Contests with Network Externalities: Theory & Evidence

Luke Boosey[*] Christopher Brown[†]

July 9, 2021

Abstract

We study competitive behavior in all-pay Tullock (1980) contests with identity-dependent externalities (IDEs) governed by a fixed network. First, we introduce a model of *network contest games*, in which the prize generates an externality—which may be positive or negative—that impacts each player directly connected by the network to the winner of the contest. We establish existence of Nash equilibria and provide sufficient conditions for uniqueness, building on recent theoretical advances for games played on networks. We then derive closed-form results, with an intuitive characterization, for regular networks and for a subclass of core-periphery structures. Second, using a controlled laboratory experiment, we provide robust empirical support for the comparative statics predictions of the model. Our experimental findings also suggest that observed patterns of mean over-investment relative to point predictions may be driven by both heterogeneous *joy of winning* and *social efficiency concerns* that emerge in the presence of IDEs. Altogether, our study provides a novel application for the theory of network games, and new insights regarding behavior in all-pay contests.

Keywords: contests, networks, identity-dependent externalities, network games, best-response potential, experiment, joy of winning

JEL classification codes: C72, C92, D72, D74, D85, Z13

[*]Department of Economics, Florida State University, 113 Collegiate Loop, Tallahassee, FL 32306-2180, USA; E-mail: `lboosey@fsu.edu`

[†]Krannert School of Management, Purdue University, 403 W State St, West Lafayette, IN 47907-2056, USA; E-mail: `cbrown.econ@gmail.com`

1 Introduction

In virtually all areas of social and economic interaction, one can find examples of agents competing with each other in pursuit of some valuable prize. Individuals and organizations will often expend significant resources on marketing, advertising, and lobbying in order to outperform their rivals and command a greater influence over market allocations or political outcomes. A long tradition of research in industrial economics, public choice, and political economy has explored competitive behavior in rent-seeking environments, R&D competition, patent races, political campaigns, and promotion tournaments. Many of these settings are modeled as *contests*, in which agents exert costly effort or make irreversible investments and the winner takes all.

A standard simplifying assumption when modeling contests is that losing agents are indifferent to the identity of the winner. However, agents may have considerably more general preferences over the possible allocations of the prize. In particular, agents who do not win the contest may care a great deal about who does, especially if the allocation of the prize affects the nature of subsequent interactions between the contestants. In the related context of winner-pay auctions, Jehiel et al. (1996) introduced the notion of *identity-dependent externalities* (or IDEs) as a way of capturing the consequences of the allocation for bidders in post-auction interactions. For example, such externalities may arise in relation to the assignment of exclusive licensing agreements (Brocas, 2003), the sale of a nuclear weapon or location of environmentally hazardous enterprises (Jehiel et al., 1996), competition for access to a cost-reducing process innovation, or the allocation of talent across teams (Das Varma, 2002).

A relatively smaller number of studies consider the implications of IDEs for *all-pay contests* (see, e.g., Linster, 1993; Esteban and Ray, 1999; Konrad, 2006; Klose and Kovenock, 2015), however, there remain many interesting questions to explore. For instance, in many settings the structure of IDEs will be governed by an underlying network of connections. As such, there are naturally arising questions regarding the impact of network structure on competitive behavior which, to date, have not been addressed by the existing literature on IDEs in auctions and contests.

For example, consider a collection of community councils lobbying a city planning committee in charge of selecting the location for a new public facility. Each community's ideal outcome would be to have the facility located within their own neighborhood. However, if the facility generates positive externalities or is more easily accessible to neighborhoods that are sufficiently close to the eventual location, it is natural to expect that lobbying activity will depend on the geographical network connecting the communities. If the externalities are sufficiently strong, or the communities sufficiently well-connected, they may engage in less lobbying activity than if it is more difficult to access a facility located outside their own neighborhood.

Alternatively, the investment decisions made by firms competing for an exclusive licensing agreement will typically depend on the rivalry structure in the firms' product market space. Firms who operate in close proximity to the winning firm may be significantly worse off than other unsuccessful firms.[1] How might the structure of product market rivalry affect rent-seeking

[1] A similar example can be made in the context of professional sporting organizations competing for the services of a talented free-agent athlete. For instance, in Major League Baseball, the Boston Red Sox (part of the American

behavior in this setting? The natural intuition in this case suggests that the negative externalities associated with the exclusive license will intensify competition among firms who are engaged in markets with more heated rivalry.

In this paper, we study the effects of network-based identity-dependent externalities on competitive behavior in all-pay contest environments. To do so, we develop and analyze a theoretical model of a *network contest game* and then test the predictions of the model in a controlled laboratory experiment. Our theoretical framework builds on recent developments to the understanding of strategic behavior in games played on networks (Bramoullé et al., 2014). We concentrate on Tullock (1980) contests, the most commonly studied formulation for *imperfectly-discriminating* all-pay contests, wherein each player's probability of winning the contest is increasing in her own effort investment, relative to the investments of others. The primary innovation of our model is the introduction of a network that governs the flow of externalities from the winning player to her neighbors.

We start by establishing the existence of a Nash equilibrium for general network structures and externalities (Theorem 1). The main challenge to existence is the fact that payoff functions in the network contest game are (like the standard contest environment) discontinuous at zero. We rely on results from Reny (1999) and Bagh and Jofre (2006) to prove existence. Then, we provide closed-form characterizations of equilibria for two broad classes of network structures: *regular* networks and (a subclass of) *core-periphery* networks.

For regular networks, there exists a symmetric equilibrium in any network contest game. Moreover, comparative statics with respect to the size of the externality and the density of the network are consistent with the intuition highlighted by the motivating examples given above. For instance, positive externalities introduce incentives for players to free ride on their neighbors' investments, leading to lower equilibrium investment. Conversely, negative externalities drive up the effective value of winning the contest, intensifying competition and increasing equilibrium investment. Each of these effects is amplified as the network becomes more densely connected, as captured by an increase in the common *degree* for regular networks. Nevertheless, the symmetric equilibrium in regular networks is typically not unique. For instance, we show that when externalities are positive and sufficiently strong, there may also exist a *specialized equilibrium*, in which some subset of the players choose to be inactive (invest nothing) in the contest.

We then show that (semi-)symmetric equilibria in our subclass of core-periphery networks also take the form of a specialized equilibrium for sufficiently strong, positive externalities. In particular, highly connected core players, facing stronger free-riding incentives than peripheral players, invest nothing in equilibrium. In contrast, when the prize allocation generates stronger negative externalities, the core players—who are more exposed by the structure of the network—increase their equilibrium investment substantially compared to the peripheral players.

League East Division) might be much happier to see a top free agent player sign a deal with the San Diego Padres (who are in the National League West Division) than with the New York Yankees, who play in the same League and Division as Boston. There are, of course, several other considerations that influence the negotiations between sporting teams and free agent athletes, including salary demands, team budgets, contract length, synergies with existing team members, and the athlete's locational preferences. Nevertheless, the point is that competition in these kinds of settings, which may include both winner-pay and all-pay components, is likely influenced by the anticipated interest and activity of rival teams.

Having already demonstrated the potential for multiple equilibria, we then provide sufficient conditions for there to be a *unique* Nash equilibrium (Theorem 2). Our characterization closely follows the seminal approach developed by Bramoullé et al. (2014) for network games with linear best replies. However, adapting their results to the network contest game turns out to be a non-trivial exercise. In particular, because best replies are non-linear in Tullock (1980) contests, the main results derived by Bramoullé et al. (2014) cannot be directly applied. Moreover, other approaches based on variational inequalities (VI) that have been applied to network games without linear best replies (see, e.g., Melo, 2018; Parise and Ozdaglar, 2019; Zenou and Zhou, 2020) also do not apply.

Nevertheless, we demonstrate that the key insights provided by Bramoullé et al. (2014) can be suitably adapted to the network contest game. For instance, a key part of our uniqueness theorem relates the size of the externalities in the network contest game to the lowest eigenvalue of the network, which also plays a crucial role in Bramoullé et al. (2014).[2] While Bramoullé et al. (2014) exploit the theory of potential games (Monderer and Shapley, 1996) to derive their results, our formulation does not admit an exact best response function. However, we establish that the network contest game is a *best-reponse potential game* (Voorneveld, 2000), which allows us to take an analogous approach. As such, our theoretical framework establishes new results extending both the well-developed literature on contest theory and the growing body of work studying strategic behavior in network games.

In the second part of the paper, we complement our theoretical analysis with an empirical contribution by testing the predictions of the model in a controlled laboratory experiment. In general, empirical analysis of interaction in networks using naturally occurring data is extremely challenging. As such, laboratory experiments can be especially useful for testing the theoretical predictions of the model and identifying additional behavioral factors that influence competition. In our experiment, subjects are placed into groups of six and assigned to positions in one of four network configurations — the complete network, a circle (or ring) network, a star network, and a core-periphery network with two core players. We then implement three different conditions that vary the size and sign of the externality: a strong negative externality, a strong positive externality (of the same magnitude as in the negative condition), and a baseline control in which the network structure is retained but externalities are set equal to zero.

Overall, our main experimental findings provide strong support for the theoretical predictions. At the aggregate level, the comparative static predictions across treatments are well supported by the observed patterns of mean investment. The lone exception is in the circle network, where the effect of the positive externalities is slightly weaker than predicted when using all rounds of the experiment. However, allowing for the effects of experience by excluding the earlier rounds, we find that even this exceptional case realigns with the predicted comparative static results. Moreover, the circle network (with positive externalities) is the only condition for which there are multiple equilibria. Thus, we also examine this condition more closely, to see whether behavior is consistent with the symmetric equilibrium, a specialized equilibrium, or neither. We find little

[2] As discussed by Bramoullé et al. (2014), the lowest eigenvalue captures the "two-sidedness" or "bipartiteness" of the graph. When the lowest eigenvalue (which is negative) is sufficiently large in magnitude, the amplification of agents' interactions increases the chances of multiple equilibria.

support for symmetric equilibrium play. For some groups, investment activity is more consistent with the predicted patterns of specialized equilibrium play. However, the coordination problem that arises with multiple equilibria prevents any clear picture from forming.

In addition, in most treatment conditions, we observe mean over-investment relative to the Nash equilibrium prediction, along with substantial variance across individuals. This finding coincides with the widespread documentation of over-investment (and over-spreading) in standard contest experiments without externalities (see, e.g., Sheremeta, 2013). A prominent explanation for this type of behavior is that individuals derive non-monetary utility from winning (commonly referred to as "joy of winning") beyond the actual monetary value of the prize. In our baseline conditions (with no externalities), we observe over-investment levels consistent with the rest of the experimental literature. We then offer some support for the "joy of winning" hypothesis, using an elicitation procedure pioneered by Sheremeta (2010) to measure subjects' preferences for winning *per se*, and showing that subjects with higher joy of winning also tend to invest more in the main contests.

In the two regular networks (complete and circle), we find that over-investment is sensitive to the externality condition. In particular, the degree of over-investment is much smaller for positive externalities than it is for negative externalities. In this case, we explain the differences between treatment conditions by arguing that joy of winning is heightened in the presence of negative externalities, and (partly) suppressed when there are positive externalities. Regression analysis provides statistical support for this argument.

In contrast, for the two core-periphery networks, the patterns of over-investment suggest a different explanation. With negative externalities, we observe no evidence of mean over-investment by the core players but substantial over-investment by the peripheral players. Conversely, with positive externalities, peripheral players exhibit little to no over-investment, on average, while the core players display significant over-investment relative to the equilibrium prediction. Unlike for the regular networks, joy of winning that is sensitive to the externality cannot fully explain these observed patterns of behavior. Instead, we show how the behavior in core-periphery networks may depend on the presence of social efficiency concerns among subjects.

The basic intuition stems from the fact that the equilibrium prediction for core-periphery networks entails a particular kind of inefficiency with regards to the aggregate flows of externalities. For instance, in the star network, the equilibrium outcomes generate (i) a high probability of widespread harm with negative externalities, and (ii) the minimal aggregate flow of benefits with positive externalities. In the former case, peripheral players with a collective concern for social efficiency may over-invest in hopes of reducing the chances that the core player wins (thereby harming everyone else). In the latter case, a concern with social efficiency might explain more restrained investment by peripheral players and greater participation by the core player, since the aggregate flow of externalities is maximized when the prize is awarded to the core player. A closer inspection of the data at the group-level offers some support for this argument, rounding out the discussion of our experimental findings.

Related literature.—Our study contributes to and draws together two separate literatures. The first of these explores the implications of identity-dependent externalities for strategic behavior in competitive environments. The second is the relatively more recent literature studying games played on networks. In addition, our work naturally relates to the vast body of theoretical and experimental research on behavior in contests.

Both the pioneering work by Jehiel et al. (1996) and the majority of the subsequent related literature are primarily concerned with optimal selling procedures in the presence of identity-dependent externalities.[3] Other related work has focused on strategic non-participation in auctions, especially with negative externalities (see, e.g., Jehiel and Moldovanu, 1996; Brocas, 2003) and explored the notion of *type-dependent* externalities (Brocas, 2013a, 2014), according to which the externality flows are correlated with the players' private valuations and not just their identities. In all-pay contest environments, there are a handful of related studies, including Konrad (2006) and Klose and Kovenock (2015), both of which characterize equilibria in the context of (perfectly-discriminating) all-pay auctions.[4] There are, however, relatively few studies that consider externalities in the context of *imperfectly-discriminating* all-pay contests. One exception is Linster (1993), who analyzes the equilibrium of a generalized Tullock contest in which the players are not indifferent to who wins the prize, if they themselves do not. He describes his model as a generalization of the results of Tullock (1980), wherein the prize is a private good, and those of Katz et al. (1990), in which the prize is a pure public good.

Another exception is Esteban and Ray (1999), who explore the relationship between equilibrium conflict and the distribution of preferences over outcomes in a lottery contest between interest groups. A crucial aspect of their model is the introduction of a "metric" over the different groups, which allows for spatial preferences over the preferred outcomes of other interest groups. While both of these studies incorporate the notion of identity-dependent externalities into a Tullock-style contest, neither draws a formal connection between these externalities and the underlying network structure that governs them. In contrast, the central objective of our study is to bring together the extant literature on identity-dependent externalities and the relatively more recent developments in the theory of network games.

Recent years have seen a substantial increase in research studying social and economic networks.[5] In particular, our study contributes to the burgeoning stream of research examining strategic behavior in games played between agents on a network. Typically, this literature examines games with linear best replies (see, e.g., the linear-quadratic utility functions in Ballester

[3]For instance, Jehiel et al. (1996, 1999) characterize the revenue-maximizing auctions for alternative information structures (including the case where externality flows are private information), Jehiel and Moldovanu (2000) study efficient auction design with externalities, while Das Varma (2002) characterizes the revenue and efficiency rankings of the standard sealed-bid and open ascending bid auction formats. See Jehiel and Moldovanu (2006) for a summary of the literature on standard, winner-pay auctions with identity-dependent externalities. In addition, Lu (2006) and Brocas (2013b) extend the analysis of the optimal auction to include the possibility of externalities between the seller and the bidders, whereas Aseff and Chade (2008) derive the optimal mechanism for a seller with multiple identical units.

[4]Konrad (2006) examines the effect of cross-shareholdings between firms competing for contracts. Klose and Kovenock (2015) prove existence of equilibria in the presence of identity-dependent externalities and characterize necessary and sufficient conditions for the existence of an equilibrium in which only two players are active bidders. They show that unless the externalities are 'small', many results from the standard all-pay auction literature regarding equilibrium strategies and payoffs no longer hold.

[5]See Jackson (2014) for a discussion of the importance of studying networks and how it can better inform our understanding of a wide range of economic behavior.

et al., 2006; Bramoullé and Kranton, 2007; Bramoullé et al., 2014). There are also approaches that consider games with non-linear best replies, including Allouch (2015) who studies the private provision of local (network-based) public goods, and Melo (2018), Parise and Ozdaglar (2019), and Zenou and Zhou (2020), each of whom apply techniques based on variational inequalities (VI) to establish existence and uniqueness in network games without assuming linear best replies. As discussed above, our model also entails non-linear best replies. However, the VI approaches adopted by Melo (2018) and Parise and Ozdaglar (2019) rely on an assumption that the objective function for each agent depends only on own action and a neighborhood aggregate, which is not satisfied in our contest game due to the dependence of the contest success function on all players' actions. Despite these challenges, we demonstrate that many of the insights offered by Bramoullé et al. (2014) for network games with linear best replies can be adapted to our network contest game.

To the best of our knowledge, the only other study to forge this connection between network games and externalities in a lottery contest game is König et al. (2017). They develop a stylized model of conflict to capture the impact of informal networks of alliances and enmities on conflict expenditures and outcomes, then apply their model to study empirically the Second Congo War. In their model, agents (groups in their setting) compete for a divisible prize in which any group's share of the prize depends on the group's relative *operational performance*, which takes the form of a generalized Tullock CSF. However, in contrast with our model, which features externalities generated by the allocation of the prize, the effort investments of other groups in König et al. (2017) feed directly into each group's operational performance through the underlying network of alliances and enmities. Each group's operational performance is increasing in own effort and the effort of its allies, and decreasing in the effort of its enemies, but there are no allocation-based spillovers.[6] While this surely captures an interesting feature of the contest technology that is absent from our model, it does not allow for the identity-dependent externalities (resulting from the allocation of the prize) that are the primary focus of our study.

Finally, for the most part, experimental research on network games has focused either on coordination problems and games with strategic complementarities (see, e.g., Keser et al., 1998; Berninghaus et al., 2002; Cassar, 2007; Gallo and Yan, 2015) or on public goods games where actions are strategic substitutes (Rosenkranz and Weitzel, 2012; van Leeuwen et al., 2019). Charness et al. (2014) examine both games of strategic complements and strategic substitutes, varying whether subjects in the experiment have complete or incomplete information about the network, in order to test the predictions of Galeotti et al. (2010) for network games.[7] Thus, the experimental literature has largely focused on games with a relatively simple structure (e.g., binary actions, linear-quadratic utility, local interaction or core-periphery networks). We provide

[6]There is also a related, though distinct literature on *conflict networks* (see, e.g., Goyal and Vigier, 2014; Franke and Öztürk, 2015; Matros and Rietzke, 2018; Kovenock and Roberson, 2018; Xu et al., 2019) and the formation of conflict networks (Hiller, 2017; Jackson and Nei, 2015). In contrast with both our model and the model in König et al. (2017), these studies typically focus on environments where the network is used to describe the structure of conflict between agents who participate in *multiple battles*.

[7]There is also a substantial amount of experimental research on cooperation in prisoners' dilemma games played on networks, and on social learning in networks, which is summarized nicely in the chapter by Choi et al. (2016) on networks in the laboratory.

the first experimental study of a contest game played on a network. As such, our findings serve to simultaneously broaden the experimental literature on network games and provide a novel extension on the rich body of work on contest experiments.

The remainder of the paper is organized as follows. Section 2 presents our theoretical model and derives the main results on existence and uniqueness of Nash equilibria in the network contest game. Specific results for the class of regular networks and a class of *core-to-periphery* structures are also provided, with several examples, in this section of the paper. Section 3 describes the design of our experiment and highlights our experimental predictions, based on the theoretical framework. The experimental findings are presented and discussed in Section 4, with concluding remarks in Section 5.

2 Theoretical Framework

2.1 Preliminaries

Consider an environment with a set of individuals $N = \{1, \ldots, n\}$ arranged in a network, described by the adjacency matrix $\mathbf{G}$, where $g_{ij} = g_{ji} = 1$ if distinct agents i and j are linked, and $g_{ij} = g_{ji} = 0$ otherwise. We follow the convention that $g_{ii} = 0$ for all $i \in N$. Each individual competes in a contest by choosing a level of investment (or effort) $x_i \geqslant 0$. Let $\mathbf{x}_{-i}$ denote the vector of investments chosen by all individuals other than i and suppose the probability of player i winning the contest is given by the Tullock (1980) lottery contest success function. That is,

$$P_i(x_i, \mathbf{x}_{-i}) = \begin{cases} \frac{1}{n}, & \text{if } \sum_{h=1}^{n} x_h = 0, \\ \frac{x_i}{\sum_{h=1}^{n} x_h}, & \text{otherwise.} \end{cases} \qquad [1]$$

The winner of the contest receives a prize $V > 0$. We assume, without loss of generality, that the value of the prize is normalized to $V = 1$. In the standard contest setting, player i's payoff from winning is $V = 1$, while the payoff from losing is 0, regardless of who among the other players wins the contest. In such a setting, it is a well-known result (see, e.g., Szidarovszky and Okuguchi, 1997) that the unique equilibrium is symmetric, given by $x_i = \bar{x}$ for all $i = 1, \ldots, n$, where

$$\bar{x} = \frac{n-1}{n^2} \qquad [2]$$

The main innovation in our model is that there are identity-dependent externalities generated by the prize that, together with the network, lead to different possible payoffs for player i when she does not win the contest.

In particular, if a player does not win the contest, her payoff depends on whether or not she is linked to the winner. The allocation of the prize to a player i imposes an externality αV, with $\alpha \in [-1, 1)$, on each agent who is connected to i; i.e., each agent j with $g_{ij} = 1$. If $g_{ij} = 0$, no externality is imposed on player j.[8] All players have the same linear cost of effort function,

[8]Notice that the model incorporates a few stylized assumptions about the externality. In particular, the externality generated by allocating the prize to player i is the same for all of player i's neighbors, and does not spillover beyond the winner's immediate neighbors. We view these as natural starting points from which the

$c(x_i) = x_i$. Thus, the expected payoff to player i from a profile of investments $(x_i, \mathbf{x}_{-i})$ is given by

$$\pi_i(x_i, \mathbf{x}_{-i}; \mathbf{G}) = P_i(x_i, \mathbf{x}_{-i}) - x_i + \alpha \sum_{j=1}^{n} g_{ij} P_j(x_j, \mathbf{x}_{-j}). \quad [3]$$

Throughout the paper, we refer to the game as a *network contest game*, represented in normal form as $\Gamma = (X_i, \pi_i)_{i=1}^n$ where $X_i = \mathbb{R}_+$ represents the strategy set for player i, and $\pi_i(\cdot)$ is the payoff function defined in [3].

2.2 Equilibrium Analysis

We start our analysis by noting that any strategy profile with only one active agent cannot be a Nash equilibrium. Indeed, for a strategy profile $\mathbf{x}$ with $x_j > 0$ and $\mathbf{x}_{-j} = \mathbf{0}$, player j's best response function is empty. Thus, we can restrict attention to strategy profiles with at least two active agents. Similarly, given $\alpha < 1$, it is also straightforward to show that $\mathbf{x} = \mathbf{0}$ is not an equilibrium.

Consider player i and fix a profile $\mathbf{x}_{-i}$ with at least one strictly positive investment. The expected payoff for player i in equation (3) can be rewritten as

$$\pi_i(x_i, \mathbf{x}_{-i}; \mathbf{G}) = \frac{x_i}{\sum_{h=1}^n x_h} - x_i + \alpha \sum_{j=1}^{n} g_{ij} \frac{x_j}{\sum_{h=1}^n x_h} \quad [4]$$

for all $x_i \geqslant 0$ and all $\mathbf{x}_{-i} \neq \mathbf{0}$. Note that $\partial^2 \pi_i / \partial x_i^2 < 0$ so that the payoff functions are strictly concave. Thus, player i's best response to $\mathbf{x}_{-i}$ is a well-defined, single-valued function given by

$$f_i(\mathbf{x}_{-i}; \alpha, G) = \max\left\{0, \left[\sum_{h \neq i} x_h (1 - \alpha g_{ih})\right]^{0.5} - \sum_{h \neq i} x_h\right\}. \quad [5]$$

As in the standard contest game, the best response functions are non-linear. As such, the main analysis of uniqueness and stability for network games developed in Bramoullé et al. (2014) cannot be directly applied. Moreover, the payoff functions do not satisfy the assumptions on the objective function required to apply the variational inequalities approach followed by Parise and Ozdaglar (2019) and Melo (2018) for network games with non-linear best replies.[9] When $\alpha = 0$, the best response functions are, as expected, the same as those for the standard contest game, for which existence and uniqueness are well established. For $\alpha \neq 0$, the issue is not quite as straightforward. We investigate the issue of uniqueness in section 2.3. To prove existence of a pure strategy Nash equilibrium, we rely on results from Reny (1999) and Bagh and Jofre

model might be generalized. We also assume that the externality parameter does not depend on the winner's identity. As such, all of the heterogeneity that arises in the model is captured by an agent's position within the network.

