\tilde{F}(t_2)^{k_2}\mathrm{d}\tilde{F}(t_3)^{k_3}$$
$$Y_2 = v\int\limits_{\overline{v}\geqslant t_1>v}\int_{t_2\geqslant t_1\geqslant t_3\geqslant v^*_{\text{FD}}}\frac{t_1}{2t_2}\mathrm{d}\tilde{F}(t_1)^{k_1}\mathrm{d}\tilde{F}(t_2)^{k_2}\mathrm{d}\tilde{F}(t_3)^{k_3}$$
$$Y_3 = \int_{\overline{v}\geqslant v\geqslant t_1\geqslant t_2\geqslant t_3\geqslant v^*_{\text{FD}}}(v-t_2)\mathrm{d}\tilde{F}(t_1)^{k_1}\mathrm{d}\tilde{F}(t_2)^{k_2}\mathrm{d}\tilde{F}(t_3)^{k_3}$$
$$Y_4 = \int_{\overline{v}\geqslant v\geqslant t_2\geqslant t_1\geqslant t_3\geqslant v^*_{\text{FD}}}(v-t_1)\mathrm{d}\tilde{F}(t_1)^{k_1}\mathrm{d}\tilde{F}(t_2)^{k_2}\mathrm{d}\tilde{F}(t_3)^{k_3}.$$

This expression can be simplified considerably, first by integrating over t_3 and summing over k_3, then summing over k_1 and k_2 and swapping the variables of integration in two of the integral terms, which gives

$$\Pi(v, v; v^*_{\text{FD}}) = v(1-q)^{nm-m} + m(m-1)(n-1)(vZ_1 + vZ_2 + vZ_3 - Z_4 - Z_5),$$

where

$$Z_1 = \int_{\overline{v}\geqslant t_1\geqslant t_2\geqslant v^*_{FD}} F(t_1)^{m-2}F(t_2)^{nm-m-1}\mathrm{d}F(t_1)\mathrm{d}F(t_2)$$
$$Z_2 = \int\limits_{t_1=v}^{\overline{v}}\int\limits_{t_2=v^*_{FD}}^{t_1}\frac{t_2}{2t_1}F(t_1)^{m-2}F(t_2)^{nm-m-2}[F(t_1)-F(t_2)]\mathrm{d}F(t_1)\mathrm{d}F(t_2)$$
$$Z_3 = \int_{v\geqslant t_1\geqslant t_2\geqslant v^*_{FD}} F(t_1)^{m-1}F(t_2)^{nm-m-2}\mathrm{d}F(t_1)\mathrm{d}F(t_2)$$

$$Z_4 = \int\limits_{t_1=v}^{\overline{v}} \int\limits_{t_2=v^*_{FD}}^{t_1} t_1 F(t_1)^{m-1} F(t_2)^{nm-m-2} \mathrm{d}F(t_1)\mathrm{d}F(t_2)$$

$$Z_5 = \int\limits_{t_1=v}^{\overline{v}} \int\limits_{t_2=v^*_{FD}}^{t_1} t_2 F(t_1)^{m-2} F(t_2)^{nm-m-1} \mathrm{d}F(t_1)\mathrm{d}F(t_2)$$

It's straightforward to show that vZ_1, vZ_3 and $-Z_5$ are all increasing in v. Taking the derivative of $vZ_2 - Z_4$ with respect to v gives

$$\frac{\mathrm{d}(vZ_2 - Z_4)}{\mathrm{d}v} = Z_2 - v \int\limits_{t_2=v^*_{\mathrm{FD}}}^{v} \frac{t_2}{2v} f(v) F(v)^{m-2} F(t_2)^{nm-m-2} [F(v) - F(t_2)] \mathrm{d}F(t_2) +$$

$$+ f(v) \int\limits_{t_2=v^*_{\mathrm{FD}}}^{v} vF(v)^{m-1} F(t_2)^{nm-m-2} \mathrm{d}F(t_2)$$

$$= Z_2 - f(v) \int\limits_{t_2=v^*_{\mathrm{FD}}}^{v} \left[\left(\frac{t_2}{2} - v \right) F(v)^{m-1} F(t_2)^{nm-m-2} - \frac{t_2}{2} F(v)^{m-2} F(t_2)^{nm-m-1} \right] \mathrm{d}F(t_2).$$