[9] They each consider games in which the objective function depends on x_i and a neighborhood aggregate, $\sum_h g_{ih} x_h$, but does not depend otherwise on x_j if $g_{ij} = 0$. In our setting, the payoff of an agent i depends on each x_j through the CSF, even if $g_{ij} = 0$.

(2006), to deal with the fact that payoff functions are discontinuous at $\mathbf{x} = 0$.

Theorem 1 (Existence)**.** *The network contest game possesses a pure strategy Nash equilibrium.*

Proof. Here, we highlight the main idea behind the proof of Theorem 1, which is detailed in Appendix A. In particular, existence follows from Theorem 3.1 in Reny (1999). In order to apply the theorem, we establish that the network contest game is compact, quasi-concave, and better-reply secure. For the latter, we show that the game is payoff secure and *weakly reciprocal upper semicontinuous* (wrusc), which is a condition introduced by Bagh and Jofre (2006), who then prove that payoff security and wrusc imply better-reply security. □

Next, we provide a characterization of equilibrium profiles. For a given profile $\mathbf{x}$, we denote the set of active agents (those for whom $x_i > 0$) by A and the set of inactive agents by $N - A$. The following lemma provides a straightforward characterization of the set of Nash equilibria for the network contest game with externality α and network $\mathbf{G}$.

Lemma 1. *An investment profile* $\mathbf{x}$ *with active agents* A *is a Nash equilibrium if and only if* $|A| \geqslant 2$ *and*

(i) *for all* $i \in A$,

$$\sum_{j \in A}(1 - \alpha g_{ij})x_j - x_i = \left(\sum_{j \in A} x_j\right)^2 \quad [6]$$

(ii) *for all* $i \in N - A$,

$$\sum_{j \in A}(1 - \alpha g_{ij})x_j \leqslant \left(\sum_{j \in A} x_j\right)^2 \quad [7]$$

It's instructive to begin by considering the *complete* network, in which each agent is linked to every other agent. In this case, every non-winning agent is always impacted (symmetrically) by the winning agent, rendering the externalities identity-*independent*.[10] As such, the difference in payoffs from winning and losing is always equal to the difference between the prize V and the externality term αV. We obtain the following intuitive result for the network contest game with the complete network.

Proposition 1. *Consider the game in which* $\mathbf{G}$ *is the complete network,* $\mathbf{K}_n$*. For any* $\alpha \in [-1, 1)$*, there exists a unique Nash equilibrium, in which all players are active and choose the symmetric investment*

$$\bar{x}_K^\alpha = \frac{(n-1)(1-\alpha)}{n^2}.$$

Since the proof is straightforward, we instead highlight the underlying intuition. In the complete network, the payoff from losing is no longer identity-dependent. The payoff does not depend on which of the other players wins, because in any outcome, the externality is imposed

[10] Note that this characterization relies on the modeling assumption that the size of the externality is homogenous across winning agents and their neighbors.

on all non-winning agents. As a result, the game can be reformulated as a standard contest without externalities but with a prize value equal to the difference between the payoff from winning and the payoff from losing, which is $V - \alpha V = V(1-\alpha)$. Redefining $\widehat{V} = V(1-\alpha)$ (and setting $V = 1$) the result follows from the fact that the unique equilibrium in a standard contest with prize $\widehat{V}$ is $(n-1)\widehat{V}/n^2$.

Although this is a special case in which the network structure eliminates the identity-dependent component of the model, the basic intuition extends naturally to *symmetric* equilibria in the class of *regular networks*.

2.2.1 Equilibria in regular networks

For the network graph $\mathbf{G}$, we let $d_i = \sum_j g_{ij}$ denote player i's degree. Then $\mathbf{G}$ is a regular network (or regular graph) of degree k if $d_i = k$ for all $i \in N$. The next result establishes existence of a symmetric equilibrium in any regular network $\mathbf{G}$ for any $\alpha \in [-1, 1)$.

Proposition 2. *Consider the game with network* $\mathbf{G}$ *and externality* $\alpha \in [-1, 1)$*. If* $\mathbf{G}$ *is a regular network of degree* $k \in \{0, \ldots, n-1\}$*, then there exists a symmetric, pure strategy Nash equilibrium,* $\mathbf{x}^* = (x^*, \ldots, x^*)$*, where*

$$x^* \quad = \quad \frac{n - 1 - \alpha k}{n^2}. \qquad [8]$$

Proof. The proof is provided (along with all subsequent proofs) in Appendix A. □

Several remarks are in order. First, as should be expected, when $\alpha = 0$ or $k = 0$ (which is the case when $\mathbf{G}$ is the empty network), we obtain $x^* = \bar{x}$, which corresponds to the standard contest with no externalities. Second, when $k = n-1$, $\mathbf{G}$ is the complete network $\mathbf{K}_n$, and we obtain $x^* = \bar{x}_K^{\alpha}$. More importantly, comparative statics with respect to α and k have natural and intuitive interpretations.

For positive externalities ($\alpha > 0$), players are more inclined to free ride off their neighbors' investments, and thus, equilibrium investment is lower than in the standard contest without externalities. For $k < n-1$, free-riding incentives are weaker than for the complete network, since the externalities are identity-dependent. For negative externalities ($\alpha < 0$), the effective value of winning the contest increases so that competition intensifies, pushing equilibrium investment higher than in the standard contest. For both positive and negative externalities, these effects are amplified as k increases, which corresponds to an increase in network density.

Although there is a unique equilibrium when $\alpha = 0$ or when $k = n-1$, the symmetric equilibrium for incomplete networks need not be unique. In particular, for many networks, when α is positive and sufficiently large, there also exists a *specialized equilibrium*, defined below, in which some agents are inactive.

Definition 1. A **specialized equilibrium** is a Nash equilibrium $\mathbf{x}^*$ in which the set of active players A forms a maximal independent set. That is, for any two players $i, j \in A$, $g_{ij} = 0$, while for every $k \in N - A$, $\sum_{j \in A} g_{kj} \geqslant 1$.

For a given network $\mathbf{G}$ and a set of active agents A, let $d_A^i = \sum_{j \in A} g_{ij}$ denote the number of active agents linked to agent $i \in N$. Then, define $d_{N-A,A} = \min_{i \in N-A} d_A^i$. Finally, let $n_A = |A|$ denote the number of active agents in A.

Proposition 3. *Consider the game with network* $\mathbf{G}$ *and externality* $\alpha \in [-1, 1)$.

(i) *There exists a specialized equilibrium,* $\mathbf{x}^*$*, with active agents* A *and inactive agents* $N - A$*, if and only if* $\alpha \geqslant \frac{1}{d_{N-A,A}}$.

(ii) *In every specialized equilibrium,* $x_i^* = \bar{x}_A$ *for all* $i \in A$*, where* $\bar{x}_A = \frac{n_A - 1}{n_A^2}$.

Proposition 3 establishes that, in fact, in any specialized equilibrium, each inactive player must be linked to at least two active players.[11] Moreover, a specialized equilibrium is symmetric for players in A. That is, each active player chooses the same investment, corresponding to the equilibrium investment in a standard contest (without externalities) among n_A agents.

Corollary 1. *Specialized equilibria do not exist for negative externalities* ($\alpha < 0$).

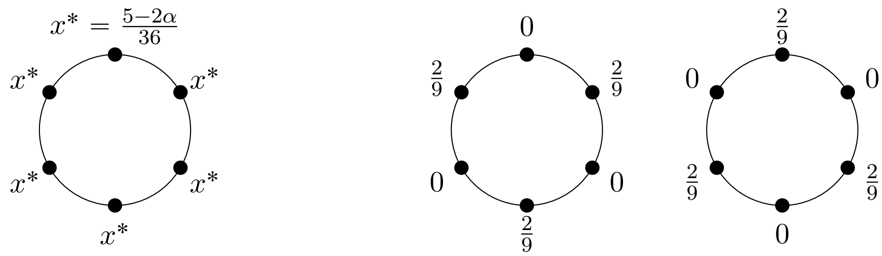
When α is sufficiently large, inactive players are content to exit the competition for the prize because they can free ride off their active neighbors and enjoy the positive externality that accrues if one of their neighbors wins. The greater the number of active neighbors, the lower the externality can be for the inactive player to opt out of the competition, but α must always be positive for a specialized equilibrium to exist.

Combining Proposition 3 with Proposition 2, it follows that for *regular networks*, there may exist multiple equilibria. Whenever the graph has a maximal independent set A with $\alpha \geqslant 1/d_{N-A,A}$, there is both a specialized equilibrium and the symmetric equilibrium with full participation.[12] In addition, in many cases, there may exist multiple specialized equilibria corresponding to different maximal independent sets of agents. To illustrate this multiplicity, we present three examples of regular networks and highlight the ranges of α for which there exist both specialized equilibria and a symmetric equilibrium with $A = N$.

Example 1 (A circle (or ring) network). In the circle network, the players are arranged around a circle and linked to the two agents on either side. Thus, the circle network is regular of degree $k = 2$. Hence, there exists a symmetric equilibrium for any $\alpha \in [-1.1)$, in which all agents are active and each invests $\bar{x}_A = \frac{5-2\alpha}{36}$; see panel (a) in Figure 1. Moreover, for $n = 6$, there are two maximal independent sets, as shown in Figure 1, panel (b). For each of these, $n_A = 3$, so that each active agent invests $\bar{x}_A = 2/9$. Furthermore, since every inactive player is linked to two active players, $d_{N-A,A} = 2$. Thus, the specialized equilibria exist if and only if $\alpha \geqslant 0.5$.

[11] Bramoullé et al. (2010) describe a maximal independent set *of order* r as a maximal independent set, A, with each node $j \in N - A$ connected to at least r nodes in A. As they note, while a maximal independent set exists for any graph, maximal independent sets of order r with $r > 1$ need not exist.

[12] Note that in some cases, such a maximal independent set may not exist. For instance, consider the circle network with $n = 5$ agents. In this network, every maximal independent set is of order at most one, meaning that there is always at least one inactive agent who is connected to only one active agent, i.e., $d_{N-A,A} = 1$. In this case, a specialized equilibrium does not exist for any $\alpha < 1$.



(a) Symmetric equilibria, $\alpha \in [-1.1)$

(b) Specialized equilibria, $\alpha \geqslant 0.5$

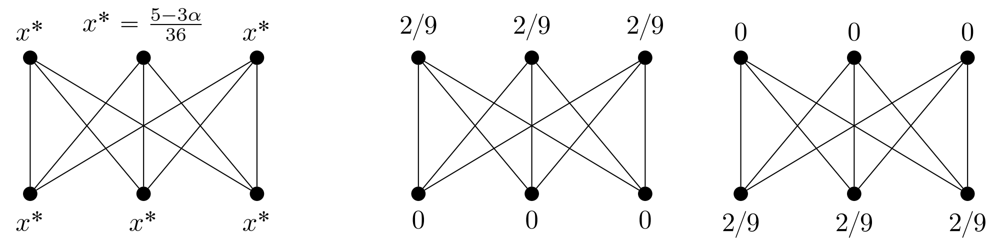
Figure 1. Equilibria in the circle network with $n = 6$ agents. Panel (a): A symmetric equilibrium with all agents active exists for any $\alpha \in [-1, 1)$. Panel (b): When $\alpha \geqslant 0.5$, there are two specialized equilibria, each characterized by a maximal independent set of three agents, with each active agent investing $\bar{x}_A = 2/9$.

Example 2 (A bipartite network). $\mathbf{G}$ is a bipartite graph if the nodes (agents) can be partitioned into two disjoint sets A and B, with $g_{ij} = 0$ for all $i, j \in A$ and $g_{kl} = 0$ for all $k, l \in B$. Figure 2 illustrates a complete bipartite graph with 6 agents. This network is regular of degree $k = 3$. Hence, there exists a symmetric equilibrium for any $\alpha \in [-1, 1)$, in which all agents are active and each invests $\bar{x}_A = \frac{5-3\alpha}{36}$; see panel (a) in Figure 2. Moreover, the three agents on the top and the three agents on the bottom represent the two maximal independent sets (as well as the two elements of the partition); see panel (b) in Figure 2. Given $n_A = 3$, each active agent invests $\bar{x}_A = 2/9$. Since the graph is a complete bipartite graph, each inactive agent in a specialized profile is linked to all of the active agents, so that $d_{N-A,A} = 3$. Thus, the specialized equilibria shown exist if and only if $\alpha \geqslant 1/3$.

Example 3 (A prism network). The prism network with $n = 6$ agents corresponds to the skeleton of a triangular prism; this network, which is regular of degree $k = 3$, is shown graphically in Figure 3. For this network, there exists a symmetric, interior equilibrium for any $\alpha \in [-1, 1)$, in which each agent invests $\bar{x}_A = \frac{5-3\alpha}{36}$; see panel (a). Additionally, for $\alpha \geqslant 0.5$, there are two specialized equilibria in which the set of active agents is a maximal independent set consisting of three agents, with each investing $\bar{x}_A = 2/9$.

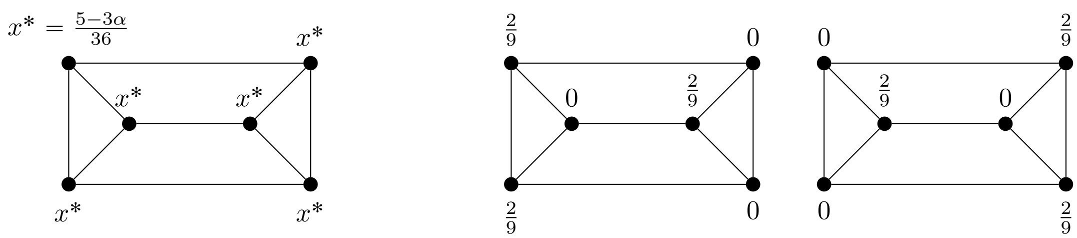
Although we have introduced specialized equilibria in the context of our analysis for regular networks, specialized equilibria are not related to any particular class of networks. As a simple demonstration, we provide an example of a line network with $n = 5$ agents, which is not regular. Note that, for a line network with an even number of agents (n even), for every maximal independent set of agents, there is always at least one inactive agent who is linked to just one active agent. Thus, by Proposition 3, there does not exist a specialized equilibrium for the line if n is even.

Example 4 (A line network). In the line network, whenever n is odd, there is a specialized equilibrium associated with the maximal independent set consisting of the endpoints of the line and every second node in between (see Figure 4). Every inactive agent is connected to two active agents, so that $d_{N-A,A} = 2$. Thus, the specialized equilibrium exists if and only if $\alpha \geqslant 0.5$.



(a) Symmetric equilibria, $\alpha \in [-1, 1)$ **(b)** Specialized equilibria, $\alpha \geqslant 1/3$

Figure 2. Equilibria in the complete bipartite network with $n = 6$ agents. Panel (a): A symmetric equilibrium with all agents active exists for any $\alpha \in [-1, 1)$. Panel (b): When $\alpha \geqslant 1/3$, there are two specialized equilibria, each characterized by a maximal independent set of three agents, with each active agent investing $\bar{x}_A = 2/9$.



(a) Symmetric equilibria, $\alpha \in [-1, 1)$ **(b)** Specialized equilibria, $\alpha \geqslant 0.5$

Figure 3. Equilibria in the prism network with $n = 6$ agents. Panel (a): A symmetric equilibrium with all agents active exists for any $\alpha \in [-1, 1)$. Panel (b): When $\alpha \geqslant 0.5$, there are two specialized equilibria, each characterized by a maximal independent set of three agents, with each active agent investing $\bar{x}_A = 2/9$.

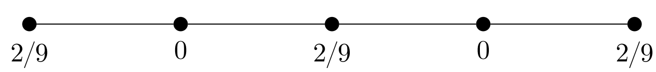


Figure 4. A specialized equilibrium for the line network with $n = 5$ agents exists if and only if $\alpha \geqslant 0.5$. The center agent and the agents at the endpoints of the line form a maximal independent set. Each active agent invest $\bar{x}_A = 2/9$.

In the next section, we shall see further examples of specialized equilibria in the context of another commonly studied class of networks – those that exhibit a core-periphery structure.

2.2.2 Equilibria in a subclass of core-periphery networks

The class of *core-periphery networks* is comprised of networks consisting of two types of agents – a set of highly connected *core* players, and a set of less connected *periphery* players. While this class of networks is very broadly defined, and contains many complex network structures, we restrict attention to a subset of the class that includes many of the most commonly studied core-periphery structures.

In particular, we define a subclass of core periphery referred to as *core-to-periphery* networks. In a *core-to-periphery* network, there are $n_1 \geqslant 1$ core players. All of the core players are connected to each other, creating a dense, or completely connected core. In addition, each core player is connected to $m \geqslant 1$ periphery players. We further assume that each periphery player is connected to a *single* core player and no other periphery players. Thus, there are $n = n_1(1+m)$ total players, comprised of $n_1 m$ periphery players, all with degree 1, and n_1 core players, each with degree $k = (n_1 - 1) + m$.

The conditions laid out in the previous paragraph are satisfied by, for instance, the *star network*, which has a single core player ($n_1 = 1$) connected to m periphery players. For all such *core-to-periphery* networks, we characterize the semi-symmetric equilibrium in which all players of the same type choose identical levels of investment. We denote the investment levels by x_c and x_p for core and periphery players, respectively.

Proposition 4. *Consider the game with network* $\mathbf{G}$ *and externality* $\alpha \in [-1, 1)$. *Suppose* $\mathbf{G}$ *is a* ***core-to-periphery*** *network with* n_1 *core players, each connected to* m *peripheral players. Then there exists a semi-symmetric, pure strategy Nash equilibrium in which every core player chooses the same investment* x_c^*, *and every peripheral player chooses the same investment* x_p^*, *where*

(i) *if* $\alpha < \frac{1}{m}$,

$$x_c^* = [1 - \alpha m]\Delta \qquad x_p^* = [1 + \alpha(n_1 - 2)]\Delta \tag{9}$$

where

$$\Delta = \frac{n_1[1 + m + \alpha m(n_1 - 3)] - [1 + \alpha(n_1 - 1 - \alpha m)]}{n_1^2[1 + m + \alpha m(n_1 - 3)]^2} \geqslant 0. \tag{10}$$

(ii) *if* $\alpha \geqslant \frac{1}{m}$,

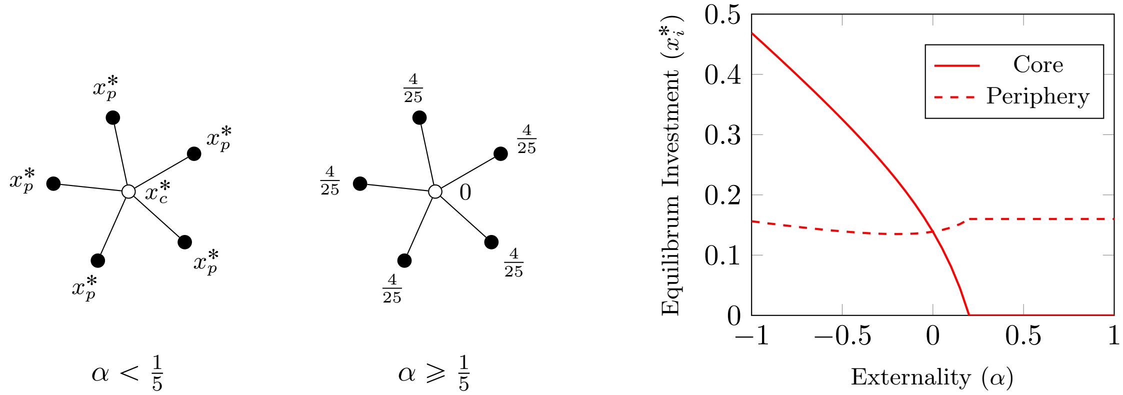
$$x_c^* = 0 \qquad x_p^* = \frac{n_1 m - 1}{(n_1 m)^2}. \tag{11}$$

Note that when $\alpha = 0$, the equilibrium investments reduce to the standard contest equilibrium,

$$x_c^* = x_p^* = \frac{n_1(1+m) - 1}{n_1^2(1+m)^2} = \frac{n-1}{n^2}. \tag{12}$$

For negative externalities and sufficiently small, positive externalities ($\alpha < 1/m$), the semi-symmetric equilibrium is interior; that is, both sets of agents are active. In addition, the semi-symmetric equilibrium investment for core players is decreasing in the externality (and strictly decreasing until they become inactive). In contrast, for periphery players, equilibrium investment is non-monotonic in α.

Moreover, for $\alpha < 0$, we have $x_c^* > x_p^*$. Intuitively, the core players are structurally more 'exposed' to the negative externality than are the less connected periphery players (who are linked only to a single core agent, by assumption). Accordingly, for $\alpha > 0$, free-riding incentives



(a) Star network, for $\alpha < \frac{1}{5}$ (left) and $\alpha \geqslant \frac{1}{5}$ (right) **(b)** Equilibrium investment for $\alpha \in [-1, 1)$

Figure 5. Semi-symmetric equilibria in the star network.

are also stronger for core players than for periphery players, so that $x_c^* < x_p^*$ in the semi-symmetric equilibrium with positive externalities.

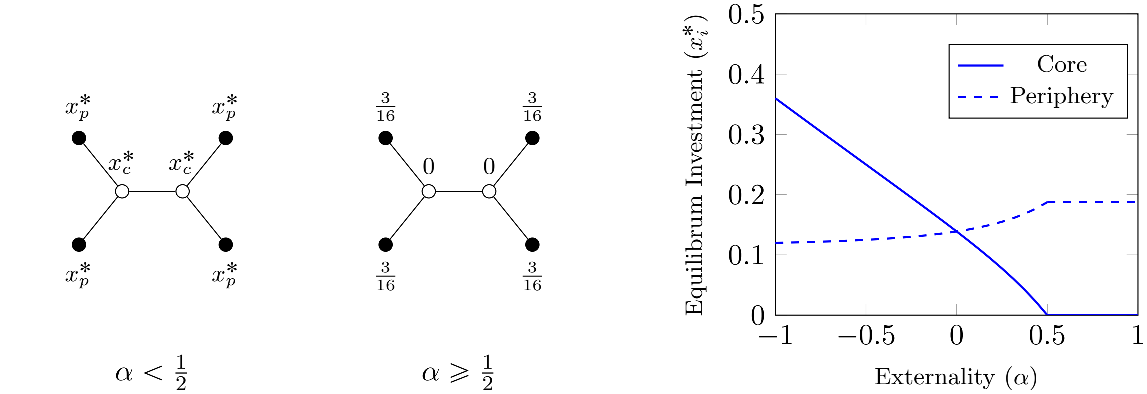
When the positive externality becomes sufficiently large ($\alpha \geqslant 1/m$), the semi-symmetric equilibrium is a *specialized equilibrium*. Free-riding incentives for the core players are sufficiently strong that they choose to be inactive in the contest. When this is the case, only the periphery players are active, and since they are not connected to each other, they form a maximal independent set and their equilibrium investment coincides with the equilibrium for a standard contest between $n_1 m$ players (i.e., the total number of periphery players). Thus, for the subclass of core-to-periphery network structures, strong positive externalities lead to polarization of competition in the semi-symmetric equilibrium. The following examples serve to illustrate the semi-symmetric equilibria in two common core-to-periphery network structures.

Example 5 (A star network). In a star network, there is a single core-player, such that $n_1 = 1$, and m peripheral players connected to the core (see Figure 5a where the core player is distinguished by the hollow node). For $m = 5$, the semi-symmetric equilibrium involves full participation when $\alpha < \frac{1}{5}$, with

$$x_c^* = \frac{5(1-5\alpha)(1-\alpha)^2}{4(3-5\alpha)^2} \qquad \text{and} \qquad x_p^* = \frac{5(1-\alpha)^3}{4(3-5\alpha)^2}.$$

When $\alpha \geqslant \frac{1}{5}$, the semi-symmetric equilibrium is a specialized equilibrium with A equal to the set of peripheral players, with $x_c^* = 0$ and $x_p^* = \frac{4}{25}$. Figure 5 shows the two cases on the network graph in panel (a) and in a graph that plots the equilibrium investment against α for both player types.

Example 6 (A core-periphery network with $n_1 = 2$). In the CP2 network (see Figure 6a), there are $n_1 = 2$ core players (distinguished by hollow nodes), each connected to 2 peripheral players.



(a) CP2 network, for $\alpha < \frac{1}{2}$ (left) and $\alpha \geqslant \frac{1}{2}$ (right) **(b)** Equilibrium investment for $\alpha \in [-1, 1)$

Figure 6. Semi-symmetric equilibria in the CP2 network.

Thus, the semi-symmetric equilibrium involves full participation when $\alpha < \frac{1}{2}$, with

$$x_c^* = \frac{(1-2\alpha)(5(1-\alpha)+2\alpha^2)}{4(3-2\alpha)^2} \qquad \text{and} \qquad x_p^* = \frac{5(1-\alpha)+2\alpha^2}{4(3-2\alpha)^2},$$

and is the specialized equilibrium with $x_c^* = 0$ and $x_p^* = \frac{3}{16}$ whenever $\alpha \geqslant \frac{1}{2}$. These equilibria are again illustrated on the network graph and plotted against α in panels (a) and (b) of Figure 6.

2.3 Uniqueness of equilibria

In this section, we provide a general treatment of uniqueness in the network contest game. Since the game does not admit linear best replies, we cannot directly apply the results from Bramoullé et al. (2014) to characterize a sufficient condition for uniqueness. However, using a similar approach, combined with direct argument, we are able to provide a similar characterization of the conditions under which the network contest game possesses a unique equilibrium.

To facilitate the exposition, we provide a general description of our approach. First, we show that while the contest game with network externalities is not an exact potential game, it is a *best-response (or best-reply) potential game* (Voorneveld, 2000). That is, there exists a function $\mathbf{P}$ (called a BR-potential) with the same best replies as the network contest game. As such, the set of Nash equilibria in the game coincide with those strategy profiles that maximize the BR-potential, $\mathbf{P}$.

Second, we partition the domain $\mathbf{X}$ of the BR-potential $\mathbf{P}$ into two subsets: $\mathbf{X}^H$, consisting of strategy profiles $\mathbf{x}$ such that $\sum_h x_h \geqslant 0.5$, and $\mathbf{X}^L$, consisting of strategy profiles $\mathbf{x}$ such that $\sum_h x_h < 0.5$. For $\mathbf{X}^H$, the BR-potential $\mathbf{P}$ is strictly concave in $\mathbf{x}$ as long as $[\mathbf{I} + \alpha\mathbf{G}]$ is positive definite, which is true if and only if $\alpha < 1/|\lambda_{min}(\mathbf{G})|$, where $\lambda_{min}(\mathbf{G})$ is the lowest eigenvalue of $\mathbf{G}$. This is the familiar sufficient condition provided by Bramoullé et al. (2014) for uniqueness in network games with linear best replies.

For $\mathbf{X}^L$, the BR-potential $\mathbf{P}$ need not be strictly concave in $\mathbf{x}$, even if $\alpha < 1/|\lambda_{min}(\mathbf{G})|$. That is, the condition that $[\mathbf{I} + \alpha\mathbf{G}]$ is positive definite does not assure that $\mathbf{P}$ is strictly concave over $\mathbf{X}^L$. Nevertheless, we show directly that if there exists a Nash equilibrium in $\mathbf{X}^L$, we must have either $\alpha > 0.5$ (if the Nash equilibrium involves at least one inactive agent) or $\alpha > 0.5(n-2)/\Delta(\mathbf{G})$, where $\Delta(\mathbf{G}) \equiv \max_i d_i$ is the maximum degree in the graph (if the Nash equilibrium involves all agents being active). Combining these conditions, we obtain the following result.

Theorem 2. *Consider the game with network* $\mathbf{G}$ *and externality* $\alpha \in [-1, 1)$*. The following three conditions are, when jointly satisfied, sufficient for there to exist a unique Nash equilibrium;*

(i) $\alpha \leqslant 0.5$*;*

(ii) $\alpha \leqslant \frac{0.5(n-2)}{\Delta(\mathbf{G})}$*; and*

(iii) $\alpha < \frac{1}{|\lambda_{min}(\mathbf{G})|}$.

Furthermore, whenever these conditions are satisfied, the unique equilibrium involves total investment $\sum_h x_h \geqslant 0.5$.

It is important to highlight that, depending on the network, one of the conditions in Theorem 2 will always imply the other two. For instance, if $\Delta(\mathbf{G}) \equiv \max_i d_i < n-1$ (i.e., if no player is directly linked to every other player), then condition (i) implies condition (ii). Otherwise, condition (ii) implies condition (i). Similarly, if in addition to $\Delta(\mathbf{G}) < n-1$ we have $|\lambda_{min}(\mathbf{G})| \geqslant 2$, then condition (iii) is sufficient on its own. In particular then, for many networks, the condition derived by Bramoullé et al. (2014) for network games with linear best replies (our condition (iii)) is also sufficient for the contest game with network externalities, in which best replies are non-linear.

It is also straightforward to see that these conditions are in general sufficient, but not necessary for uniqueness. Consider the complete network with $\alpha \in (0.5, 1)$. The lowest eigenvalue is -1, so that condition (iii) is always satisfied. However, conditions (i) and (ii) are (clearly) not satisfied. Nevertheless, as shown in Proposition 1, there exists a unique equilibrium for all values of $\alpha \in [-1, 1)$.

In order to prove Theorem 2, we first introduce the definition of a best-response potential game (Voorneveld, 2000) and the BR-potential function, $\mathbf{P}$.

Definition 2. A game $\Gamma = (X_i, \pi_i)_{i=1}^n$ with strategy space $\mathbf{X} = X_1 \times \ldots \times X_n$ and payoff functions $\pi_i : \mathbf{X} \to \mathbb{R}$ for players $i \in N = \{1, \ldots, n\}$ is called a **Best-Response potential game (BR-potential game)** if there exists a function $\mathbf{P} : X \to \mathbb{R}$ such that

$$\arg\max_{x_i \in X_i} \mathbf{P}(x_i, \mathbf{x}_{-i}) \quad = \quad \arg\max_{x_i \in X_i} \pi_i(x_i, \mathbf{x}_{-i}) \qquad [13]$$

for any $i \in N$ and any $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$. The function $\mathbf{P}$ is called a **BR-potential** for Γ.

Next, we construct a BR-potential for the contest game with network externalities. Note that, for any $\mathbf{x} \in \mathbf{X}$, we let $|A(\mathbf{x})|$ denote the number of nonzero entries in the vector $\mathbf{x}$ (i.e., the set of active agents under profile $\mathbf{x}$). In addition, let $X_{tot} = \sum_h x_h$ be the sum of investments for the profile $\mathbf{x}$.

Lemma 2. *The following function, $\mathbf{P}$, is a BR-potential for the network contest game.*

$$\mathbf{P}(x_1, \ldots, x_n) \quad = \quad \begin{cases} \sum\limits_{j<k} (1 - \alpha g_{jk}) x_j x_k - \frac{1}{3}(X_{tot})^3 & \text{if } |A(\mathbf{x})| \geqslant 2, \\ -\frac{1}{3} x_j \left[\max\limits_{i \neq j} (1 - \alpha g_{ij}) \right]^2 & \text{if } |A(\mathbf{x})| = 1 \text{ and } x_j > 0, \\ -\frac{1}{3} \frac{n-1}{n} & \text{if } |A(\mathbf{x})| = 0. \end{cases} \qquad [14]$$

The proof, provided in Appendix A, involves showing that the best responses coincide with those of the game, and closely follows the approach used by Ewerhart (2017) for the standard contest game without externalities.[13] Then, by Proposition 2.2 of Voorneveld (2000), a strategy profile $\mathbf{x}$ is a Nash equilibrium of the game if and only if it maximizes the BR-potential, $\mathbf{P}$. Therefore, if there exists a unique maximizer for $\mathbf{P}$, it is also the unique Nash equilibrium of the network contest game.

Thus, to complete the proof of Theorem 2, we derive conditions to ensure that $\mathbf{P}$ has a unique maximizer. For network games with linear best replies, Bramoullé et al. (2014) establish that a sufficient condition for the potential function (which is an exact potential function, rather than a BR-potential) to be strictly concave is that $[\mathbf{I} + \alpha\mathbf{G}]$ is positive definite. In contrast, positive definiteness of $[\mathbf{I} + \alpha\mathbf{G}]$ is *not* sufficient to ensure strict concavity of $\mathbf{P}$ for the network contest game.

To get around the issue, we partition the domain $\mathbf{X}$ into two subsets, $\mathbf{X}^H$ and $\mathbf{X}^L$, as described above. On the subdomain $\mathbf{X}^H$, the condition that $[\mathbf{I} + \alpha\mathbf{G}]$ is positive definite ensures strict concavity of $\mathbf{P}$ over $\mathbf{X}^H$. The same is not necessarily true on the subdomain $\mathbf{X}^L$. However, we can establish conditions for uniqueness by following direct argument. Suppose there is an equilibrium profile $\mathbf{x} \in \mathbf{X}^L$. This means $X_{tot} < 0.5$. If there is at least one inactive player in $\mathbf{x}$, we show that it must be the case that $\alpha \geqslant 1 - X_{tot} > 0.5$. If all players are active in $\mathbf{x}$, we show that it must be the case that $\alpha > (n-2)/(2\Delta(\mathbf{G}))$. Thus, if both conditions are violated, existence of a Nash equilibrium implies that all Nash equilibria are contained in $\mathbf{X}^H$, where the condition on the minimum eigenvalue is sufficient for uniqueness.

3 The Experiment

3.1 Design of the experiment

The basic decision environment in our experiment is a network contest game with $n = 6$ players. The game is setup according to the model described in Section 2. Each individual is given a fixed endowment, $\omega = 800$ tokens and asked to choose how much to invest in a project. Within each group, only one player's project can be successful, and the probability that player i's project is successful is given by the Tullock (1980) lottery contest success function in equation (1). We

[13] Moreover, setting $\alpha = 0$ yields the same BR-potential he constructs.

Table 1. Summary of experimental treatments.

Network	Treatment Order (Blocks 2–4)	Sessions	Groups	Subjects
Complete	NPB, PNB, BNP, BPN	4	10	60
Circle	NPB, PNB, BNP, BPN	4	11	66
Star	NPB (2), PNB (2), BNP, BPN	6	18	108
CP2	NPB (2), PNB (2), BNP, BPN	6	16	96

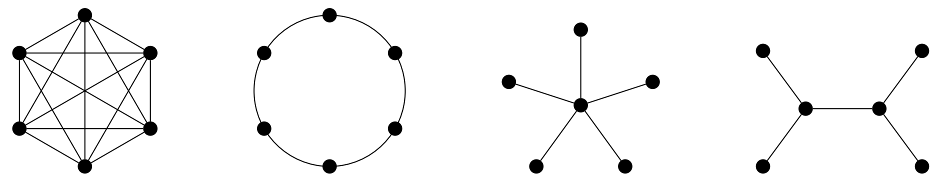
set the value of the prize to be $V = 500$ tokens and assume that the value of the externality, αV, is proportional to the prize. The resulting material payoffs to player i, accounting for the network structure $\mathbf{G}$ and the externality parameter α are $1300 - x_i$ if player i wins the contest, $800 + 500\alpha - x_i$ if player i does not win, but is directly linked to the winner by $\mathbf{G}$, and $800 - x_i$ if player i does not win and is not directly linked to the winner.

We introduce two sources of treatment variation. First, we examine the four network structures shown in Figure 7, varied across sessions (i.e., between subjects). The Complete and Circle networks are both *regular* networks (with degree $k = 5$ and $k = 2$, respectively). The Star and CP2 networks are both *core-periphery* networks. Second, we examine three values of the externality parameter, α, in every session (i.e., within subjects). The first value, $\alpha = 0$, represents the baseline environment with no externality. The other two values capture a (strong) negative externality ($\alpha = -0.8$) and a (strong) positive externality ($\alpha = 0.8$). Altogether, this generates 12 treatment conditions, distinguished by the network and the externality parameter. For notational convenience, we will occasionally describe the different treatment conditions by attaching a B (for Baseline) to indicate $\alpha = 0$, N (for Negative) to indicate $\alpha = -0.8$, or P (for Positive) to indicate $\alpha = 0.8$, at the end of the network name. For example, Complete-B refers to the Complete network with baseline $\alpha = 0$, while Star-P refers to the Star network with positive externality, $\alpha = 0.8$.

3.2 Procedures

Each session consisted of four blocks, with multiple rounds in each block. In all sessions, Block 1 consisted of 10 rounds with $\alpha = 0$ (the Baseline condition). For the other three blocks, we implemented the Negative condition (15 rounds), the Positive condition (15 rounds), and another Baseline condition (10 rounds), varying the order of the three conditions across sessions. Table 1 summarizes the treatment design, number of sessions, and number of independent groups. For the Complete and Circle networks, we ran one session each of NPB (i.e., Negative in Block 2, Positive in Block 3, Baseline in Block 4), PNB, BNP, and BPN. For the Star and CP2 networks, we ran two sessions each of NPB and PNB, and one session each of BNP and BPN. The additional sessions of NPB and PNB ensured that we were able to collect enough observations for the core player in the Star network.

In total, we conducted 20 sessions at the XS/FS laboratory at Florida State University (FSU). Subjects could only participate in one session. The experiment was implemented using z-Tree (Fischbacher, 2007), with a total of 330 subjects, randomly recruited via ORSEE (Greiner, 2015)



(a) Complete **(b)** Circle **(c)** Star **(d)** CP2

Figure 7. The set of networks

from a sub-population of FSU students who had all pre-registered to receive announcements about participation in experiments.

At the beginning of each session, subjects were randomly divided into groups of six. Groups were fixed across all rounds and all blocks in every session. Participants were seated randomly at private computer terminals and given a set of written instructions. The experimenter then read the instructions aloud to facilitate common understanding.[14] Participants completed a short set of control questions to ensure they understood the instructions. The instructions were framed in terms of a general externality, X. Then, before each block, the experimenter announced the value of X $(= \alpha V)$ and reminded participants of the way payoffs are calculated. Participants were not informed about the number of blocks or the details of any future blocks until after the previous block was completed.

Each player in a group was randomly assigned a letter ID from A to F. The letter ID and the position in the network were fixed across the entire experiment. In each round, players were shown the network, with their own ID and position highlighted. In addition, their direct neighbors in the network were highlighted in yellow, while those members of the group with whom they were not connected were shown in black. They were also reminded about the externality at the top of the screen, and prompted to enter the number of tokens they would like to invest in their project. After all players had made their decisions, an interim summary screen displayed a table showing all players' investments, the total investment, and the corresponding probability of winning.[15] After a few moments, the same screen was updated to also show the player the letter ID of the winner, whether or not they were affected by the externality (if they were not the winner), and the calculation of their payoffs for the round.

Before the four blocks that constituted the main part of the experiment, we also elicited subjects' attitudes towards risk, ambiguity, and losses, using a list-style procedure similar to the methods used by Holt and Laury (2002) and Sutter et al. (2013). After the four blocks were completed, subjects were rematched into new groups of 6 subjects for a single decision round. They were given the same endowment of 800 tokens and asked to choose a project investment, just as in the four main blocks. In contrast with the other four blocks, there were no network connections (and thus no externalities) and the winner received a prize of 0 tokens. This part of the experiment was designed to provide a measure of each subject's *joy of winning*, following

[14] A copy of the experimental instructions (for the Circle network) are provided in Appendix C.

[15] In the experiment, winning was not explicitly mentioned. Rather, we referred to the player's own project being the successful one.

Table 2. Equilibrium predictions by treatment condition.

Network (position)	Externality parameter		
	Negative	Baseline	Positive
COMPLETE	125.00	69.44	13.89
CIRCLE – *Symmetric*	91.67	69.44	47.22
CIRCLE – *Specialized*			
(active)	–	–	111.11
(inactive)	–	–	0.00
STAR (core)	206.63	69.44	0.00
STAR (peripheral)	74.39	69.44	80.00
CP2 (core)	157.89	69.44	0.00
CP2 (peripheral)	60.73	69.44	93.75

the approach introduced by Sheremeta (2010).

At the end of the experiment, subjects were paid for one randomly chosen period from each block, for the single decision round in the joy of winning task, and for one (randomly selected) of the risk, loss, or ambiguity aversion elicitation tasks. Tokens were converted to US dollars according to the exchange rate 400 tokens = \$1. Average earnings (including \$7 show-up fee) amounted to \$17.46.

3.3 Experimental predictions

We formulate our predictions based on the analysis in Section 2. Table 2 summarizes the equilibrium predictions for each treatment condition. First, for all networks, when $\alpha = 0$ (Baseline), the unique equilibrium investment is symmetric across positions and corresponds to the standard equilibrium investment for a contest with 6 players and a prize of $V = 500$. Furthermore, in the COMPLETE network, in light of Proposition 1, the unique equilibrium is the symmetric one corresponding to a standard contest with prize value equal to $V(1-\alpha)$. Accordingly, the equilibrium investment declines sharply as the externality increases from $\alpha = -0.8$ (Negative), to $\alpha = 0$ (Baseline), to $\alpha = 0.8$ (Positive).

In the CIRCLE network, there is a unique equilibrium for the Negative condition and the Baseline condition. However, for the Positive externality condition, there exists both a symmetric equilibrium with $x^* = 47.22$ and a pair of specialized equilibria, each consisting of three active players who invest $\bar{x}_A = 111.11$ and three inactive players. The two specialized equilibria can be obtained by switching the sets of active and inactive players, since both are maximal independent sets. This poses a potential coordination problem for the players in CIRCLE-P, since players may hold different beliefs about whether or not they are playing the symmetric equilibrium, the specialized equilibrium in which they are active, or the specialized equilibrium in which they are inactive. In our analysis, we consider the possibility that players are able to coordinate over time, and investigate whether play within groups is consistent with the maximal independent set characteristic of a specialized equilibrium.

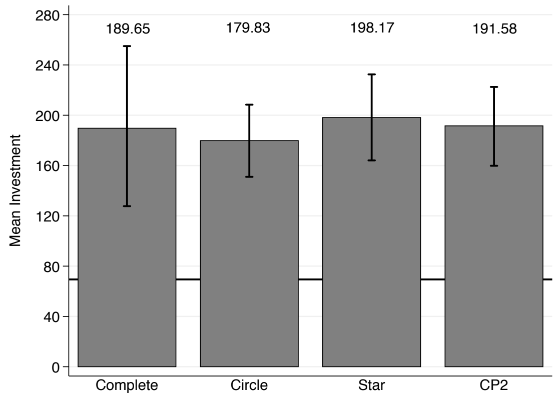
Compared with the Complete network, the predicted investment in the symmetric equilibrium exhibits a much flatter decline as the externality increases. Thus, in line with the comparative statics results discussed in Section 2.2.1, the effects of the externality (positive or negative) are increasing in the degree k for regular networks. Similarly, if subjects perfectly implement one of the specialized equilibria in Circle-P, the predicted average equilibrium investment is 55.55, which is higher than in the symmetric equilibrium. Of course, if subjects fail to coordinate on an equilibrium in Circle-P, the comparison with Complete-P is more complicated. On the one hand, in the specialized equilibrium, payoffs are higher for inactive players than active players, which might be expected to exacerbate the coordination failure as subjects clamor to occupy the more lucrative role as an inactive agent. On the other hand, if subjects do not anticipate the (material) benefits of being inactive, or are influenced by other motivations (joy of winning, social preference), failure to coordinate may manifest in higher average investment than expected under any of the equilibria, especially in the earlier rounds.

For the Star and CP2 networks, we examine the core player(s) and peripheral players separately. In both networks, the equilibrium is unique for all three externality conditions, is interior when $\alpha = -0.8$ and $\alpha = 0$, and specialized (with the core players inactive) when $\alpha = 0.8$. Thus, the equilibrium investment for the core player(s) is very high in the Negative condition, but equals zero in the Positive condition, reflecting their incentive to free ride in the specialized equilibrium. In contrast, for the peripheral players, equilibrium investment is similar across all values of α. Nevertheless, equilibrium investment in Star is higher than the baseline with both the negative and positive externality, while it is lower (higher) than the baseline with a negative (positive) externality in CP2.

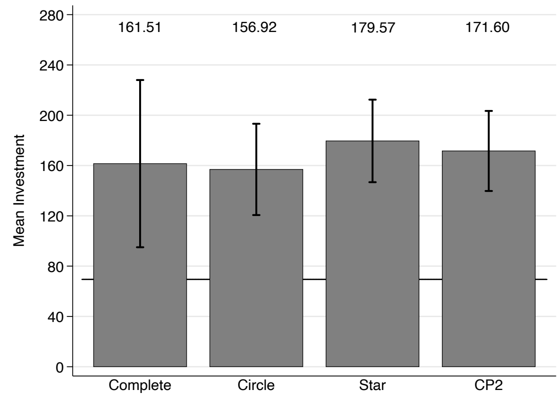
4 Results of the Experiment

The results are organized as follows. First, we report aggregate results concerning mean investment levels across networks and externality conditions. We concentrate on the comparative statics predictions in order to highlight the main treatment effects. In addition, we conduct a closer examination of behavior in the Circle-P condition, where there is both a symmetric and specialized equilibrium. We then shift our attention to the evidence regarding over-investment relative to the point predictions, and discuss alternative behavioral explanations for the differences between observed and predicted investment patterns.

Throughout this section, we rely on non-parametric tests for treatment comparisons and on the wild cluster bootstrap method (Cameron et al., 2008) for post-estimation hypothesis tests on regression coefficients. When the relevant test is not indicated, the reported p-values correspond to a Wald test (with wild cluster bootstrap) comparing the estimated constant in a linear regression to the NE prediction. Furthermore, in all figures, error bars indicate 95% wild cluster bootstrap confidence intervals.



(a) All rounds



(b) Final six rounds

Figure 8. Mean investment levels in the Baseline condition ($\alpha = 0$) from Block 1, by network. The solid reference line indicates the NE point prediction (69.44). Error bars indicate 95% wild cluster bootstrap confidence intervals.

4.1 Aggregate Results

Baseline investment. We first compare the mean investment level across networks in Block 1, where $\alpha = 0$. In this case, the network is payoff irrelevant and thus there should be no systematic differences across networks. Figure 8 shows that the mean Block 1 investment is 189.65 in Complete, 179.83 in Circle, 198.17 in Star, and 191.58 in CP2. Consistent with the prediction, we find no significant differences between networks (Kruskal-Wallis test, $p = 0.89$; also, for all pairwise comparisons between networks using the Wilcoxon ranksum test, $p > 0.401$). However, there is substantial over-investment, on average, relative to the Nash Equilibrium prediction (69.44) in all networks, which is consistent with the experimental literature on standard contests.

Result 1. *Mean investment in the Baseline condition (Block 1) does not differ across networks.*

Figure B.1 in Appendix B shows that in all networks, mean investment is trending down towards the Nash Equilibrium point prediction over the course of Block 1, which is consistent with some learning by the subjects as they gain experience with the strategic environment. Thus, in order to account for experience, we also replicate the analysis using only the final six rounds of the block (see Figure 8b). The corresponding mean investments are 161.51 in Complete, 156.92 in Circle, 179.57 in Star, and 171.60 in CP2, which are also not significantly different from each other (Kruskal-Wallis test, $p = 0.772$; for all pairwise comparisons using the Wilcoxon ranksum test, $p > 0.322$). Although these investment levels are lower than when we use all 10 rounds, over-investment relative to the NE prediction persists.