Since $v \geqslant t_2$, the integrand is everywhere negative, which ensures that $vZ_2 - Z_4$ is increasing in v. It follows that $\Pi(v, v; v^*_{\mathrm{FD}})$ is increasing in v for $v \geqslant v^*_{\mathrm{FD}}$, as desired.

A.4 Equation (13)

Start by performing integration over t_3 and summation over k_3 in (12). Integration over t_3 is on $[v^*, t_2]$ in the first integral and on $[v^*, t_1]$ in the second integral, producing

$$p_{\mathrm{FD}}(v^*) = (1-q)^{nm-m} + (n-1) \sum_{k_1=0}^{m-1} \sum_{k_2=1}^{m} \sum_{k_3=0}^{nm-2m} \binom{m-1}{k_1} \binom{m}{k_2} \binom{nm-2m}{k_3} \times$$

$$\times\, q^{k_1+k_2+k_3} (1-q)^{nm-1-k_1-k_2-k_3} \left[\int_{t_1 \geqslant t_2 \geqslant v^*} \left(1 - \frac{t_2}{2t_1}\right) \tilde{F}(t_2)^{k_3} \mathrm{d}\tilde{F}(t_1)^{k_1} \mathrm{d}\tilde{F}(t_2)^{k_2} + \right.$$

$$\left. + \int_{t_2 \geqslant t_1 \geqslant v^*} \frac{t_1}{2t_2} \tilde{F}(t_1)^{k_3} \mathrm{d}\tilde{F}(t_1)^{k_1} \mathrm{d}\tilde{F}(t_2)^{k_2} \right] =$$

$$= (1-q)^{nm-m} + (n-1) \sum_{k_1=0}^{m-1} \sum_{k_2=1}^{m} \binom{m-1}{k_1} \binom{m}{k_2} \times$$

$$\times\, q^{k_1+k_2} (1-q)^{2m-1-k_1-k_2} \left[\int_{t_1 \geqslant t_2 \geqslant v^*} \left(1 - \frac{t_2}{2t_1}\right) F(t_2)^{nm-2m} \mathrm{d}\tilde{F}(t_1)^{k_1} \mathrm{d}\tilde{F}(t_2)^{k_2} + \right.$$

$$\left. + \int_{t_2 \geqslant t_1 \geqslant v^*} \frac{t_1}{2t_2} F(t_1)^{nm-2m} \mathrm{d}\tilde{F}(t_1)^{k_1} \mathrm{d}\tilde{F}(t_2)^{k_2} \right].$$

Next, we sum over k_1 and k_2, and swap the variables of integration in the second integral:

$$p_{\mathrm{FD}}(v^*) = (1-q)^{nm-m} + (n-1)\left[\int_{t_1 \geqslant t_2 \geqslant v^*} \left(1 - \frac{t_2}{2t_1}\right) F(t_2)^{nm-2m} \mathrm{d}F(t_1)^{m-1} \mathrm{d}F(t_2)^m + \right.$$
$$\left. + \int_{t_1 \geqslant t_2 \geqslant v^*} \frac{t_2}{2t_1} F(t_2)^{nm-2m} \mathrm{d}F(t_2)^{m-1} \mathrm{d}F(t_1)^m \right].$$

Separate the first integral into two parts and combine its second part with the second integral:

$$p_{\mathrm{FD}}(v^*) = (1-q)^{nm-m} + (n-1)\int_{t_1 \geqslant t_2 \geqslant v^*} F(t_2)^{nm-2m} \mathrm{d}F(t_1)^{m-1} \mathrm{d}F(t_2)^m + \tag{20}$$
$$+ m(m-1)(n-1)\int_{t_1 \geqslant t_2 \geqslant v^*} \frac{t_2}{2t_1} F(t_1)^{m-2} F(t_2)^{nm-m-2} [F(t_1) - F(t_2)] \mathrm{d}F(t_1) \mathrm{d}F(t_2).$$

Notice that the last term is equal to $A(v^*)$ defined in (13).