Treatment comparisons. Next, we compare mean investment across networks and externality conditions using the data collected in Blocks 2–4. Table 3 reports the mean investment for each network and each externality condition, alongside the corresponding Nash equilibrium

Table 3. Summary statistics for mean investment in Blocks 2-4 by treatment condition

	Externality					
	Negative ($\alpha = -0.8$)		Baseline ($\alpha = 0$)		Positive ($\alpha = 0.8$)	
Network	Observed	[NE]	Observed	[NE]	Observed	[NE]
COMPLETE	180.28	[125.00]	123.36	[69.44]	53.85	[13.89]
CIRCLE	168.90	[91.67]	114.72	[69.44]	103.06	[47.22]
						[55.55]†
STAR						
core	256.06	[206.63]	161.43	[69.44]	71.79	[0.00]
peripheral	179.16	[74.39]	143.15	[69.44]	126.14	[80.00]
CP2						
core	213.03	[157.89]	125.33	[69.44]	102.28	[0.00]
peripheral	168.37	[60.73]	137.36	[69.44]	140.09	[93.75]

Notes: † denotes the average equilibrium investment in the specialized equilibria for CIRCLE-P.

(NE) point predictions. As is typical in contest experiments, and consistent with behavior in Block 1, we observe considerable over-investment (over-dissipation) relative to the NE in all conditions. However, we postpone a more detailed discussion of over-investment until Section 4.3. In the rest of this section, we concentrate on treatment comparisons and comparative static predictions. We begin by considering the two regular networks, COMPLETE and CIRCLE, before turning our attention to the core-periphery structures, STAR and CP2.

4.1.1 Regular networks

Figure 9a illustrates the mean investment reported in Table 3 (using all rounds) for each externality condition in the two regular networks, COMPLETE and CIRCLE.

Within-network comparisons. Comparisons within network are generally consistent with the comparative static predictions. In both networks, investment is highest for the negative externality (180.28 in COMPLETE, 168.90 in CIRCLE) and lowest for the positive externality (53.85 in COMPLETE, 103.06 in CIRCLE), as predicted in Section 3.3. Using the mean investment across all rounds of a block within each independent group as a single observation, we observe significant differences for all pairwise comparisons between externality conditions in the COMPLETE network (Wilcoxon Signed-Rank tests, $p = 0.047$ for N vs. B, $p = 0.005$ for N vs. P, $p = 0.007$ for P vs. B). Similarly, in the CIRCLE network, we observe significantly higher mean investment in the negative condition compared with the other two conditions (Wilcoxon Signed-Rank tests, $p = 0.016$ for N vs. B, $p = 0.008$ for N vs. P). However, mean investment in the CIRCLE network is not significantly different between the baseline (zero externality) and positive externality conditions (Wilcoxon Signed-Rank test, $p = 0.374$), consistent with Figure 9a. We summarize our findings in the following two results.

Result 2. *For the* COMPLETE *network, mean investment is strictly decreasing in the externality level, in line with the comparative static predictions.*

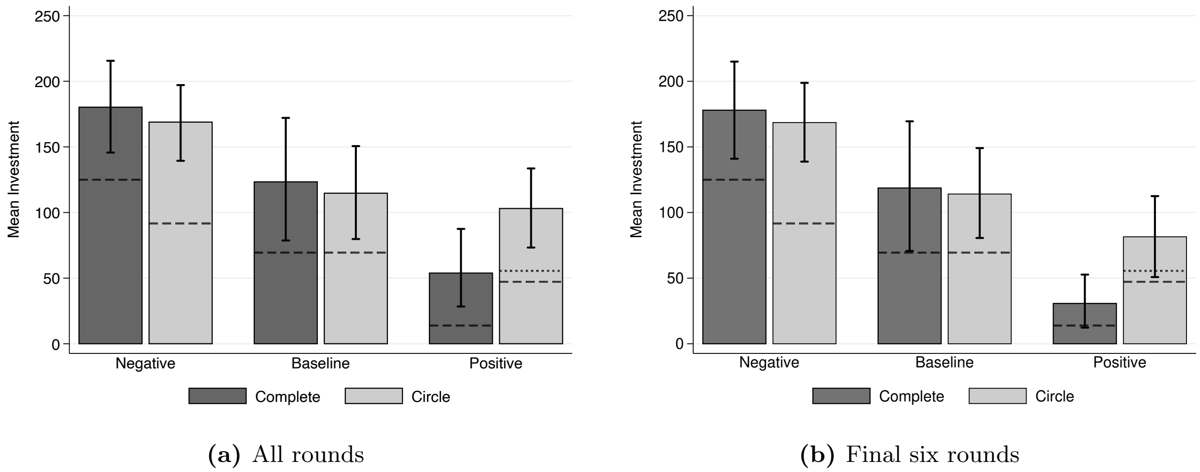


Figure 9. Mean investment levels by externality in the Regular networks. Dashed lines indicate symmetric NE point predictions, while the short-dashed line for Circle-P indicates specialized NE point prediction. Error bars indicate 95% wild cluster bootstrap confidence intervals.

Result 3. *For the* Circle *network, mean investment is significantly higher with the negative externality than with either of the other externality conditions. However, mean investment is no different with the positive externality than with zero externality.*

The second part of the result for Circle may be driven in part by the multiplicity of equilibria in the positive externality condition. In particular, while the symmetric equilibrium predicts lower investment in Circle-P than in Circle-B, the difference is quite small.[16] Furthermore, the predicted difference is even smaller if we posit that subjects play a specialized equilibrium (69.44 vs. 55.55). In addition, the likelihood of some coordination failure, especially in the early rounds, could explain why average investment remains at a similar level when α increases from 0 to 0.8. Given that it is the only condition in which there are multiple equilibria, we investigate the Circle-P condition in more detail in Section 4.2.

We also examine investment using the final six rounds of each block. Figure 9b shows that for the Negative and Baseline externality conditions, the mean investment levels using the final six rounds are no different than those reported in Table 3 and Figure 9a, which use all rounds. For both the Complete and Circle networks, over-investment relative to the NE prediction remains statistically significant. However, for the Positive externality condition, the mean investment over the final six rounds is considerably lower than it is using all rounds. In fact, over the final six rounds, over-investment in the Complete-P condition is only marginally significant ($p = 0.0925$). Similarly, while the difference between mean investment and the symmetric equilibrium in Circle-P over the final six rounds is still significant ($p = 0.032$), the difference relative to the average predicted investment in a specialized equilibrium is only marginally significant ($p = 0.0954$). Finally, when using only the final six rounds of each block, the comparative static prediction that investment in Circle-B is higher than in Circle-P is

[16]Note, however, that the difference is no smaller than the difference between Circle-N and Circle-B.

now supported (Wilcoxon Signed-Rank test, $p = 0.0076$). Thus, if we allow for learning (or experience) to take place in each block, we can remove the qualified support for the comparative static predictions in the CIRCLE network altogether.

Between-network comparisons. Next, we hold fixed the externality condition and compare investment levels *between* the two regular networks. As expected, in the Baseline condition, we find no significant differences, while in the Positive condition, investment is significantly higher in CIRCLE than in COMPLETE, which is consistent with both the symmetric and specialized equilibria for CIRCLE-P.[17] However, contrary to the theoretical prediction, mean investment in the Negative condition is not significantly different between COMPLETE and CIRCLE (Wilcoxon Ranksum test using group-level means, $p = 1.000$ using all rounds, $p = 0.622$ using the final six rounds).

Result 4. *Average investment is significantly lower in* COMPLETE-P *than in* CIRCLE-P*, and is not significantly different between* COMPLETE-B *and* CIRCLE-B*, consistent with the predictions. In contrast, contrary to the theoretical prediction, average investments in* COMPLETE-N *and* CIRCLE-N *are not significantly different from each other.*

4.1.2 Core-Periphery Networks

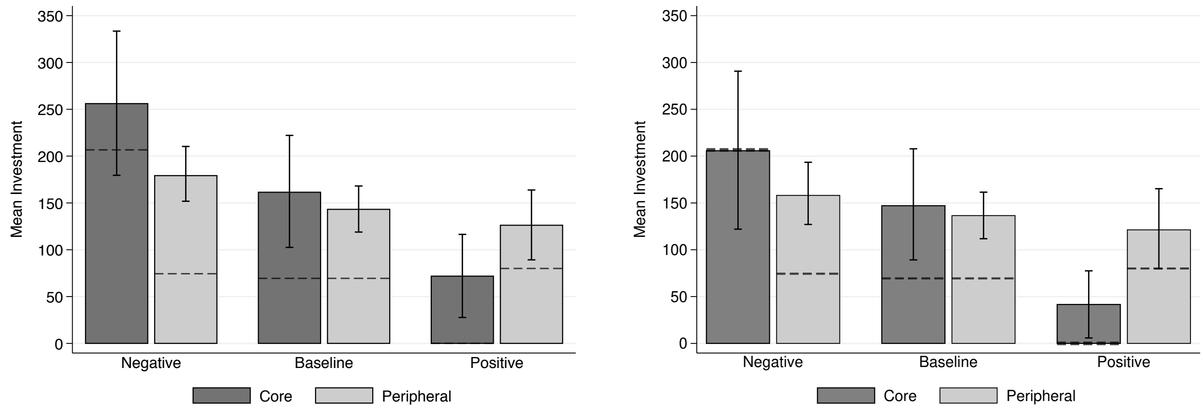
For the STAR and CP2 networks, we compare mean investment levels separately for core players and peripheral players. We concentrate first on the impacts of the externality condition in the STAR network, then in the CP2 network, before turning our attention to the comparison between networks while holding the externality fixed.

Within-network comparisons. Figure 10a shows the mean investment across externality conditions for the core player and the peripheral players in the STAR network. Core players invest significantly more than the NE point predictions in the Baseline and Positive conditions, but not in the Negative condition. Nevertheless, the comparisons between Negative, Baseline, and Positive for core players are all in line with the comparative statics predictions. Specifically, investment in Negative is higher than in Baseline (Wilcoxon Signed-Rank test, $p = 0.018$) and Positive ($p < 0.001$), and investment in Baseline is higher than in Positive ($p = 0.004$).

For peripheral players, the NE investment levels are very similar across the three externality conditions (cf. Table 2). However, Figure 10a shows that the mean investment is, in fact, slightly higher in Negative than in Baseline and Positive, which do not differ from one another.[18] The comparative static results for core players are all robust to using only the final six rounds of each block (see Figure 10b). However, the differences between mean investment of peripheral players for the different externality conditions are no longer statistically significant when we

[17]For the Wilcoxon Ranksum test, using mean investment over all rounds for a single group as one observation, we have $p = 1.000$ for Baseline and $p = 0.029$ for Positive. Nothing substantive changes if we use the final six rounds, with $p = 0.994$ for Baseline and $p = 0.006$ for Positive.

[18]Wilcoxon Signed-Rank tests, $p = 0.043$ for N vs. B, $p = 0.003$ for N vs. P, $p = 0.223$ for P vs. B.



(a) All rounds

(b) Final six rounds

Figure 10. Mean investment levels by externality in the Star network. Dashed lines indicate NE point predictions. Error bars indicate 95% wild cluster bootstrap confidence intervals.

restrict attention to the final six rounds.[19] Our next result summarizes these findings for the Star network.

Result 5. *For the* Star *network,*

(i) mean investment by the core players is strictly decreasing in the externality level, in line with the comparative static predictions;

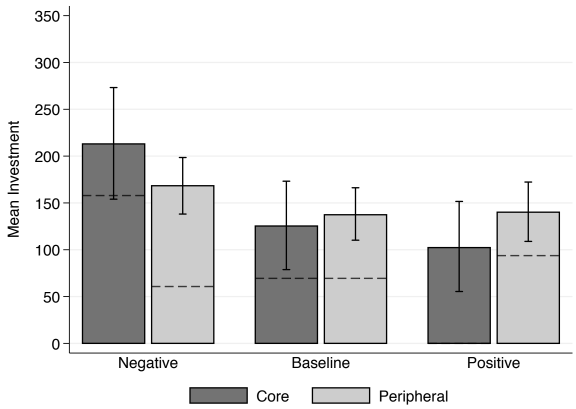
(ii) mean investment by the peripheral players is significantly higher with the negative externality than with the other two externality conditions when using all rounds, but does not differ between the three externality conditions when using the final six rounds of each block.

One interesting feature of the data that is highlighted by Figure 10a is the differential over-investment by peripheral players across externality conditions. In particular, we find that the percentage over-investment by peripheral players declines from 140.8% in Negative, to 106.11% in Baseline, to 57.7% in Positive.[20] We explore this feature in greater detail in Section 4.3, especially since, as we show next, a similar pattern arises for peripheral players in the CP2 network.

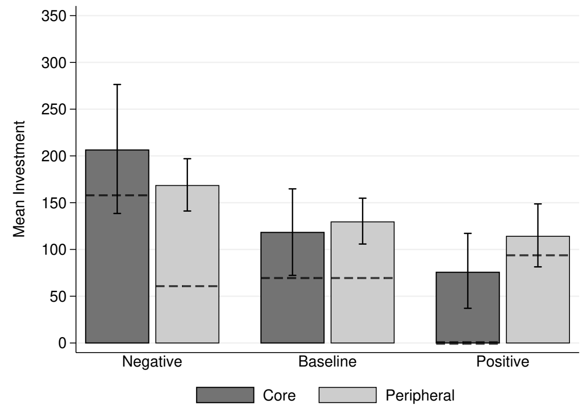
Figure 11a shows the mean investment in the CP2 network. As in the Star network, the core players invest significantly more than the NE point predictions in the Baseline and Positive conditions, but not the Negative condition. Nevertheless, the comparisons between externality conditions are all consistent with the comparative statics predictions for the core players. Mean investment in Negative is higher than in both Baseline (Wilcoxon Signed-Rank test, $p = 0.007$) and Positive ($p = 0.006$), while investment in Baseline is higher than in Positive ($p = 0.030$).

[19] Wilcoxon Signed-Rank tests, $p = 0.396$ for N vs. B, $p = 0.085$ for N vs. P, $p = 0.122$ for P vs. B.

[20] Note, however, that when using only the final six rounds of each block, the over-investment rates for Negative (112.5%) and Baseline (96.5%) are similar, though both are much larger than the rate for Positive (51.6%). In fact, the difference between mean investment and the NE prediction for peripheral players in Star-P is only marginally significant ($p = 0.051$).



(a) All rounds



(b) Final six rounds

Figure 11. Mean investment levels by externality in the CP2 network. Dashed lines indicate NE point predictions. Error bars indicate 95% wild cluster bootstrap confidence intervals.

Figure 11b shows that each of these comparisons is also robust to using only the final six rounds of each block ($p < 0.01$ for each pairwise comparison).

The pattern of behavior for peripheral players in CP2 is also very similar to what we observe in the Star network. Mean investment is higher in Negative than in Baseline ($p = 0.010$) but only marginally higher than in Positive ($p = 0.079$), while Baseline and Positive are not significantly different ($p = 0.796$). Moreover, using only the final six rounds actually widens the difference between investment in the Negative condition and investment in the other two externality conditions by the peripheral players (Wilcoxon Signed-Rank tests, $p < 0.001$ for N vs. B; $p = 0.008$ for N vs. P; $p = 0.234$ for P vs. B).

Result 6. *For the* CP2 *network,*

(i) mean investment by the core players is strictly decreasing in the externality level, in line with the comparative static predictions;

(ii) mean investment by the peripheral players is significantly higher with the negative externality than with the other two externality conditions, and does not differ between the baseline and positive externality.

Alongside these results, we observe the same feature of behavior in CP2 as in the Star network with regards to the percentage over-investment by peripheral players. Over-investment falls from 177.2% in Negative, to 97.8% in Baseline, to 49.4% in Positive. Using the final six rounds only, the over-investment rates are 177.3% in Negative, 86.4% in Baseline, and just 21.7% in Positive. Furthermore, the difference between mean investment by peripheral players and the NE prediction in the CP2-P condition is not statistically significant ($p = 0.234$).

Thus, in both of our core-periphery network structures, for the peripheral players, we observe extremely high over-investment rates in the presence of negative externalities, intermediate rates of over-investment in the absence of any externalities, and lower (even statistically insignificant)

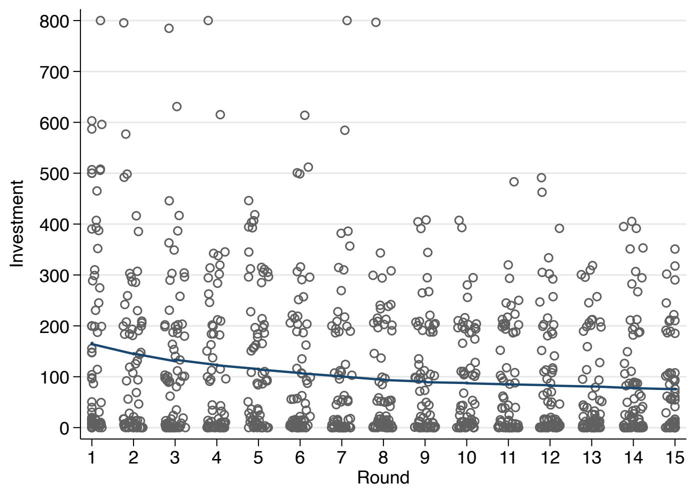


Figure 12. Scatterplot of investment by round in Circle-P with Lowess smoother (bandwidth = 0.5).

over-investment rates in the presence of positive externalities.

Between-network comparisons. Finally, we compare behavior of the core players and peripheral players in the Star and CP2 networks, holding the externality level fixed. Core players' mean investment is less in CP2 than in Star when the externality is negative and in the baseline condition, and higher in CP2 than in Star when the externality is positive, but none of the differences is statistically significant (Wilcoxon Ranksum test, $p = 0.370$ for Negative, $p = 0.309$ for Baseline, and $p = 0.124$ for Positive). Similarly, mean investment by the peripheral players is not different between Star and CP2 for any of the externality conditions (Wilcoxon Ranksum test, $p = 0.581$ for Negative, $p = 0.605$ for Baseline, and $p = 0.448$ for Positive).

4.2 Symmetric vs. specialized equilibrium play in Circle-P

In this section, we provide a closer examination of the patterns of investment behavior in the Circle-P condition. In particular, we investigate whether play is consistent with either the symmetric equilibrium or the specialized equilibria that arise for the Circle network when $\alpha = 0.8$. As a starting point, Figure 12 shows a scatterplot of all the data in Circle-P by round. Some of the notable features of the data are the concentration of observations at or around zero (in all rounds) and the gradual decline (and compression) of observations across rounds.

Focusing on the final six rounds of the data (rounds $10-15$), there is still a considerable amount of heterogeneity, although the majority of the observations are no greater than 200. The clustering of observations around zero and the (smaller) clusters around 100 and 200 offer some hope for the emergence of specialized equilibria. However, as in most contest experiments, the spread of investment levels across individuals is hardly encouraging evidence of symmetric equilibrium play.

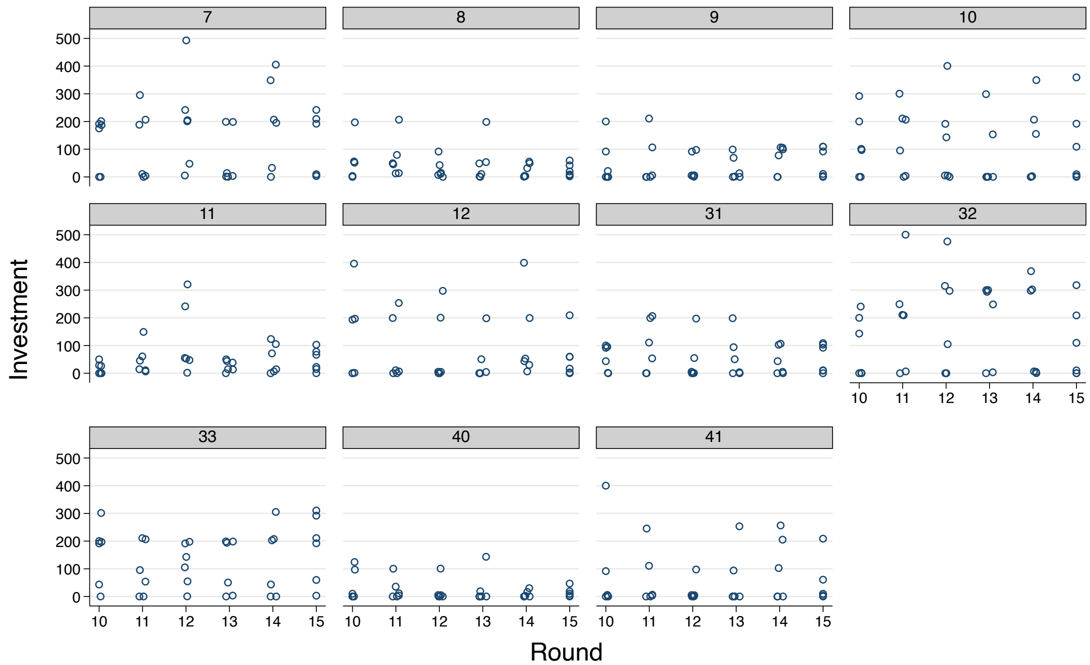


Figure 13. Scatterplot of investments in rounds 10 – 15 by group in the Circle-P condition.

In order to explore the evidence regarding specialized equilibrium play, we examine the data at the group level. Our analysis concerns three key features of specialized equilibria. First, in any specialized equilibrium of the Circle-P condition, there are three active agents who form a maximal independent set. Second, the three active agents invest just over 100 tokens into the contest (equilibrium investment is 111.11). Third, the other three agents, who are inactive, invest zero. Realistically, the subjects face a coordination problem, due to the fact that there are two specialized equilibria in the circle with $n = 6$.

We begin by examining individual investment levels in each independent group, in order to provide an initial test of consistency with specialized equilibrium play. In order to allow for the possibility that players gain experience and require time to coordinate, we focus in the main text on just the final six rounds (rounds 10 – 15).[21] Figure 13 plots the investment choices for each player over the final six rounds, for each independent group in the Circle-P condition. From the figure, there appears to be heterogeneity across groups. For instance, in some groups (group IDs 7, 9, 10, 12, 31, 32, and 41), there are consistently two to four players who are inactive (or invest close to nothing) and at least two players who invest significantly more (most often 100 tokens or more). At least in terms of investment *levels*, the pattern in these groups is consistent with specialized equilibrium play. In other groups (8, 11, and 40), the investments are typically more clustered together (consistent with more symmetric play), with only an occasional (single) high investor who separates from the other five group members.

[21] For each group, we also plot the individual investments by each player over all 15 rounds (see Figure B.2 in Appendix B).

One of the limitations of the plots in Figure 13 is that they contain no information regarding the configurations of active players within the network. In order to address this aspect of specialized equilibrium play, we next ask how often the subjects choosing the three highest investments in a group form a maximal independent set. In the Circle-P condition, there are 11 groups and 15 rounds. Out of the resulting 165 observations, there are only 18 instances in which the three highest investments are all *strictly* higher than the others and come from agents who form a maximal independent set. If we allow for the possibility that there are ties at the median (so that the 3rd and 4th highest investments are equal), there are 49 instances (out of 165) in which the three highest investments come from agents who form a maximal independent set. This suggests that even if groups are choosing investments consistent with a specialized equilibrium, they are rarely successful in perfectly coordinating on which sets of agents are active and which are inactive.

Another way to examine the consistency of play with a specialized equilibrium is to compare average investment by each maximally independent set of agents in the circle. Consider the two subsets of agents, denoted by their position labels in the experiment, $M_A = \{A, D, E\}$ and $M_B = \{B, C, F\}$.[22] For each of the two subsets M_A and M_B, we compute the average investment by its constituent members in each round. Then, in Figure 14, we plot the average investments for M_A and M_B in the final six rounds, for each independent group in Circle-P.[23] Consistent with the heterogeneity across groups observed in Figure 13, we observe a mix of patterns between the two maximal independent sets. For instance, in groups 7, 31, and 41, average investment is consistently higher for players in the maximal independent set M_B than for those in M_A, whereas in groups 9 and 32, average investment is higher for players in M_A than for those in M_B. In contrast, in groups 8, 11, 33, and 40 (all of which exhibited more clustering in Figure 13), the mean investment levels are roughly similar across the two maximal independent sets.

Nevertheless, even when there is a gap between average investment for the two maximal independent sets, it may not necessarily reflect specialized equilibrium play. To assess whether or not it does, it is useful to compare the patterns observed in Figure 14 for the Circle-P condition with the average investment by the same groups in the Circle-N and Circle-B conditions. Figure B.4 in Appendix B shows the corresponding plots for the Circle-N condition. For several of the groups (including 9, 10, 12, 31, and 32), there are similar gaps between average investment for the two maximal independent sets. Similarly, Figure B.5 shows that for the Circle-B condition, there are comparable gaps between average investment for M_A and M_B in groups 12, 31, 32, and 41. Together, these findings suggest that the gaps may be driven by factors other than the presence of a specialized equilibrium, since no such equilibria exist for the Circle-N or Circle-B conditions.

One possibility is that there are different types of subjects, some of whom are inclined to over-invest in the contest (for instance, due to high non-monetary utility - or 'joy' - of winning), and others for whom the resulting best response is to remain inactive. We discuss this idea alongside other possible influences as part of the analysis of over-investment, in the next subsection.

[22]These two subsets correspond to the two maximal independent sets given the labeling used in the experiment – see Figure C.1 in the Experimental Instructions (Appendix C).

[23]The corresponding plots with all 15 rounds included are presented in Figure B.3 in Appendix B.

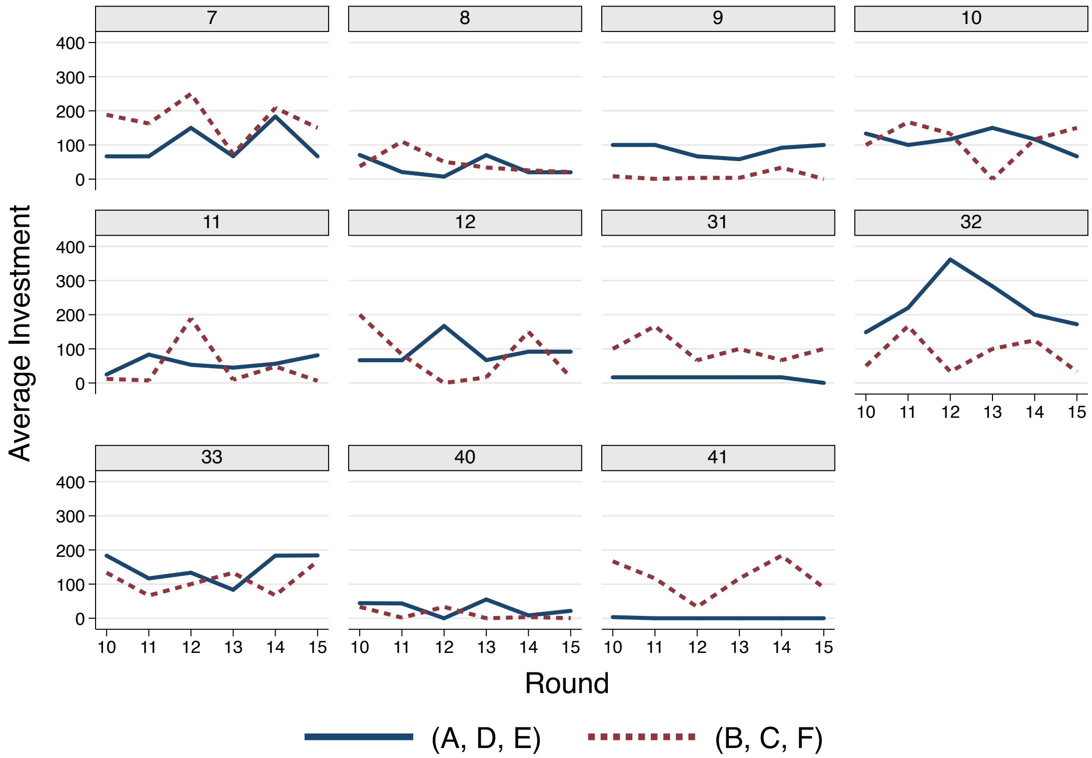


Figure 14. Average investment by maximal independent sets $M_A = \{A, D, E\}$ and $M_B = \{B, C, F\}$ over the final six rounds in the CIRCLE-P condition. Each cell represents one independent group.

4.3 Explaining the Patterns of Over-Investment

Mean over-investment relative to the NE point prediction has been widely documented in standard lottery contest experiments without externalities (see, e.g., Sheremeta, 2013). Given the wealth of evidence, several alternative explanations have been proposed for over-investment, and these have been well summarized by Dechenaux et al. (2015) and tested systematically by Sheremeta (2016). One such explanation is that individuals derive some non-monetary utility from winning *per se*, commonly referred to as the 'joy of winning', beyond the actual value of the prize (Goeree et al., 2002; Sheremeta, 2010; Brookins and Ryvkin, 2014; Boosey et al., 2017). A second explanation contends that individuals who care about status or relative payoffs may invest more in a contest (see, e.g., Hehenkamp et al., 2004; Mago et al., 2016).

A third common explanation for overbidding is based on the argument that individuals are boundedly rational and subject to making mistakes. A standard approach to model the noise associated with these mistakes is the Quantal Response Equilibrium (QRE) framework (McKelvey and Palfrey, 1995). In this framework, players mix over the available strategy space in a way that places higher weights on strategies that are better-performing (generate higher expected payoffs) in response to the behavior of other players.[24] Yet others have argued that individuals are subject to judgmental biases, such as non-linear probability weighting, or the hot hand fallacy, which may lead to higher investment than the standard NE prediction (Parco et al., 2005; Amaldoss and Rapoport, 2009; Sheremeta, 2011).

In this section, we consider the evidence of (mean) over-investment relative to NE predictions and discuss some alternative behavioral considerations that can help to explain the patterns that we observe. For each treatment condition, we briefly review the aggregate patterns that were highlighted in the previous section, to help organize the subsequent discussion.

4.3.1 Baseline condition

Recall that in the Baseline condition ($\alpha = 0$), the network structure is theoretically irrelevant. Focusing on Block 1, Figure 8 showed that average investment is significantly above the NE point prediction in all four networks. Moreover, the amount of over-investment does not differ significantly across networks. Neither of these results change when using only the final six rounds of each block.

We also examine the Baseline conditions implemented in Blocks 2 and 4 of the experiment. Figure B.6 in Appendix B shows the mean investment levels for each network, pooling together the sessions from Blocks 2 and 4.[25] Overall, we find no significant differences between networks (Kruskal-Wallis test, $p = 0.142$), although pairwise comparisons using the Wilcoxon ranksum test suggest that investment in the STAR network are higher than in the COMPLETE ($p = 0.084$) and CIRCLE ($p = 0.053$) networks.[26] We summarize these observations with the following result.

Result 7. *In the Baseline condition, over-investment levels are similar across networks, and consistent with the robust evidence of over-investment in standard contest experiments.*

Another well-documented finding in the contest experiments literature is that mean over-investment is accompanied by considerable variance (or overspreading), with many subjects investing less than the NE while others substantially over-invest. Following the approach introduced by Sheremeta (2010), at the end of the main experiment we elicited a measure of subjects' non-monetary value of winning (their *joy of winning*), by asking them to choose an investment for a contest with a prize of zero. The data obtained from our experimental elicitation of joy of winning reveals that about 22% of subjects submitted non-zero levels of investment for a prize with value zero. While the majority of these were relatively small, about 6% of investments in this part of the experiment were larger than 80 tokens (10% of their total endowment) and about 4% were larger than 200 tokens (25% of their total endowment). In Table 4, we report the percentage of subjects who chose different ranges of investment in the zero prize contest, alongside the mean Baseline (Block 1) investment for the subjects in each range. The main

[24] Supporting evidence for this approach is reported in Sheremeta (2011), Chowdhury et al. (2014), Lim et al. (2014), and Brookins and Ryvkin (2014).

[25] In these later Baseline blocks, the network structure is, similarly, theoretically irrelevant; although, it is possible for Block 4, that individuals' investment behavior is influenced by previous exposure to the Negative and Positive conditions.

[26] We obtain virtually identical results when using only the final six rounds.

Table 4. Distribution of Joy Investment (investments in the zero prize contest) and corresponding Block 1 investments

Joy Investment	Percent of subjects	Mean Baseline Investment (Block 1)
0	77.88%	175.18
1	11.21%	206.95
2–15	3.03%	240.74
16–100	3.63%	259.14
> 100	4.22%	346.20

takeaway from Table 4 is that the mean Baseline investment in Block 1 is higher for subjects who choose higher "joy investments". In particular, subjects who invested more than 100 in the zero prize contest also invested nearly twice as much (on average) in the Baseline condition as subjects who invested zero in the contest with no prize.

We also estimate a mixed effects model regressing Baseline investment in Block 1 on investment in the zero prize contest (Joy Investment). We estimate the model in the following equation,

$$\text{Invest}_{it} = \beta_0 + \beta_1 \text{Joy Investment}_i + \gamma(1/t) + u_{0i} + u_{1i}\text{Joy Investment} + \epsilon_{it}, \qquad [15]$$

allowing for subject-level fixed effects and between-subject heterogeneity in the effect of Joy Investment (random slope coefficients), and include a time trend (the reciprocal of the round number) to account for learning during Block 1.[27] Standard errors are clustered at the group level, to account for the dependence across individuals. The results are reported in Table 5, both overall and separately for each network.

Overall, the relationship between a subject's Joy Investment and her investment levels in the Baseline condition is positive and highly significant. Disaggregating the data by network, we find that the effect is heterogeneous, although significantly positive for each case except the Star network. There is significant variance in the slope coefficients overall and for the Complete and CP2 networks, whereas the estimation results for the Circle and Star networks are no different than from a standard random effects regression. The time trend is also significant (except in the Circle network), consistent with the evidence that investment in Block 1 declines (non-linearly) with experience in each block. We summarize our findings in the following result.

Result 8. *Subject-level investment in the Baseline condition (Block 1) is increasing in the elicited measure of the subject's joy of winning.*

4.3.2 Regular networks

Next we consider over-investment in the regular networks, for the Negative and Positive externality conditions. For regular networks, Figure 9 provides clear evidence of mean over-investment

[27] A likelihood ratio test confirms that including random slopes with respect to the explanatory variable 'Joy Investment' significantly improves the fit of the model when the data are pooled across networks. The same is true for the Complete and CP2 networks on their own, although not for the Circle or Star networks, where the coefficient on Joy Investment does not exhibit any subject-level variation.

Table 5. Multilevel mixed effects model with between subjects random slopes, regressing Baseline Investment (from Block 1) on Joy Investment.

Dependent variable: Individual investment in round t					
	Overall	COMPLETE	CIRCLE	STAR	CP2
Joy Investment	0.396***	2.676***	0.538***	−0.060	0.302**
	(0.138)	(0.479)	(0.044)	(0.151)	(0.145)
$1/t$	75.104***	126.198***	54.882	61.847***	71.990***
	(13.436)	(32.436)	(39.499)	(20.533)	(21.108)
Constant	161.366***	134.461***	155.277***	181.118***	159.233***
	(9.753)	(31.197)	(17.755)	(16.374)	(16.749)
$\widehat{\sigma}^2_{u_0}$ (Constant)	17180.50***	18450.80***	14992.30***	18599.59***	14237.84***
$\widehat{\sigma}^2_{u_1}$ (Joy Investment)	0.125***	2.761***	–	–	0.067***
$\widehat{\sigma}^2_{\epsilon}$ (Residual)	26409.22***	28985.74***	25251.39***	26042.89***	25844.08***
Groups	55	10	11	18	16
Observations	3300	600	660	1080	960
Wald $\chi^2(2)$	40.44***	45.58***	161.53***	9.10**	18.19***

Standard errors clustered at the group level in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

in all conditions (including the Baseline in Blocks 2–4), although it is only marginally significant over the final six rounds of COMPLETE-P and CIRCLE-P. One possible explanation for the differences between conditions could be that subjects' joy of winning is elevated in the presence of negative externalities, and diminished in the presence of positive externalities. We examine this possibility by estimating a multilevel mixed effects model for investment over the final six rounds of each block. The full specification of interest is the following model,

$$\begin{aligned}\text{Invest}_{it} = {} & \beta_0 + \beta_1 \text{Joy Investment}_i + \beta_2 \text{Neg} + \beta_3 \text{Pos} + \boldsymbol{\gamma}\mathbf{X} + \alpha_1(\text{Neg} \times \text{Joy Investment}_i) \\ & + \alpha_2(\text{Pos} \times \text{Joy Investment}_i) + u_{0i} + u_{1i}\text{Joy Investment}_i + \epsilon_{it}, \end{aligned} \qquad [16]$$

where Neg and Pos are dummy variables for the externality condition, $\mathbf{X}$ is a vector consisting of the individual elicited measures of ambiguity aversion (AA), risk aversion (RA), and loss aversion (LA). We allow for both random intercepts (corresponding to Baseline investment) and random slope coefficients on Joy Investment. The results are reported in Table 6. In the first column, we estimate the model under the restrictions that $\alpha_1 = \alpha_2 = 0$ and $u_{1i} = 0$. That is, we exclude interactions between Joy Investment and the externality condition, and exclude random slope coefficients on Joy Investment. In the second column, we allow for subject-level random slope coefficients on Joy Investment, and in the third column, we further include interactions between Joy Investment and the externality condition.

In all three columns, Joy Investment has a strongly significant positive effect on investment. The third column mirrors the result obtained for the Baseline condition in Block 1, that Joy Investment has a significant positive effect on Baseline investment, but for the Baseline condition implemented during the main part of the experiment (Blocks 2–4). More importantly, the

Table 6. Multilevel mixed effects model with between subjects random slopes, regressing Investment (final six rounds) on Joy Investment and externality condition; Regular networks.

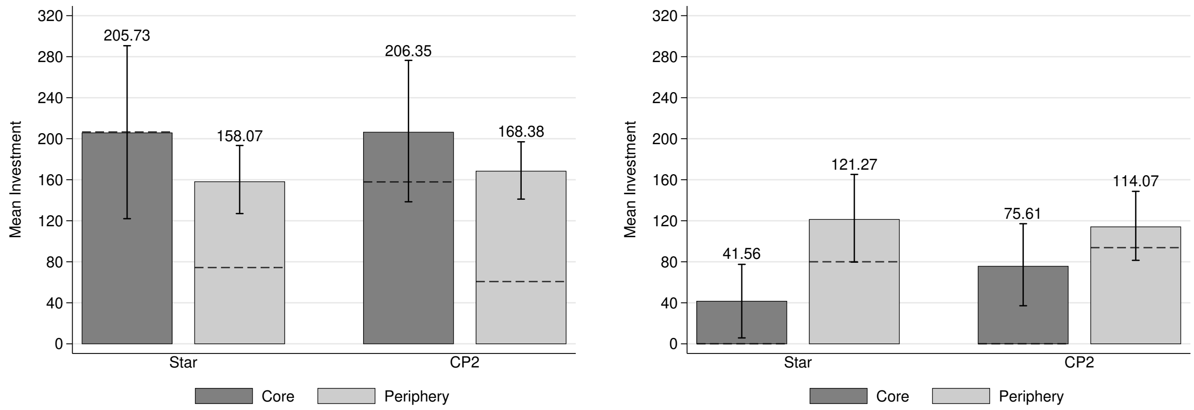
Dependent variable: Individual investment in round t			
	Regular	Regular	Regular
Joy Investment	0.44**	0.63***	0.68***
	(0.21)	(0.23)	(0.25)
Neg	56.76***	56.76***	55.94***
	(13.61)	(13.61)	(13.51)
Pos	−58.97***	−58.97***	−56.39***
	(12.26)	(12.26)	(12.31)
AA	−5.33*	−5.68**	−5.68**
	(2.83)	(2.77)	(2.77)
RA	−2.78	−3.56*	−3.56*
	(1.78)	(1.83)	(1.83)
LA	−1.60	−1.92	−1.92
	(1.96)	(1.98)	(1.98)
Joy Invest × Neg	–	–	0.06
			(0.08)
Joy Invest × Pos	–	–	−0.20***
			(0.06)
Constant	154.81***	163.98***	163.39***
	(25.90)	(27.57)	(27.40)
$\widehat{\sigma}_{u_0}$ (Constant)	90.89***	88.06***	88.08***
$\widehat{\sigma}_{u_1}$ (Joy Investment)	–	0.319***	0.319***
$\widehat{\sigma}_{\epsilon}$ (Residual)	134.42***	134.42***	134.13***
Groups	21	21	21
Observations	2268	2268	2268
Wald $\chi^2(6)$	130.70***	145.71***	172.43***

Robust standard errors, adjusted for clustering at the group level, in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

effect of Joy Investment is no different when the externality is negative (since the coefficient estimate $\widehat{\alpha}_1 = 0.06$ is not significantly different from zero), but is significantly weaker when the externality is positive ($\widehat{\alpha}_2 = -0.20$).[28] Thus, we find support for the hypothesis that joy of winning has a significantly weaker influence on investment in regular networks when there are positive externalities than it does when there are negative externalities or no externalities at all.

Result 9. *In the regular networks, non-monetary utility of winning has a strongly significant positive effect on investment in the Baseline and Negative externality conditions. However, the effect in the Positive condition is significantly weaker than in the other conditions and is not significantly different from zero.*

[28] In fact, the effect of Joy Investment in the Positive condition is not significant. That is, we cannot reject the hypothesis that $\beta_1 + \alpha_2 = 0$, with $p = 0.102$.



(a) Negative **(b)** Positive

Figure 15. Mean investment levels in the final six rounds of the Negative ($\alpha = -0.8$) and Positive ($\alpha = 0.8$) conditions, by network. Dashed lines indicate NE point predictions. Error bars indicate 95% wild cluster bootstrap confidence intervals.

4.3.3 Core-Periphery networks

We turn next to the two core-periphery networks. Figure 15 highlights three key patterns of over-investment across externality conditions for the different types of player in Star and CP2. First, we observe significant mean over-investment by the core players in the Positive conditions, where they are predicted to be inactive. Second, there is no mean over-investment by core players in the Negative conditions, for either network. Third, peripheral players exhibit considerably higher over-investment rates in the Negative condition than in the Positive condition, for which mean over-investment loses significance over the final six rounds.[29]

Result 10. *In the core-to-periphery networks, we observe a stark reversal in the patterns of mean over-investment for the Negative and Positive externalities.*

(i) *With negative externalities, peripheral players exhibit mean over-investment while core players' investments are in line with the NE prediction.*

(ii) *With positive externalities, core players exhibit significant mean over-investment, while peripheral players' mean investment levels are close to the NE prediction.*

The same as for the regular networks, one might argue that the patterns of over-investment by peripheral players are consistent with non-monetary utility of winning that is sensitive to the externality condition. That is, if joy of winning is elevated in the presence of negative externalities, and diminished in the presence of positive externalities, it may explain, at least in part, why the peripheral players over-invest by substantially more in the Negative condition than they do in the Positive condition. We estimate the same model as in Equation [16], separately for core players and peripheral players. The results are reported in Table 7.

[29]For completeness, refer to Figures 10 and 11 for the mean investment levels using all rounds.

Table 7. Multilevel mixed effects model with between subjects random slopes, regressing Investment (final six rounds) on Joy Investment and externality condition; by player type in Core-Periphery networks.

Dependent variable: Individual investment in round t						
	Pooled		STAR		CP2	
	Peripheral	Core	Peripheral	Core	Peripheral	Core
Joy Investment	0.33*	0.86**	0.15	4.17***	0.40	0.71**
	(0.18)	(0.38)	(0.21)	(0.95)	(0.26)	(0.31)
Neg	24.71**	94.95***	19.56	48.15**	32.34***	116.10***
	(12.14)	(26.28)	(19.19)	(22.77)	(10.95)	(39.70)
Pos	−14.86	−56.76***	−15.98	−88.21***	−13.16	−29.01**
	(11.25)	(13.44)	(17.63)	(20.13)	(10.72)	(14.08)
Joy Invest × Neg	0.13	−0.59***	0.10	2.34***	0.15	−0.64***
	(0.11)	(0.08)	(0.15)	(0.38)	(0.16)	(0.09)
Joy Invest × Pos	−0.01	−0.29***	0.04	−3.83***	−0.05	−0.31***
	(0.08)	(0.05)	(0.05)	(0.61)	(0.12)	(0.05)
AA	2.54	0.71	3.63*	4.05	1.21	−6.53
	(1.70)	(3.32)	(2.20)	(3.17)	(2.93)	(4.09)
RA	−0.38	3.94	0.98	13.34***	−2.36	−6.53**
	(2.30)	(4.54)	(2.84)	(3.75)	(4.11)	(2.62)
LA	−1.11	−4.28**	−2.53	−7.69***	0.83	−2.33
	(1.71)	(1.90)	(2.13)	(2.31)	(2.45)	(1.81)
Constant	140.05***	119.92***	152.04***	95.99***	127.07***	175.93***
	(24.06)	(35.48)	(27.58)	(35.25)	(41.45)	(34.48)
$\widehat{\sigma}_{u_0}$ (Constant)	89.76***	78.04***	96.62***	53.43***	74.98***	68.45***
$\widehat{\sigma}_{u_1}$ (Joy Investment)	0.37***	0.65**	0.19***	–	0.46***	0.47***
$\widehat{\sigma}_{\epsilon}$ (Residual)	144.35***	143.01***	144.31***	124.52***	144.22***	147.69***
Groups	34	34	18	18	16	16
Observations	2772	900	1620	324	1152	576
Wald $\chi^2(8)$	53.87***	218.47***	72.34***	3057.92***	140.67***	483.51***

Robust standard errors, adjusted for clustering at the group level, in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In the first two columns, we pool the STAR and CP2 networks together. For peripheral players, Joy Investment has a positive, significant effect on investment in the Negative and Positive conditions, and a marginally significant effect in Baseline, but the effects do not differ significantly between the three externality conditions. In contrast, for the core players, while the effect is significant in the Baseline condition (Wald test, $p = 0.022$), we find no statistically significant effect in the Negative ($p = 0.521$) or Positive ($p = 0.147$) conditions.

The remaining columns examine the STAR network and CP2 network separately. For the STAR network, the effect of Joy Investment for peripheral players is only statistically significant for the Negative externality condition.[30] In contrast, for the core players, Joy Investment has a

[30] We reject the null hypothesis that $\beta_1 + \alpha_1 = 0$ (Wald test, $p = 0.029$) for the Negative condition. For the

strongly significant impact in the Baseline condition, an even stronger impact in the Negative condition, and a weaker, statistically insignificant effect in the Positive condition (Wald test, $p = 0.718$). The results are similar for the peripheral players in the CP2 network, where the effect is significant only in the Negative condition (Wald test, $p = 0.028$). However, for the core players, the effect is only significant in the Baseline condition (Wald test, $p = 0.022$), although it is stronger in the Positive condition than in the Negative condition.[31] We summarize these findings as follows.

Result 11. *In the core-periphery networks, non-monetary utility of winning*

(i) *has a significant positive effect on peripheral players' investment in the Negative condition, but does not affect their behavior in the other externality conditions;*

(ii) *has a strongly significant effect on investment by the core players in the Baseline conditions and in* STAR-N*, but no significant effect on core players' investment in the other core-periphery treatment conditions.*

Thus, the same argument we appeal to for regular networks (cf. Result 9) does not receive the same support in the context of core-periphery networks. Joy of winning does not appear to explain the differential patterns of over-investment by the peripheral players in the different externality conditions. Furthermore, the argument clearly does not explain the observed investment patterns of the core players. Instead, we argue that subjects take into consideration the impact of the allocative externalities on *social efficiency*.

Social efficiency concerns. Consider the following line of reasoning. In the core-periphery networks, a greater number of neighbors are impacted when the prize is allocated to a core player. As a result, the sum of payoffs can be altered (perhaps quite significantly) by the allocation.[32] For instance, consider the STAR network, in which there is just one core player. In the Negative externality condition, the unique equilibrium involves the core player winning with higher probability than an individual peripheral player.[33] However, when the core player wins, every other player is impacted severely. If a peripheral player wins the contest instead, the only player who suffers is the core player.