Finally, consider the first two terms:

$$\begin{aligned}
&(1-q)^{nm-m} + (n-1)\int_{t_1 \geqslant t_2 \geqslant v^*} F(t_2)^{nm-2m} \mathrm{d}F(t_1)^{m-1} \mathrm{d}F(t_2)^m \\
&= F(v^*)^{nm-m} + (m-1)\int_{v^*}^{\overline{v}} [F(t_1)^{nm-m} - F(v^*)^{nm-m}] F(t_1)^{m-2} \mathrm{d}F(t_1) \\
&= F(v^*)^{nm-m} + (m-1)\left[\frac{1 - F(v^*)^{nm-1}}{nm-1} - \frac{F(v^*)^{nm-m} - F(v^*)^{nm-1}}{m-1}\right] \\
&= F(v^*)^{nm-m} + \frac{m-1}{nm-1} + \frac{m(n-1)}{nm-1} F(v^*)^{nm-1} - F(v^*)^{nm-m} = p(v^*),
\end{aligned}$$

where $p(v^*)$ is defined in (8).

A.5 Equation (16) and proof of Proposition 4

Start by summing up over k_1, k_2 and k_3 in (15):

$$B_{\mathrm{FD}} = n(n-1)\int_{t_1 \geqslant t_2 \geqslant t_3 \geqslant v^*_{\mathrm{FD}}} \left(\frac{t_2}{2} + \frac{t_2^2}{2t_1}\right) \mathrm{d}F(t_1)^m \mathrm{d}F(t_2)^m \mathrm{d}F(t_3)^{nm-2m}.$$

Next, perform integration over t_3 on $[v^*_{\mathrm{FD}}, t_2]$:

$$B_{\mathrm{FD}} = m^2 n(n-1)\int_{t_1 \geqslant t_2 \geqslant v^*_{\mathrm{FD}}} \left(\frac{t_2}{2} + \frac{t_2^2}{2t_1}\right) F(t_1)^{m-1} F(t_2)^{nm-m-1} \mathrm{d}F(t_1) \mathrm{d}F(t_2).$$

Subtracting aggregate investment in the case of no disclosure, Eq. (9), obtain

$$\frac{B_{\mathrm{FD}}-B_{\mathrm{ND}}}{m^2n(n-1)}=\int_{t_1\geqslant t_2\geqslant v^*_{\mathrm{FD}}}\left(\frac{t_2}{2}+\frac{t_2^2}{2t_1}\right)F(t_1)^{m-1}F(t_2)^{nm-m-1}\mathrm{d}F(t_1)\mathrm{d}F(t_2)$$
$$-\int_{t_1\geqslant t_2\geqslant v^*}t_2F(t_2)^{nm-2}\mathrm{d}F(t_1)\mathrm{d}F(t_2)$$
$$\geqslant\int_{t_1\geqslant t_2\geqslant v^*}t_2F(t_2)^{nm-m-1}\left[\left(\frac{1}{2}+\frac{t_2}{2t_1}\right)F(t_1)^{m-1}-F(t_2)^{m-1}\right]\mathrm{d}F(t_1)\mathrm{d}F(t_2)$$
$$\geqslant\int_{t_1\geqslant t_2\geqslant v^*}t_2F(t_2)^{nm-m-1}\left[\frac{t_2}{t_1}F(t_1)^{m-1}-F(t_2)^{m-1}\right]\mathrm{d}F(t_1)\mathrm{d}F(t_2).$$

The first inequality follows from the fact that $v^*_{\mathrm{FD}}\leqslant v^*$, and the second is obtained by replacing $\frac{1}{2}$ with $\frac{t_2}{2t_1}\leqslant\frac{1}{2}$.