Thus, from a social efficiency standpoint, the total harm is minimized if the prize is allocated to a peripheral player. If subjects share some concern for social efficiency, we might expect to see the peripheral players collectively over-invest, so as to reduce the chances of the widespread harm that will arise in the event that the core player wins the contest. This can be interpreted as an alternative explanation to the joy of winning hypothesis, or as a foundation for the elevated

other two conditions, we fail to reject, with $p > 0.335$.

[31] We fail to reject the null hypotheses that $\beta_1 + \alpha_1 = 0$ ($p = 0.839$) or $\beta_1 + \alpha_2 = 0$ ($p = 0.245$).

[32] In contrast, in the regular networks, the structure of the network is such that the flow of externalities, while identity-dependent, has no effect on the sum of payoffs (holding fixed the effort investments), since regardless of which agent is allocated the prize, the number of neighbors (who are impacted by the externality) is the same. As such, social efficiency concerns should not come into play.

[33] The core player's equilibrium probability of winning is about 0.357, while each peripheral player wins with approximately 0.129 probability.

joy of winning on the part of the peripheral players in STAR-N. A similar concern with social efficiency may mitigate the joy of winning for the core players in the STAR-N condition, leading them to reduce their investment levels in consideration of others. However, even those core players who are unconcerned with the harm they may inflict upon others may rationally reduce their investment, as a best response to the over-investment by peripheral players.

An analogous argument can be made to explain the opposite patterns of over-investment observed in the presence of positive externalities. In this case, the total externality flows are maximized when the core player wins the contest. However, in equilibrium, the probability that the core player wins is zero, since the equilibrium investment profile involves the core player choosing to be inactive. Thus, equilibrium and efficiency are in direct conflict with each other. If the core and peripheral players care about social efficiency, they may be able to coordinate on an investment profile in which the peripheral players remain inactive, allowing the core player to win the contest and provide externality benefits to all. Indeed, we find some evidence in support of these patterns for some of the groups in the STAR-P condition.

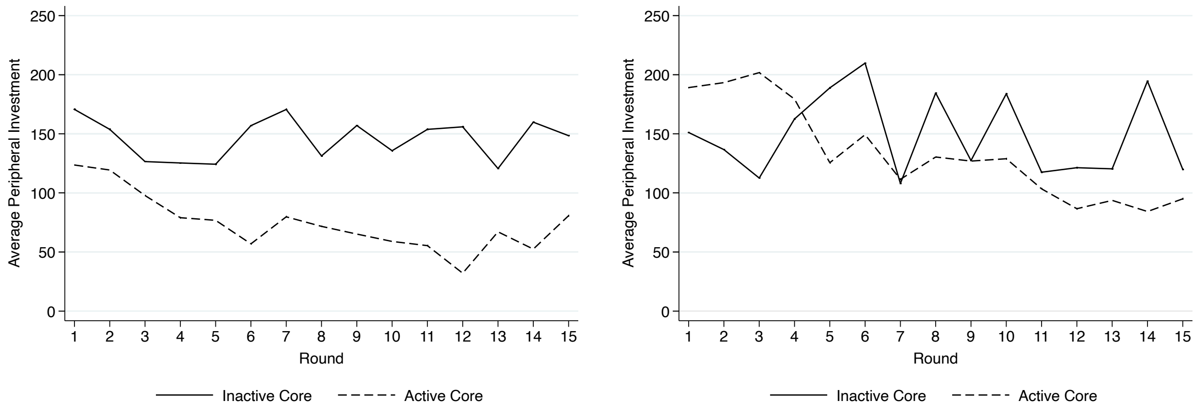
Figure 16 displays average investment levels among peripheral players over time for the Positive externality condition, depending on the median investment level (across all rounds) of the core player(s). In the case of the STAR network, we calculate the median investment level of the lone core player across all 15 rounds in STAR-P.[34] We find that 13 of 18 core players have a median investment level of one or less and we classify these individuals as *inactive*; the remaining five core players all have median investment levels of 100 tokens or more, and so are classified as *active*. In the case of the CP2 network, we first calculate the sum of the core players' investments in each round and then calculate the median of the aggregate investment by core players, collectively, across the 15 rounds. We find that four out of 16 cores have a median aggregate investment of five or less and classify these cores as *inactive*; the remaining 12 cores all have median aggregate investment levels of 120 or more and are classified as *active*.

In STAR-P, we see a clear disparity between the average investment levels among peripheral players when the core player is active versus inactive. Specifically, peripheral players invest, on average, at a much lower level when the core player in their group is active in the contest than when the core player is inactive.[35] This is consistent with a preference among the peripheral players for a more socially efficient outcome in which the core player wins the contest and all peripheral players benefit from the positive externality flows. When the core player is inactive, peripheral players' investments are, on average, well above the NE prediction across all rounds, consistent with the impact that joy of winning appears to have in the Baseline condition.

In CP2-P, average investments by peripheral players in groups with active cores appear to be converging to a slightly lower level in later rounds compared to those groups with inactive cores. However, the disparity is not nearly as stark as in the STAR network. This is not especially surprising, since there remains some conflict between the two sides of the CP2 network. Specifically, unlike in the STAR network, the peripheral players are not guaranteed to benefit when an active core player wins the contest. Only when the prize is allocated to the core player

[34] Figure B.7 in Appendix B provides boxplots for each individual core player in the STAR-P and CP2-P conditions, using all rounds.

[35] Furthermore, their mean investment is also below the NE prediction for most of the final 10 rounds.



(a) Star-P

(b) CP2-P

Figure 16. Average investment level among peripheral players in Star-P and CP2-P, separated by whether the core player(s) are *active* or *inactive*.

to whom the peripheral player is linked does the player enjoy the positive externality.

Examining the data at the group-level provides some additional clarity regarding the patterns of behavior that influence the observed levels of over-investment. Figure 17 plots the average investments of individuals by type in the final six rounds of the Star-N condition, for each independent group. For half of the groups, the core player consistently invests more than the average of the peripheral players' investments (see group IDs 14, 16, 21, 22, 23, 29, 30, and 38), in line with the equilibrium profile. However, there are also groups in which the core player is inactive, while the peripheral players average significantly positive investments (e.g., see group IDs 13, 24, and 39, and to a lesser degree, group IDs 15, 17, and 18, where the core player invests less than both the NE prediction and the average of the peripheral players).

Similarly, in the CP2 network (see Figure 18), there are several groups in which the core players invest (on average) more than the peripheral players (for example, group IDs 43, 44, 46, 53, and 59). Yet, in most other groups, mean investment by the peripheral players is comparable to, or even slightly above, the mean investment by the core players. Indeed, for six of the groups, the average core player investment is below the NE prediction over the final six rounds.

Altogether, the heterogeneity among groups in the Star and CP2 networks serves to illustrate that the different patterns of mean over-investment for core and peripheral players in the Negative externality condition may be driven by the differential impact of social efficiency concerns in several groups.

Figure 19 plots the average investments of individuals by type in the final six rounds of the Star-P condition, for each independent group. First, we observe that the most common pattern is a core player investing at or near zero in every round, accompanied by positive average investment levels among the peripheral players. We see this type of behavior, which most resembles the equilibrium predictions in 12 of the 18 groups (those with group IDs 13, 16, 17, 18, 19, 20, 23, 24, 28, 29, 30, and 39). However, another pattern that emerges is the

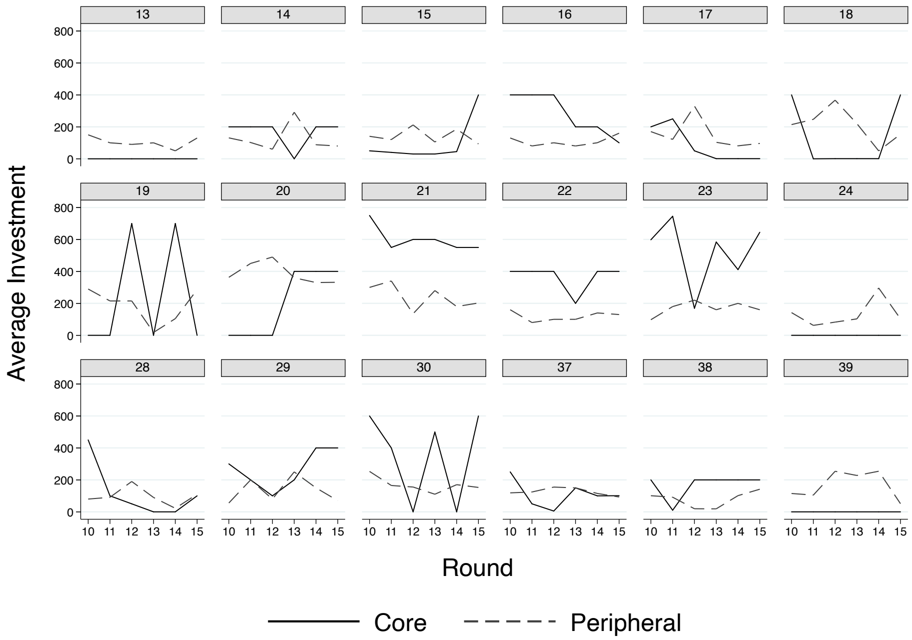


Figure 17. Average investment by player type over the final six rounds in the Star-N condition. Each cell represents one independent group.

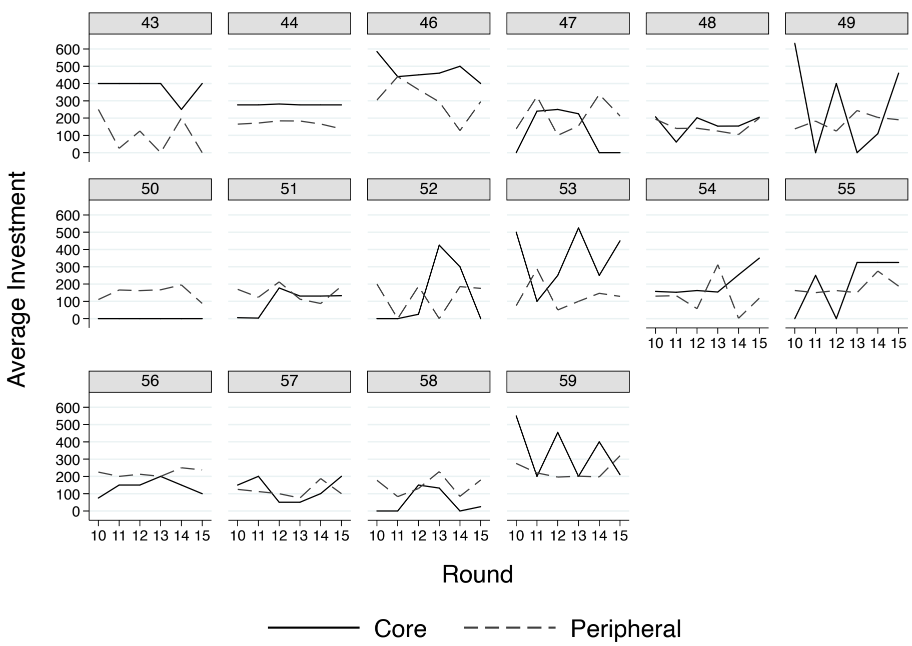


Figure 18. Average investment by player type over the final six rounds in the CP2-N condition. Each cell represents one independent group.

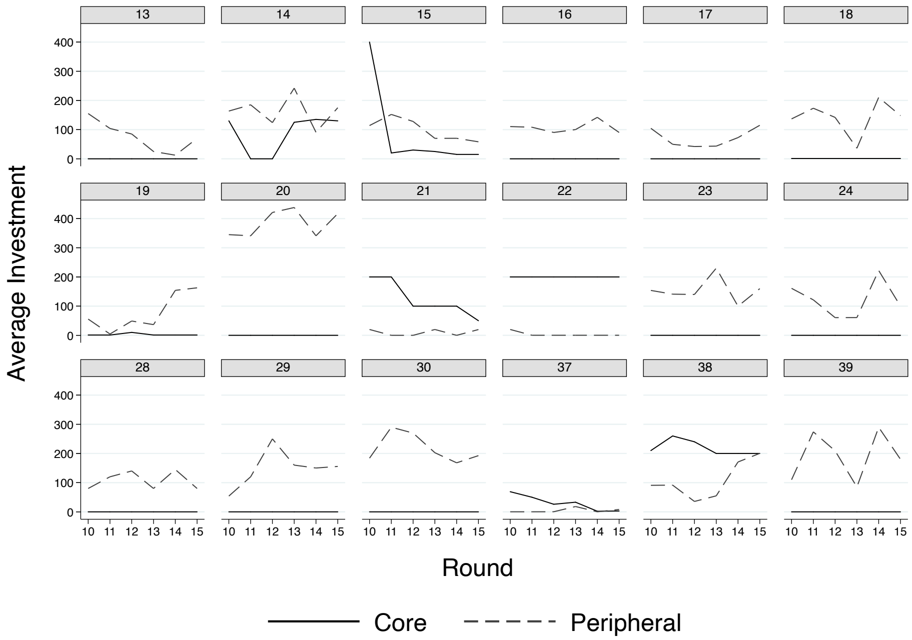


Figure 19. Average investment by player type over the final six rounds in the Star-P condition. Each cell represents one independent group.

one in which the core player is active (investing strictly positive amounts) while the peripheral players average at or near zero investment. This type of pattern, which is more consistent with coordination on a more socially efficient outcome, can be clearly seen in groups 21 and 22, and to a lesser degree, in group 37.

Similarly, Figure 20 plots the average investments of individuals by type in the final six rounds of the CP2-P condition, for each independent group. As in the Star network, one of the prominent patterns is, consistent with the NE prediction, for the core players to invest nothing while the peripheral players compete against each other. These groups are characterized by zero (or near-zero) core investments and positive peripheral players' investments (for instance, see groups with IDs 47, 50, 53, 55, 58, and 59). Another pattern involves relatively higher (average) investment by the core players, and lower investment by the peripheral players, as in groups 43 and 51. However, several other groups display a mixture of behavior, with both core and peripheral players competing actively even over the final six rounds. As discussed above, this is not especially surprising, since it may correspond to groups in which the peripheral players on one side of the CP2 network and the core player on the other side of the CP2 network compete against each other.

The following result summarizes our findings regarding the impact of social efficiency concerns on over-investment patterns in the core-periphery networks.

Result 12. *In the core-to-periphery networks, the aggregate patterns of mean over-investment are driven by a mixture of group-level patterns - some groups converge to investment profiles*

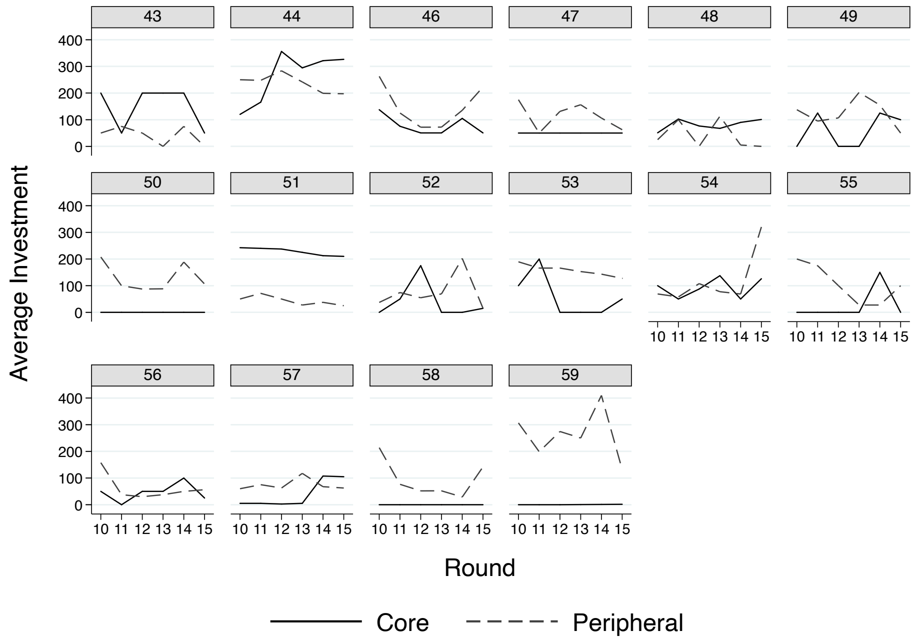


Figure 20. Average investment by player type over the final six rounds in the CP2-P condition. Each cell represents one independent group.

consistent with the NE prediction, while others exhibit behavior consistent with a concern for social efficiency. For negative (positive) externalities, this heterogeneity combines to decrease (increase) the mean investment of core players and increase (decrease) the mean investment of peripheral players.

5 Conclusion

In this paper, we introduce and analyze a model of contests with identity-dependent externalities that are governed by a network. Our theoretical results simultaneously broaden the scope of traditional contest theory and extend the network games literature to a setting in which players have non-linear best replies. The model allows for positive and negative externalities, stemming from the allocation of the prize, that impact the payoffs of all players directly connected to the winner of the contest. We prove the existence of equilibria in general, and characterize sufficient conditions—related to the structure of the network—for uniqueness. For two broad classes of networks (regular and core-to-periphery), we provide closed-form results and show that the comparative statics align with the intuition from our motivating examples. Our framework can serve as a basis for studying a wide range of competitive situations, whether between firms or other organizations, individuals connected in a social network, or lobbyists with preferences over a multi-dimensional policy space.

In order to test the predictions of the model, we conducted a laboratory experiment in which we systematically varied both the network and the externalities. The experimental findings lend considerable support to the comparative statics predictions of the model. Furthermore, we observe mean over-investment in most treatment conditions, consistent with the existing experimental literature on contests. Nevertheless, the particular patterns of over-investment depend on the network, the externality condition and, in the core-periphery structures, the player's position within the network. We provide supporting evidence for the influence of two behavioral phenomena—joy of winning, and social efficiency concerns—that appear to play an important role in the network contest game.

There are several directions in which our research may be extended. Our theoretical framework is relatively stylized—for instance, we limit attention to contests in which the externality flows are all of the same size and sign and the identity-dependence is driven entirely by the structure of the network. In future work, it may be interesting to generalize the model to allow for both positive and negative externalities within the same network, or to allow for link-specific externality flows. A related extension might be to allow for externalities that travel beyond the winner's immediate neighborhood, but with diluted impact proportional to the distance traveled. From an empirical perspective, it would be useful to explore the impact of externalities in other, potentially larger, network structures. Finally, more work is needed to better understand the importance of social efficiency concerns and joy of winning for competitive behavior when there are network-based identity-dependent externalities.

References

Allouch, N. (2015). On the private provision of public goods on networks. *Journal of Economic Theory 157*, 527–552.

Amaldoss, W. and A. Rapoport (2009). Excessive expenditure in two-stage contests: Theory and experimental evidence. *Game Theory: Strategies, Equilibria, and Theorems. Hauppauge, NY: Nova Science Publishers*.

Aseff, J. and H. Chade (2008). An optimal auction with identity-dependent externalities. *The RAND Journal of Economics 39*(3), 731–746.

Bagh, A. and A. Jofre (2006). Reciprocal upper semicontinuity and better reply secure games: a comment. *Econometrica 74*(6), 1715–1721.

Ballester, C., A. Calvó-Armengol, and Y. Zenou (2006). Who's who in networks. wanted: The key player. *Econometrica 74*(5), 1403–1417.

Berninghaus, S. K., K.-M. Ehrhart, and C. Keser (2002). Conventions and local interaction structures: experimental evidence. *Games and Economic Behavior 39*(2), 177–205.

Boosey, L., P. Brookins, and D. Ryvkin (2017). Contests with group size uncertainty: Experimental evidence. *Games and Economic Behavior 105*, 212–229.

Bramoullé, Y. and R. Kranton (2007). Public goods in networks. *Journal of Economic Theory 135*(1), 478–494.

Bramoullé, Y., R. Kranton, and M. D'Amours (2010). Strategic interaction and networks.

Bramoullé, Y., R. Kranton, and M. D'Amours (2014). Strategic interaction and networks. *American Economic Review 104*(3), 898–930.

Brocas, I. (2003). Endogenous entry in auctions with negative externalities. *Theory and Decision 54*(2), 125–149.

Brocas, I. (2013a). Optimal allocation mechanisms with type-dependent negative externalities. *Theory and decision 75*(3), 359–387.

Brocas, I. (2013b). Selling an asset to a competitor. *European Economic Review 57*, 39–62.

Brocas, I. (2014). Countervailing incentives in allocation mechanisms with type-dependent externalities. *Journal of Mathematical Economics 50*, 22–33.

Brookins, P. and D. Ryvkin (2014). An experimental study of bidding in contests of incomplete information. *Experimental Economics 17*(2), 245–261.

Cameron, A. C., J. B. Gelbach, and D. L. Miller (2008). Bootstrap-based improvements for inference with clustered errors. *The Review of Economics and Statistics 90*(3), 414–427.

Cassar, A. (2007). Coordination and cooperation in local, random and small world networks: Experimental evidence. *Games and Economic Behavior 58*(2), 209–230.

Charness, G., F. Feri, M. A. Meléndez-Jiménez, and M. Sutter (2014). Experimental games on networks: Underpinnings of behavior and equilibrium selection. *Econometrica 82*(5), 1615–1670.

Choi, S., E. Gallo, and S. Kariv (2016). Networks in the laboratory.

Chowdhury, S. M., R. M. Sheremeta, and T. L. Turocy (2014). Overbidding and overspreading in rent-seeking experiments: Cost structure and prize allocation rules. *Games and Economic Behavior 87*, 224–238.

Das Varma, G. (2002). Standard auctions with identity-dependent externalities. *RAND Journal of Economics*, 689–708.

Dechenaux, E., D. Kovenock, and R. M. Sheremeta (2015). A survey of experimental research on contests, all-pay auctions and tournaments. *Experimental Economics 18*(4), 609–669.

Esteban, J. and D. Ray (1999). Conflict and distribution. *Journal of Economic Theory 87*(2), 379–415.

Ewerhart, C. (2017). The lottery contest is a best-response potential game. *Economics Letters 155*, 168–171.

Fischbacher, U. (2007). z-tree: Zurich toolbox for ready-made economic experiments. *Experimental economics 10*(2), 171–178.

Franke, J. and T. Öztürk (2015). Conflict networks. *Journal of Public Economics 126*, 104–113.

Galeotti, A., S. Goyal, M. O. Jackson, F. Vega-Redondo, and L. Yariv (2010). Network games. *The Review of Economic Studies 77*(1), 218–244.

Gallo, E. and C. Yan (2015). Efficiency and equilibrium in network games: An experiment.

Goeree, J. K., C. A. Holt, and T. R. Palfrey (2002). Quantal response equilibrium and overbidding in private-value auctions. *Journal of Economic Theory 104*(1), 247–272.

Goyal, S. and A. Vigier (2014). Attack, defence, and contagion in networks. *The Review of Economic Studies 81*(4), 1518–1542.

Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with ORSEE. *Journal of the Economic Science Association 1*(1), 114–125.

Hehenkamp, B., W. Leininger, and A. Possajennikov (2004). Evolutionary equilibrium in tullock contests: spite and overdissipation. *European Journal of Political Economy 20*(4), 1045–1057.

Hiller, T. (2017). Friends and enemies: A model of signed network formation. *Theoretical Economics 12*(3), 1057–1087.

Holt, C. A. and S. K. Laury (2002). Risk aversion and incentive effects. *American Economic Review 92*(5), 1644–1655.

Jackson, M. O. (2014). Networks in the understanding of economic behaviors. *Journal of Economic Perspectives 28*(4), 3–22.

Jackson, M. O. and S. Nei (2015). Networks of military alliances, wars, and international trade. *Proceedings of the National Academy of Sciences 112*(50), 15277–15284.

Jehiel, P. and B. Moldovanu (1996). Strategic nonparticipation. *The RAND Journal of Economics*, 84–98.

Jehiel, P. and B. Moldovanu (2000). Auctions with downstream interaction among buyers. *The RAND Journal of Economics 31*(4), 768–791.

Jehiel, P. and B. Moldovanu (2006). Allocative and informational externalities in auctions and related mechanisms. In R. Blundell, W. K. Newey, and T. Persson (Eds.), *Advances in Economics and Econometrics: Volume 1: Theory and Applications, Ninth World Congress*, Volume 1, Chapter 3. Cambridge: Cambridge University Press.

Jehiel, P., B. Moldovanu, and E. Stacchetti (1996). How (not) to sell nuclear weapons. *The American Economic Review*, 814–829.

Jehiel, P., B. Moldovanu, and E. Stacchetti (1999). Multidimensional mechanism design for auctions with externalities. *Journal of economic theory 85*(2), 258–293.

Katz, E., S. Nitzan, and J. Rosenberg (1990). Rent-seeking for pure public goods. *Public Choice 65*(1), 49–60.

Keser, C., K.-M. Ehrhart, and S. K. Berninghaus (1998). Coordination and local interaction: experimental evidence. *Economics Letters 58*(3), 269–275.

Klose, B. and D. Kovenock (2015). The all-pay auction with complete information and identity-dependent externalities. *Economic Theory 59*(1), 1–19.

König, M. D., D. Rohner, M. Thoenig, and F. Zilibotti (2017). Networks in conflict: Theory and evidence from the great war of africa. *Econometrica 85*(4), 1093–1132.

Konrad, K. A. (2006). Silent interests and all-pay auctions. *International Journal of Industrial Organization 24*(4), 701–713.

Kovenock, D. and B. Roberson (2018). The optimal defense of networks of targets. *Economic Inquiry 56*(4), 2195–2211.

Lim, W., A. Matros, and T. L. Turocy (2014). Bounded rationality and group size in tullock contests: Experimental evidence. *Journal of Economic Behavior & Organization 99*, 155–167.

Linster, B. G. (1993). A generalized model of rent-seeking behavior. *Public Choice 77*(2), 421–435.

Lu, J. (2006). When and how to dismantle nuclear weapons.

Mago, S. D., A. C. Samak, and R. M. Sheremeta (2016). Facing your opponents social identification and information feedback in contests. *Journal of Conflict Resolution 60*(3), 459–481.

Matros, A. and D. Rietzke (2018). Contests on networks. *Available at SSRN 3179547*.

McKelvey, R. D. and T. R. Palfrey (1995). Quantal response equilibria for normal form games. *Games and Economic Behavior 10*(1), 6–38.

Melo, E. (2018). A variational approach to network games.

Monderer, D. and L. S. Shapley (1996). Potential games. *Games and economic behavior 14*(1), 124–143.

Parco, J. E., A. Rapoport, and W. Amaldoss (2005). Two-stage contests with budget constraints: An experimental study. *Journal of Mathematical Psychology 49*(4), 320–338.

Parise, F. and A. Ozdaglar (2019). A variational inequality framework for network games: Existence, uniqueness, convergence and sensitivity analysis. *Games and Economic Behavior 114*, 47–82.

Reny, P. J. (1999). On the existence of pure and mixed strategy nash equilibria in discontinuous games. *Econometrica 67*(5), 1029–1056.

Rosenkranz, S. and U. Weitzel (2012). Network structure and strategic investments: An experimental analysis. *Games and Economic Behavior 75*(2), 898–920.

Sheremeta, R. M. (2010). Experimental comparison of multi-stage and one-stage contests. *Games and Economic Behavior 68*(2), 731–747.

Sheremeta, R. M. (2011). Contest design: An experimental investigation. *Economic Inquiry 49*(2), 573–590.

Sheremeta, R. M. (2013). Overbidding and heterogeneous behavior in contest experiments. *Journal of Economic Surveys 27*(3), 491–514.

Sheremeta, R. M. (2016). Impulsive behavior in competition: Testing theories of overbidding in rent-seeking contests. *ESI Working Paper 16-21*. http://digitalcommons.chapman.edu/esi_working_papers/199/.

Sutter, M., M. G. Kocher, D. Glätzle-Rützler, and S. T. Trautmann (2013). Impatience and uncertainty: Experimental decisions predict adolescents' field behavior. *American Economic Review 103*(1), 510.

Szidarovszky, F. and K. Okuguchi (1997). On the existence and uniqueness of pure nash equilibrium in rent-seeking games. *Games and Economic Behavior 18*(1), 135–140.

Tullock, G. (1980). Efficient rent-seeking. In J. M. Buchanan, R. D. Tollison, and G. Tullock (Eds.), *Towards a theory of a rent-seeking society*, pp. 97–112. Texas A & M Univ Press, College Station.

van Leeuwen, B., A. Ramalingam, D. R. Arjona, and A. Schram (2019). Centrality and cooperation in networks. *Experimental economics 22*(1), 178–196.

Voorneveld, M. (2000). Best-response potential games. *Economics letters 66*(3), 289–295.

Xu, J., Y. Zenou, and J. Zhou (2019). Networks in conflict: A variational inequality approach. *Available at SSRN 3364087*.

Zenou, Y. and J. Zhou (2020). Network games made simple. Technical report, mimeo, Monash University and National University of Singapore.

A Proofs

Proof of Theorem 1. We prove existence by applying Theorem 3.1 in Reny (1999). For completeness, we restate the theorem here, using our own notation for consistency.

Theorem (Theorem 3.1, Reny (1999))**.** *If $\Gamma = (X_i, \pi_i)_{i=1}^n$ is compact, quasiconcave, and better-reply secure, then it possesses a pure strategy Nash equilibrium.*

Let $\Gamma = (X_i, \pi_i)_{i=1}^n$ denote the normal-form of the network contest game. Note that while $X_i = \mathbb{R}_+$ for each $i \in N$, we can, without loss of generality, restrict the agents' strategies to compact subsets of $\mathbb{R}_+$. To see why, note that since $\alpha \geqslant -1$, $P_i \leqslant 1$, and $d_i = \sum_h g_{ih} \leqslant n-1$, all strategies $x_i > 1 + (n-1) = n$ are strictly dominated by $x_i = 0$. Thus, we can restrict the strategy sets to $\widehat{X}_i = [0, n]$, which is compact. Next, we note that each agent i's payoff function is concave, and thus also quasiconcave, in x_i. It remains to show that Γ is *better reply secure.* To do so, we first introduce some relevant definitions and another result by Bagh and Jofre (2006) that extends on Reny (1999).

Definition A.1. In the game $\Gamma = (X_i, \pi_i)_{i=1}^n$, player i can *secure* a payoff of $\alpha \in \mathbb{R}$ at $x \in X$ if there exists $y_i \in X_i$ such that $\pi_i(y_i, x'_{-i}) \geqslant \alpha$ for all x'_{-i} in some open neighborhood of x_{-i}.

Definition A.2. A game $\Gamma = (X_i, \pi_i)_{i=1}^n$ is *payoff secure* if for every $x \in X$ and every $\varepsilon > 0$, each player i can secure a payoff of $\pi_i(x) - \varepsilon$ at x.

Let $\Lambda = \{(\mathbf{x}, \pi) \in \mathbf{X} \times \mathbb{R}^n \mid \pi_i(\mathbf{x}) = \pi_i, \forall\, i\}$ denote the graph of the vector of payoff functions for the game and let $\overline{\Lambda}$ denote the closure of Λ in $\mathbf{X} \times \mathbb{R}^n$. Finally, define the frontier of Λ to be the set of points in $\overline{\Lambda}$ but not in Λ, denoted by $\mathrm{Fr}\Lambda = \overline{\Lambda} \backslash \Lambda$. The following definition is from Bagh and Jofre (2006).

Definition A.3. A game $\Gamma = (X_i, \pi_i)_{i=1}^n$ is *weakly reciprocally upper semicontinuous* (wrusc) if, for any $(\mathbf{x}, \pi) \in \mathrm{Fr}\Lambda$, there is a player i and $\widehat{x}_i \in X_i$ such that $\pi_i(\widehat{x}_i, x_{-i}) > \pi_i$.

Having defined payoff security and wrusc, we then appeal to the following result from Bagh and Jofre (2006).

Proposition A.1 (Proposition 1, Bagh and Jofre (2006))**.** *If the game $\Gamma = (X_i, \pi_i)_{i=1}^n$ is payoff secure and wrusc, then it is better reply secure.*

To prove that Γ is payoff secure and wrusc, we follow a similar approach to Bagh and Jofre (2006) in their Example 3, which considers (a generalized form of) the standard contest game with Tullock (1980) contest success function.

(i) First, we show that the game is payoff secure. Note that payoffs are continuous except at $\mathbf{x} = \mathbf{0}$, where they are given by
$$\pi_i(\mathbf{0}) = \frac{1 + \alpha d_i}{n}$$
where d_i is player i's degree in the network. Then note that for $\tilde{x}_i > 0$, we have $\pi_i(\tilde{x}_i, \mathbf{0}) = 1 - \tilde{x}_i$, which is higher than $\pi_i(\mathbf{0})$ if $\tilde{x}_i < (n - 1 - \alpha d_i)/n$. Since $d_i \leqslant n-1$ and $\alpha < 1$, the right hand side is strictly positive, so that such a $\tilde{x}_i > 0$ can be found. Then, since $\pi_i(\cdot)$ is

continuous at $(\tilde{x}_i, \mathbf{0})$, there is a neighborhood V of $\mathbf{x}_{-i} = \mathbf{0}$ such that $\pi_i(\tilde{x}_i, \mathbf{x}'_{-i}) > \pi_i(0, \mathbf{0})$ for all $\mathbf{x}'_{-i} \in V$. Thus, the game is payoff secure at the point $\mathbf{x} = \mathbf{0}$. Payoff security at all other $\mathbf{x}$ is straightforward.

(ii) Second, we show that the game is wrusc. In this game (as in the standard contest game), the only points in FrΛ must be points of the form $(\mathbf{0}, \pi)$ where $\pi_i = \lim_{\mathbf{x}^k \to \mathbf{0}} \pi_i(\mathbf{x}^k)$ for all i. Note that

$$\begin{aligned}\sum_{i=1}^{n} \pi_i(\mathbf{x}^k) &= \sum_{i=1}^{n} P_i(\mathbf{x}^k) - \sum_{i=1}^{n} x_i + \alpha \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} P_j(\mathbf{x}^k) \\ &= 1 - \sum_{i=1}^{n} x_i + \alpha \sum_{i=1}^{n} d_i P_i(\mathbf{x}^k) \\ &\leqslant 1 - \sum_{i=1}^{n} x_i + \alpha(n-1)\end{aligned}$$

where the inequality follows from the fact that $d_i \leqslant n-1$ for all i and $\sum_{i=1}^{n} P_i(\mathbf{x}^k) = 1$. As such, $\lim_{\mathbf{x}^k \to \mathbf{0}} \sum_{i=1}^{n} \pi_i(\mathbf{x}^k) \leqslant 1 + \alpha(n-1)$ and thus, there exists some i for whom

$$\pi_i \leqslant \frac{1 + \alpha(n-1)}{n}.$$

Notice that $\lim_{x_i \to 0} \pi_i(x_i, \mathbf{0}) = 1$ and therefore, there exists $\hat{x}_i > 0$ such that $\pi_i(\hat{x}_i, \mathbf{0}) > \pi_i$, because $\alpha < 1$ ensures that $(1 + \alpha(n-1))/n < 1$. It follows that the game is wrusc.

Together, payoff security and wrusc imply the game is better reply secure, and applying Theorem 3.1 from Reny (1999), there exists a pure strategy Nash equilibrium. □

Proof of Proposition 2. Suppose that $A = N$ (that is, all agents are active). From Lemma 1, only condition (i) needs to be satisfied. Summing equation [6] for all n active players and rearranging gives

$$(n-1) \sum_{i=1}^{n} x_i - \alpha \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} x_j = n \left(\sum_{i=1}^{n} x_i \right)^2$$

and positing $x_i = x$ for all i yields

$$\begin{aligned}(n-1)nx - \alpha nkx &= n(nx)^2 \\ (n-1) - \alpha k &= n^2 x\end{aligned}$$

from which x^* follows. □

Proof of Proposition 3. Both parts of the proposition follow directly. From condition (i) in Lemma 1, it follows from the fact that $g_{ij} = 0$ for all $i, j \in A$ in a specialized equilibrium, that

$x_i = \sum_{j \in A} x_j - (\sum_{j \in A} x_j)^2$ for all $i \in A$, which implies that all active players must be choosing the same investment $\bar{x}_A = \frac{n_A - 1}{n_A^2}$. Therefore, total investment is given by $X_A = \sum_{j \in A} x_j = (n_A - 1)/n_A$. Then, for the second condition in Proposition 1 to be satisfied, it must be the case that for all $i \in N - A$,

$$\begin{aligned} & \frac{n_A - 1}{n_A^2}(n_A - \alpha d_A^i) \leqslant \frac{(n_A - 1)^2}{n_A^2} \\ \iff \quad & n_A - \alpha d_A^i \leqslant n_A - 1 \\ \iff \quad & \alpha \geqslant \frac{1}{d_A^i} \end{aligned}$$

Taking $d_{N-A,A}$ to be the minimum of d_A^i over all $i \in N - A$ ensures that the inequality is satisfied for all inactive players. □

Proof of Proposition 4. Suppose both types are active, $A = N$ and consider condition (i) from Proposition 1. For each peripheral player, equation [6] reduces to

$$(n_1 - \alpha)x_c + (n_1 m - 1)x_p = (n_1 x_c + n_1 m x_p)^2$$

while for each core player, it simplifies to

$$(n_1 - 1)(1 - \alpha)x_c + (n_1 m - \alpha m)x_p = (n_1 x_c + n_1 m x_p)^2.$$

From this, we obtain

$$x_c(1 + \alpha(n_1 - 2)) = (1 - \alpha m)x_p.$$

Substituting into the condition for the core players and solving yields the solution $x_c = (1 - \alpha m)\Delta$ and $x_p = (1 + \alpha(n_1 - 2))\Delta$, where

$$\Delta = \frac{n_1 \left[1 + m + \alpha m(n_1 - 3)\right] - \left[1 + \alpha(n_1 - 1 - \alpha m)\right]}{n_1^2 \left[1 + m + \alpha m(n_1 - 3)\right]^2} \geqslant 0.$$

For x_c to be strictly positive, we must have $\alpha < \frac{1}{m}$. Thus, a semi-symmetric equilibrium with full participation exists only when α is not too large. Once $\alpha \geqslant \frac{1}{m}$, there is a semi-symmetric equilibrium which is also a *specialized equilibrium* in which the core players are all inactive, while the peripheral players, who form a maximal independent set, invest the standard equilibrium investment for a contest between $n_1 m$ individuals (where $n_1 m$ is the number of peripheral players in the network). □

Proof of Theorem 2. The network contest game is a best-response potential game (Voorneveld, 2000). Lemma 2 provides a BR-potential for the game, $\mathbf{P}$. Then, by Proposition 2.2 of Voorneveld (2000), the profile $\mathbf{x}$ is a Nash equilibrium of the network contest game if and only if it maximizes the BR-potential, $\mathbf{P}$. The remainder of the proof establishes conditions under which there is a unique maximizer for $\mathbf{P}$.

Recall that $\mathbf{P}$ is strictly concave if $\nabla^2\mathbf{P}$ is negative definite. Before deriving the Hessian for $\mathbf{P}$, note that we can restrict the search for maxima to investment profiles $\mathbf{x}$ with $|A(\mathbf{x})| \geqslant 2$, since we have already established that there are no Nash equilibria in which fewer than 2 players are active. Thus, for any such $\mathbf{x}$, the diagonal elements of the Hessian $\nabla^2\mathbf{P}$ are given by

$$\frac{\partial^2\mathbf{P}}{\partial x_i^2} = -2\sum_{h=1}^{n} x_h$$

while the cross-partial terms are symmetric and given by

$$\frac{\partial^2\mathbf{P}}{\partial x_i \partial x_j} = \frac{\partial^2\mathbf{P}}{\partial x_j \partial x_i} = (1-\alpha g_{ij}) - 2\sum_{h=1}^{n} x_h.$$

Rewriting in matrix form and using $X_{tot} = \sum_h x_h$ gives

$$\nabla^2\mathbf{P} = \big(1-2X_{tot}\big)\mathbf{J} - [\mathbf{I}+\alpha\mathbf{G}], \tag{17}$$

where $\mathbf{J}$ denotes the $n \times n$ matrix of ones. Note that even if $\mathbf{I}+\alpha\mathbf{G}$ is positive definite, if $X_{tot} < 0.5$ and is small enough, the Hessian need not be negative definite. Our approach to get around this problem is to partition the domain into two subsets, $\mathbf{X}^H$ and $\mathbf{X}^L$.

(i) If we restrict the domain of $\mathbf{P}$ to the set $\mathbf{X}^H$ of vectors $\mathbf{x}$ such that $X_{tot} \geqslant 0.5$, it is readily verified that $\mathbf{P}$ is strictly concave on the restricted domain if $\mathbf{I}+\alpha\mathbf{G}$ is positive definite, which is equivalent to the condition that $\alpha < 1/|\lambda_{min}(\mathbf{G})|$.

(ii) Nevertheless, this condition is not sufficient to establish strict concavity on the subdomain $\mathbf{X}^L$, which is composed of strategy profiles $\mathbf{x}$ with $X_{tot} < 0.5$. Instead, we proceed by direct argument. Suppose that $\mathbf{x}$ is a Nash equilibrium with $X_{tot} < 0.5$.

   (a) Then, if there is any inactive player, k, we must have

   $$X_{tot} - \alpha\sum_{h=1}^{n} g_{kh}x_h \leqslant \big(X_{tot}\big)^2.$$

   Rearranging, we obtain

   $$\alpha \geqslant \frac{X_{tot}\big(1-X_{tot}\big)}{\sum_{h=1}^{n} g_{ih}x_h},$$

   and since $g_{ih} \leqslant 1$ for all i, h, it follows that

   $$\alpha \geqslant 1 - X_{tot} > 0.5.$$

   Thus, if there is an equilibrium with an inactive player, such that $X_{tot} = \sum_h x_h < 0.5$, it must be the case that $\alpha > 0.5$.

   (b) Then, suppose there is no inactive player for $\mathbf{x}$ with $X_{tot} < 0.5$. Then, for all n

players, we must have

$$x_i + \alpha \sum_{h=1}^{n} g_{ih} x_h \quad = \quad X_{tot}(1 - X_{tot}).$$

Summing over all i, obtain

$$\begin{aligned}
& \alpha \sum_{i=1}^{n} \sum_{h=1}^{n} g_{ih} x_h = X_{tot}(n(1 - X_{tot}) - 1) \\
\Rightarrow \quad & \alpha \sum_{h=1}^{n} x_h \sum_{i=1}^{n} g_{ih} > X_{tot} \left(\frac{n-2}{2} \right) \\
\Rightarrow \quad & \alpha \sum_{h=1}^{n} d_h x_h > X_{tot} \left(\frac{n-2}{2} \right) \\
\Rightarrow \quad & \alpha \Delta(\mathbf{G}) X_{tot} > X_{tot} \left(\frac{n-2}{2} \right) \\
\Rightarrow \quad & \alpha > \frac{n-2}{2\Delta(\mathbf{G})},
\end{aligned}$$

where the second line follows from $1 - X_{tot} > 0.5$, and the fourth line from the fact that $\Delta(\mathbf{G})$ is the maximum degree of $\mathbf{G}$.

It follows that if $\alpha \leqslant 0.5$ and $\alpha \leqslant 0.5(n-2)/\Delta(\mathbf{G})$, there cannot be a Nash equilibrium in $\mathbf{X}^L$. By existence of an equilibrium, there must exist at least one equilibrium in $\mathbf{X}^H$. If we also have that $\alpha < 1/|\lambda_{min}(\mathbf{G})|$, then $[\mathbf{I} + \alpha \mathbf{G}]$ is positive definite, $\mathbf{P}$ is strictly concave on $\mathbf{X}^H$, and there exists a unique Nash equilibrium, $\mathbf{x} \in \mathbf{X}^H$, such that $X_{tot} \geqslant 0.5$. □

Proof of Lemma 2. We proceed by cases. Fix a player i.

Case 1. Suppose $\mathbf{x}_{-i}$ has at least two strictly positive components. Then, for any x_i, $A(x_i, \mathbf{x}_{-i}) \geqslant 2$. It follows from [14] that

$$\frac{\partial \mathbf{P}}{\partial x_i} = \sum_{h \neq i} (1 - \alpha g_{ih}) x_h - X_{tot}^2$$

and

$$\frac{\partial^2 \mathbf{P}}{\partial x_i^2} = -2X_{tot} < 0.$$

It follows that $x_i \in \arg\max \mathbf{P}(x_i, \mathbf{x}_{-i})$ if and only if

$$x_i \Big(\sum_{h \neq i} (1 - \alpha g_{ih}) x_h - X_{tot}^2 \Big) = 0$$

which implies

$$x_i = \max \left\{ 0, \sqrt{\sum_{h \neq i} (1 - \alpha g_{ih}) x_h} - \sum_{h \neq i} x_h \right\},$$

which is exactly the best response function $f_i(\mathbf{x}_{-i}, \alpha, \mathbf{G})$ derived in [5].

Case 2. Next, suppose $x_j > 0$ is the only positive component of $\mathbf{x}_{-i}$. From [14],

$$x_i > 0 \Rightarrow \mathbf{P}(x_i, \mathbf{x}_{-i}) = x_i x_j (1 - \alpha g_{ij}) - \frac{1}{3}(x_i + x_j)^3$$

whereas

$$x_i = 0 \Rightarrow \mathbf{P}(x_i, \mathbf{x}_{-i}) = -\frac{1}{3} x_j \big[\max_{h \neq j} (1 - \alpha g_{hj}) \big]^2.$$

Taking the limit as x_i approaches 0 from above, we have $\lim_{x_i \to 0} \mathbf{P}(x_i, \mathbf{x}_{-i}) = -\frac{1}{3} x_j^3$, which is strictly greater than $\mathbf{P}(0, \mathbf{x}_{-i})$ if and only if

$$x_j < \max \big(1 - \alpha g_{ij}\big).$$

Multiplying through by x_j, player i's best response is interior at some $x_i > 0$ if and only if

$$x_j^2 < \max \big(1 - \alpha g_{ij}\big) x_j,$$

and is $x_i = 0$ otherwise, which again coincides with the best response function in [5].

Case 3. Finally, suppose $\mathbf{x}_{-i} = \mathbf{0}$. If $x_i > 0$, then

$$\mathbf{P}(x_i, \mathbf{0}) = -\frac{1}{3} x_i \big[\max_{h \neq i} (1 - \alpha g_{ih}) \big]^2,$$

which approaches zero (from below) as x_i approaches zero from above. In contrast, $x_i = 0$ implies $\mathbf{P}(\mathbf{0}) = -\frac{1}{3} \frac{(n-1)}{n} < 0$. As such, a maximizer does not exist for $\mathbf{P}$, just as the best response function for π_i is empty when $\mathbf{x}_{-i} = \mathbf{0}$.

By means of the three cases, we have verified that for an arbitrary player i, the set of maximizers for $\mathbf{P}$ given any $\mathbf{x}_{-i}$ coincide with the best responses according to the payoff functions π_i. Thus, $\mathbf{P}$ is a BR-potential for Γ. $\square$

B Additional figures and tables

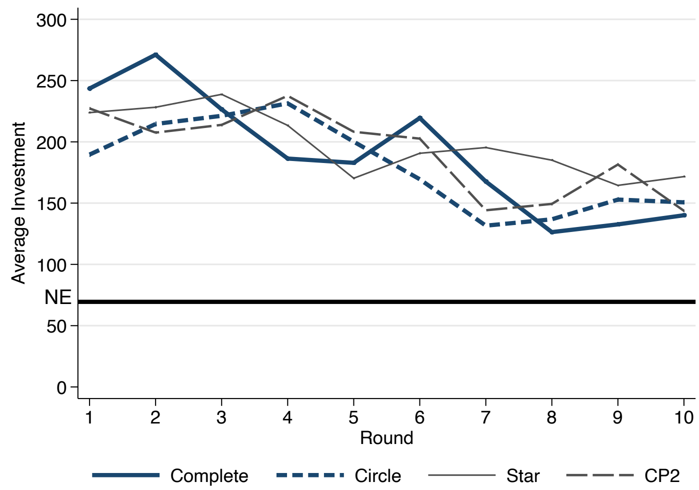


Figure B.1. Mean investment levels in the Baseline condition ($\alpha = 0$) from Block 1, by network and across rounds. The solid reference line indicates the NE point prediction (69.44).