Note that $t_1\geqslant t_2$ everywhere in the domain of integration. Thus, in order to show that the expression above is positive it suffices to show that $\frac{F(t)^{m-1}}{t}$ is increasing in t, which is equivalent to the assumption on the elasticity of $F(\cdot)$ in the proposition.

A.6 Proof of Proposition 5

By integrating over t_1, Eq. (18) can be simplified as follows:

$$B^{\max}_{\mathrm{ND}}=m(n-1)\int t_2F(t_2)^{nm-2}[1-F(t_2)^{nm}]\mathrm{d}F(t_2)$$
$$=\frac{m(n-1)}{nm-1}\int t\mathrm{d}F(t)^{nm-1}-\frac{m(n-1)}{2nm-1}\int t\mathrm{d}F(t)^{2nm-1}.$$

When either $n\to\infty$ or $m\to\infty$ (or both), each of the integrals converges to $\overline{v}$. The coefficients in front of the integrals converge to 1 and $\frac{1}{2}$, respectively, when $n\to\infty$ for a fixed m; and to $\frac{n-1}{n}$ and $\frac{n-1}{2n}$, respectively, when $m\to\infty$ for a fixed n. Thus, $B^{\max}_{\mathrm{ND}}$ converges to $\frac{\overline{v}}{2}$ when $n\to\infty$ for a fixed m; and to $\frac{(n-1)\overline{v}}{2n}$ when $m\to\infty$ for a fixed n.

A lower bound for $B^{\max}_{\mathrm{FD}}$ can be obtained by replacing t_1 with $\overline{v}$ in Eq. (19). Integrating over t_1 similar to the above, we obtain

$$B^{\max}_{\mathrm{FD}}>mn(n-1)\int\left(\frac{t_2}{2}+\frac{t_2^2}{6\overline{v}}\right)F(t_2)^{nm-m-1}[1-F(t_2)^m]\mathrm{d}F(t_2)$$
$$=n\int\left(\frac{t}{2}+\frac{t^2}{6\overline{v}}\right)\mathrm{d}F(t)^{(n-1)m}-(n-1)\int\left(\frac{t}{2}+\frac{t^2}{6\overline{v}}\right)\mathrm{d}F(t)^{nm}.$$

When $m\to\infty$, for n fixed, each of the integrals converges to $\frac{2\overline{v}}{3}$, and hence the lower bound of $B^{\max}_{\mathrm{FD}}$ converges to $\frac{2\overline{v}}{3}$ as well.

Consider now the limit $n \to \infty$, for m fixed. For brevity, let $\gamma(t) = \frac{t}{2} + \frac{t^2}{6\overline{v}}$. We can then write

$$B_{\text{FD}}^{\max} > n\mathbb{E}(\gamma(Y_{(n-1:n-1)})) - (n-1)\mathbb{E}(\gamma(Y_{(n:n)})), \tag{21}$$

where $Y_{(r:n)}$ are the order statistics from a sample of i.i.d. random variables distributed according to $F(\cdot)^m$. Note that $\gamma(\cdot)$ is a polynomial function; therefore, we can apply the recurrence relation for moments of order statistics (David and Nagaraja, 2003, Section 3.4):

$$(n-r)\mathbb{E}(\gamma(Y_{(r:n)})) + r\mathbb{E}(\gamma(Y_{(r+1:n)})) = n\mathbb{E}(\gamma(Y_{(r:n-1)})).$$

For $r = n-1$, this gives

$$\mathbb{E}(\gamma(Y_{(n-1:n)})) + (n-1)\mathbb{E}(\gamma(Y_{(n:n)})) = n\mathbb{E}(\gamma(Y_{(n-1:n-1)})),$$

and (21) then implies

$$B_{\text{FD}}^{\max} > \mathbb{E}(\gamma(Y_{(n-1:n)})),$$

where the right-hand side converges to $\frac{2\overline{v}}{3}$. This establishes the claim.