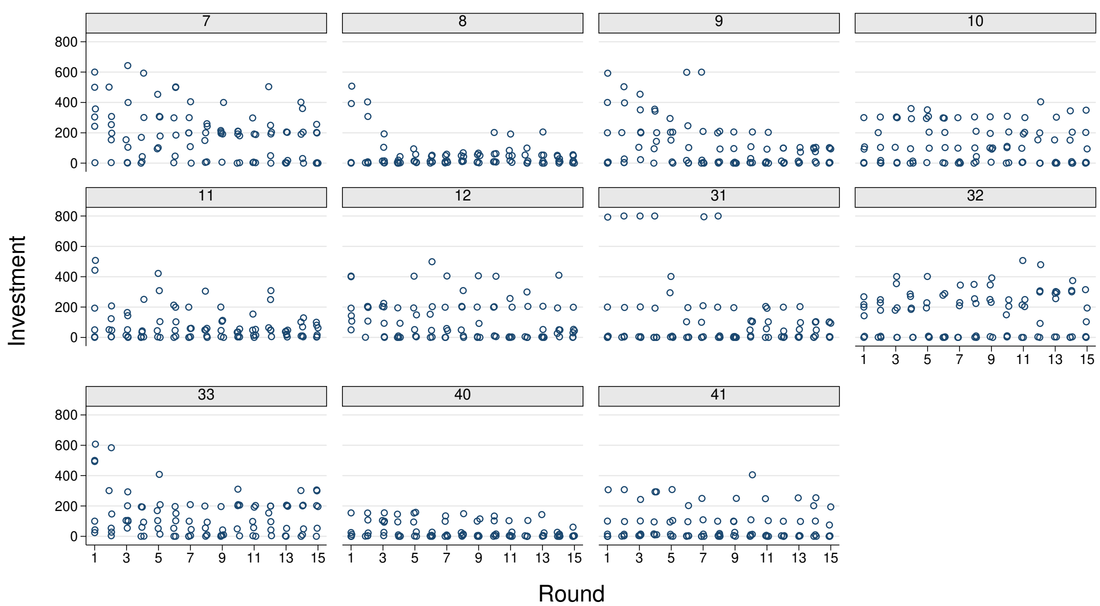


Figure B.2. Scatterplot of investments in all rounds by group in the CIRCLE-P condition.

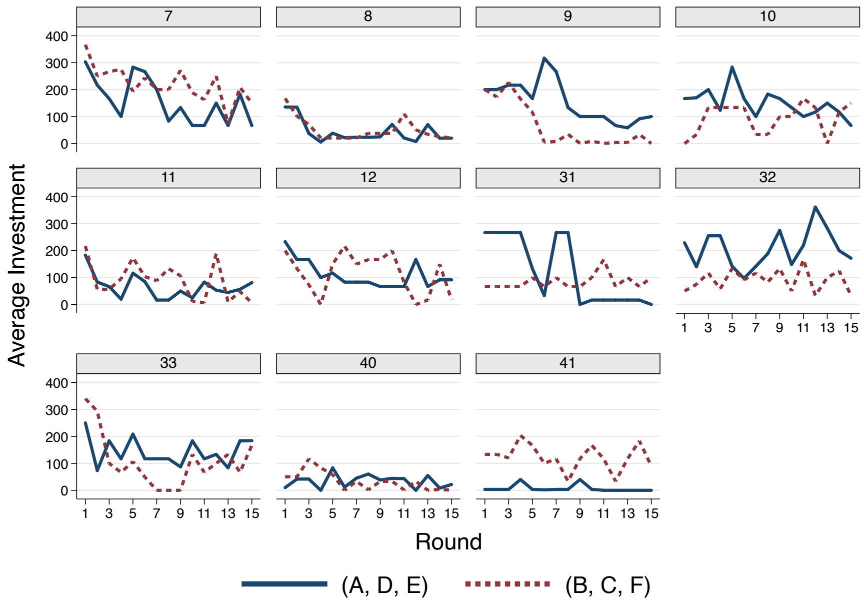


Figure B.3. Average investment by maximal independent sets $M_A = \{A, D, E\}$ and $M_B = \{B, C, F\}$ over all 15 rounds in the CIRCLE-P condition. Each cell represents one independent group.

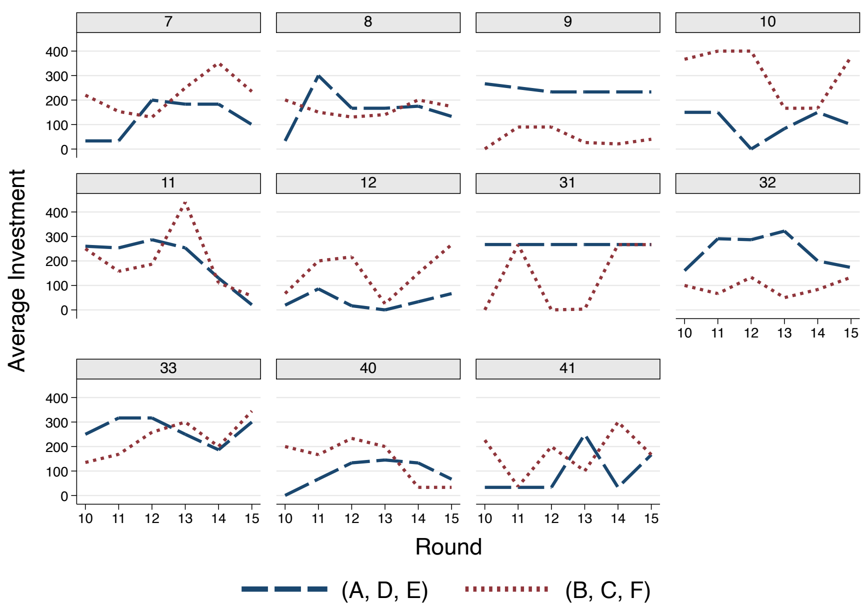


Figure B.4. Average investment by maximal independent sets $M_A = \{A, D, E\}$ and $M_B = \{B, C, F\}$ over the last rounds in the CIRCLE-N condition. Each cell represents one independent group.

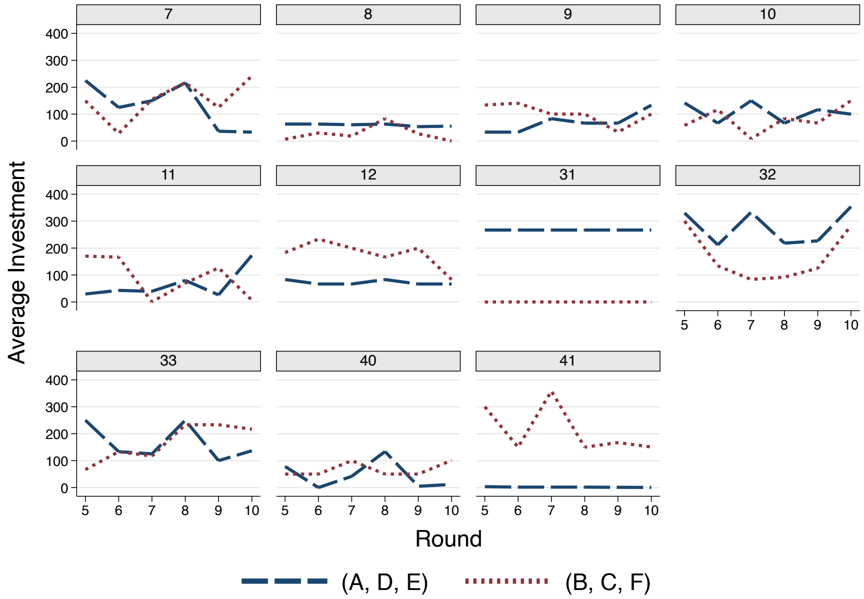


Figure B.5. Average investment by maximal independent sets $M_A = \{A, D, E\}$ and $M_B = \{B, C, F\}$ over the last rounds in the Circle-B condition. Each cell represents one independent group.

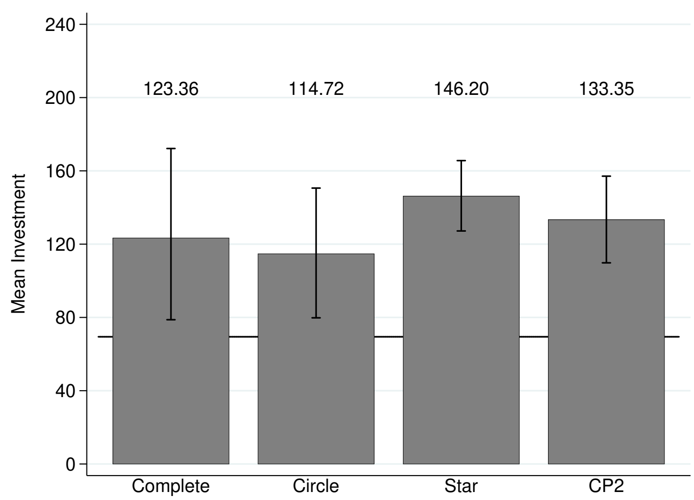
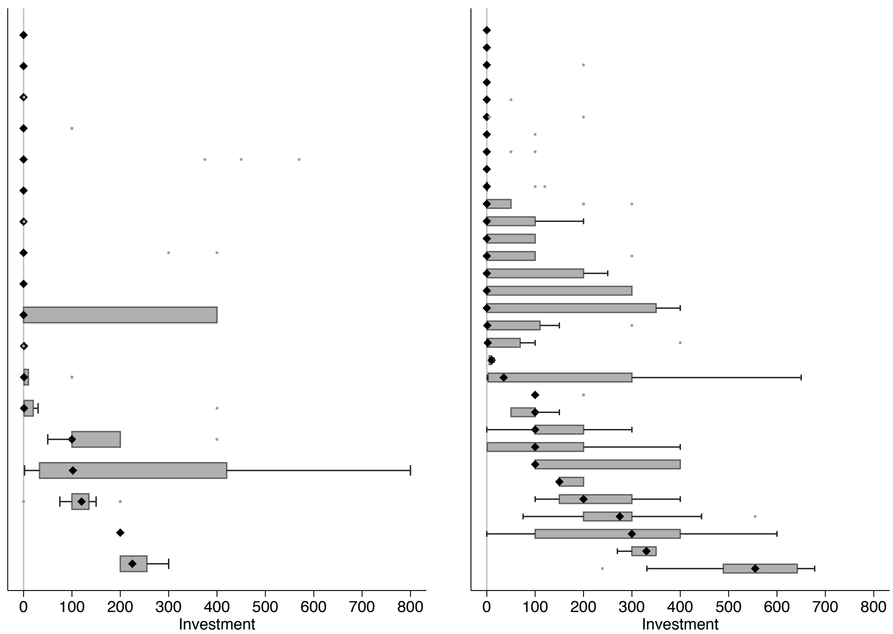


Figure B.6. Mean investment levels in the Baseline condition ($\alpha = 0$) from Blocks 2 and 4, by network. The solid reference line indicates the NE point prediction (69.44). Error bars indicate 95% wild cluster bootstrap confidence intervals.



(a) Star **(b)** CP2

Figure B.7. Boxplots of investment choices by core players in the positive externality rounds. Subjects are sorted by median investment (indicated by a black diamond). The vertical line at zero indicates the baseline Nash equilibrium prediction.

C Sample Experimental Instructions (Circle network)

Thank you for participating in today's experiment. I will read through the script so that everyone receives the same information. Please remain quiet and do not communicate with other participants during the experiment. Raise your hand if you have any questions and an experimenter will come to you to answer the question privately.

For your participation in today's experiment, you will receive the show-up fee of $7. In addition, during the experiment, you will have the opportunity to earn more money. Your additional earnings will depend on the decisions you make and on the decisions made by other participants. At the end of the experiment, you will be paid anonymously by check. No other participant will be informed about your payment.

The experiment consists of multiple parts. The instructions for subsequent parts will be given only after each previous part is completed. Below you will find the instructions for Part 1.

Part 1 Instructions

In this part, you will be asked to make three decisions. **One** of these three decisions will be randomly chosen at the end of the experiment and that decision will be used to calculate your actual earnings for Part 1.

The basic setups for the three decisions are similar. In each case, you will see a list of 20 choices between lotteries and sure amounts of money. Lotteries will always be on the left, and sure amounts of money on the right. The lists will be ordered such that you will prefer the lottery to the sure amount of money in the choice at the top of the list. As you go down the list, you will tend to like the lotteries less and less as compared to the sure amounts. At some point, you will be willing to switch from preferring a lottery to preferring the corresponding sure amount of money. At the point where you are willing to switch, please click on the SWITCH HERE button.

When you click on a SWITCH HERE button, lotteries will be your choice everywhere above that line, and sure amounts of money will be your choice everywhere below that line. All of the 20 choices that you generate will be highlighted. If you want to change your decision, simply click on another SWITCH HERE button. When you are ready to finalize your decision, click SUBMIT.

After you have made your decision, one of the 20 choices will be selected randomly. If your decision for that choice is a sure amount of money, you will earn that amount of money. If your decision for that choice is the lottery, then the outcome of the lottery will be determined according to the listed probabilities and your earnings will be equal to that outcome.

You will not be informed about your earnings from this part of the experiment until the very end of the session today, after you have completed all parts of the experiment.

Are there any questions before you begin making your decisions?

Part 2 Instructions

All amounts in this part of the experiment are expressed in **tokens**. The exchange rate is 400 tokens = $1.

This part of the experiment consists of a sequence of 10 decision rounds. At the beginning of round 1, you will be randomly assigned to a group consisting of 6 participants, including you. You will remain in this group for the duration of this part. That is, you will interact with the same 5 other participants in all 10 rounds.

Your group

Before round 1, you and the 5 other participants in your group will be randomly assigned to positions in the network graph shown in Figure C.1 below. One person will be assigned to each position. Each position is labeled with a letter ID, from A to F. Positions, and therefore also the letter IDs, will remain fixed for the duration of this part. In the network graph, a straight line between two positions indicates that players at those positions are "connected".

During the decision rounds, the network graph will be shown on the screen. Your own position will be highlighted in red. The players you are connected to will be highlighted in yellow, while those you are not connected to (if there are any) will be shown in black.

For example, Figure C.1 shows the network graph from player A's perspective. Thus, player A's position will be displayed in red, while the positions for player B and player C will be displayed in yellow. All of the other players' positions will be displayed in black, since only player B and player C are connected by an edge to player A in this network graph.

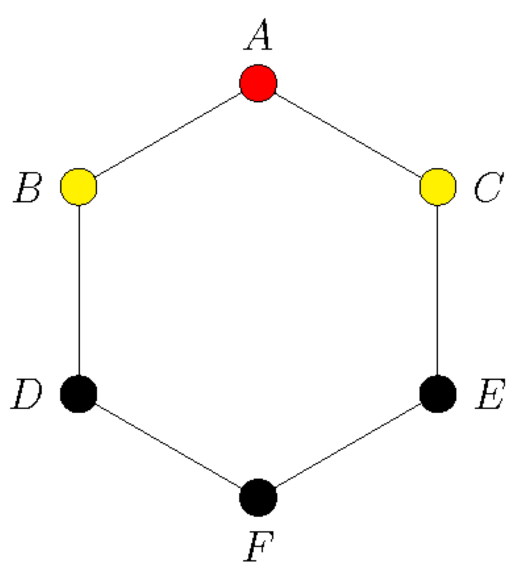


Figure C.1. The network graph - as viewed by player A

Your decision

In each round, you will be given an endowment of 800 tokens. You may use these tokens to make decisions in the round. Specifically, during the round, you can invest any integer number of tokens, from 0 to 800, into a project. Other participants in your group will face the same decision, with the same endowment of 800 tokens. After everyone has chosen a project investment, one participant in the group will be declared

the winner, based on the following procedure. The probability that you are the winner is given by:

$$\frac{\text{Number of tokens you invested in your project}}{\text{Sum of the tokens invested in projects by all participants in your group}}$$

The computer program will determine the winner according to the probabilities calculated in this way.

Consider the following two examples.

Example 1: Suppose you invested 100 tokens in your project, while the other five participants in your group invested 150 tokens, 80 tokens, 100 tokens, 120 tokens, and 250 tokens, respectively. Then, the sum of the tokens invested in projects by all participants in your group will be $(100+150+80+100+120+250) = 800$ tokens. The probability you are the winner is then

$$\frac{100}{800} = \frac{1}{8} = 0.125 = 12.50\%$$

Example 2: For this example, suppose you invested 300 tokens in your project, while the other five participants in your group invested 20 tokens, 30 tokens, 0 tokens, 200 tokens, and 50 tokens, respectively. Then, the sum of the tokens invested in projects by all participants in your group will be $(300 + 20 + 30 + 0 + 200 + 50) = 600$ tokens. The probability you are the winner is then

$$\frac{300}{600} = \frac{1}{2} = 0.5 = 50.00\%$$

Your earnings

In each decision round, the winner will receive a prize of **500 tokens**. All participants (including the winner) must pay their project investments.

In addition, the earnings for each participant who is **connected to the winner** will be changed by X tokens. In general, X can be positive, negative, or zero.

Thus, your earnings in a given round are determined as follows:

If you are the winner:

+800 (endowment)
+500 (prize)
− (tokens you invested)

1300 − **(tokens you invested)**

If you are not the winner:

but are connected to the winner:	and are not connected to the winner:
+800 (endowment)	+800 (endowment)
+0 (no prize)	+0 (no prize)
$+X$ (change in earnings)	+0 (no change in earnings)
− (tokens you invested)	− (tokens you invested)
$800 + X -$ **(tokens you invested)**	800 − **(tokens you invested)**

Example 3: Suppose you are the winner and your project investment was 100 tokens. Then your earnings for the round will be $1300 - 100 = 1200$ tokens.

Alternatively, suppose you are not the winner, and you **ARE NOT** connected to the winner. If your project investment was 100 tokens, then your earnings for the round will be $800 - 100 = 700$ tokens.

Finally, suppose you are not the winner, but that you **ARE** connected to the winner. Moreover, suppose $X = +200$. That is, the earnings of each player connected to the winner are **increased** by 200 tokens. If your project investment was 100 tokens, then your earnings for the round will be $800 + 200 - 100 = 900$ tokens.

If, instead, $X = -200$, the earnings of each player connected to the winner are **decreased** by 200 tokens. Thus, if your project investment was 100 tokens, your earnings for the round will be $800 - 200 - 100 = 500$ tokens.

Control Questions

In a moment, you will be asked to complete some control questions shown on the screen. These questions are only to help you understand the instructions - they will not affect your earnings. After several minutes, we will walk through the answers together, then move on to the next set of questions. After these are completed, we will continue with the instructions.

Feedback

After all participants have made their decisions, you will be shown the individual project investments for each participant in your group, the sum of all tokens invested in projects by participants in your group, and your probability of winning. Then, after the program determines the winner, the screen will display the position of the winner, whether or not you are connected to the winner, and a calculation of your earnings for the round.

Summary

Part 2 will consist of 10 decision rounds. In each round, you and the other participants in your group will choose project investments. The probability that your project wins depends on the share of your own project investment out of the total number of tokens invested by all participants in your group. Only one participant can be the winner in a given round. All participants must pay their project investments out of the endowment (800 tokens). The winner will receive a prize of 500 tokens. For any participant who does not win, but is connected to the winner, earnings will be changed by X tokens.

As a reminder, in the network graph shown on the screen, your position will be shown in red. The positions of the players with whom you are connected will be shown in yellow (in addition to being linked with your position by an edge). The positions of players who you are not connected to (if there are any) will be shown in black.

In Part 2, $X = 0$ for all 10 decision rounds. That is, the earnings for a participant who does not win, but is connected to the winner will not be adjusted.

To make this clear, your earnings in any decision round will be given by:

1300 – (tokens you invested)	if you are the winner,
800 – (tokens you invested)	if you are not the winner, but are connected to the winner
800 – (tokens you invested)	if you are not the winner, and are not connected to the winner

At the end of the experiment, you will be paid for **one** randomly chosen decision round from Part 2. Each of the 10 decision rounds in this part is equally likely to be selected.

Part 3 Instructions

The instructions for Part 3 are almost identical to the instructions for Part 2. However, Part 3 will consist of a sequence of 15 decision rounds. Your group, the network graph, and your position will be the same as in Part 2.

In Part 3, $X = -400$ **for all 15 decision rounds. That is, the earnings for a participant who does not win, but is connected to the winner will be decreased by 400 tokens.**

To make this clear, your earnings in any decision round will be given by:

1300 − (tokens you invested)	if you are the winner,
400 − (tokens you invested)	if you are not the winner, but are connected to the winner
800 − (tokens you invested)	if you are not the winner, and are not connected to the winner

At the end of the experiment, you will be paid for **one** randomly chosen decision round from Part 3. Each of the 15 decision rounds in this part is equally likely to be selected.

Part 4 Instructions

The instructions for Part 4 are almost identical to the instructions for Part 3. Part 4 will also consist of a sequence of 15 decision rounds. Your group, the network graph, and your position will be the same as in Parts 2 and 3.

In Part 4, $X = +400$ **for all 15 decision rounds. That is, the earnings for a participant who does not win, but is connected to the winner will be increased by 400 tokens.**

To make this clear, your earnings in any decision round will be given by:

1300 – (tokens you invested)	if you are the winner,
1200 – (tokens you invested)	if you are not the winner, but are connected to the winner
800 – (tokens you invested)	if you are not the winner, and are not connected to the winner

At the end of the experiment, you will be paid for **one** randomly chosen decision round from Part 4. Each of the 15 decision rounds in this part is equally likely to be selected.

Part 5 Instructions

The instructions for Part 5 are **exactly** identical to the instructions for Part 2. Thus, it will consist of a sequence of 10 decision rounds. Your group, the network graph, and your position will be the same as in Parts 2, 3, and 4.

In Part 5, as in Part 2, $X = 0$ for all 10 decision rounds. That is, the earnings for a participant who does not win, but is connected to the winner will not be adjusted.

To make this clear, your earnings in any decision round will be given by:

$1300 -$ (tokens you invested)	if you are the winner,
$800 -$ (tokens you invested)	if you are not the winner, but are connected to the winner
$800 -$ (tokens you invested)	if you are not the winner, and are not connected to the winner

At the end of the experiment, you will be paid for **one** randomly chosen decision round from Part 5. Each of the 10 decision rounds in this part is equally likely to be selected.

Part 6 Instructions

This part of the experiment consists of a single decision round. The basic setup is similar to the setup for Parts 2, 3, 4, and 5.

Before the round begins, you will be randomly rematched into a new group of 6 participants. In addition, there is no network graph connecting the participants for this part. However, you will still be randomly assigned a letter ID from A to F.

You and the other participants in your group will be given an endowment of 800 tokens each and asked to choose project investments. As in previous parts, the probability that your project wins depends on the share of your own project investment out of the total number of tokens invested by all participants in your group. All participants must pay their project investments out of the endowment.

There are two main differences from previous parts. The first is that **in this part, the winner will receive a prize of <u>0 tokens</u>.** The second is that, since there is no network graph connecting participants, **there is no adjustment** X to be made to the earnings of participants who are connected to the winner.

To make this clear, your earnings for this part (1 decision round only) will be given by:

800 − (tokens you invested)	if you are the winner,
800 − (tokens you invested)	if you are not the winner

After all participants have made their decisions, you will be shown the individual project investments for each participant in your group, the sum of all tokens invested in projects by participants in your group, and your probability of winning. Then, after the program determines the winner, the screen will display the letter ID of the winner, whether or not that is you, and a calculation of your earnings.