PUBLIC OPINION POLLS, VOTER TURNOUT, AND WELFARE: AN EXPERIMENTAL STUDY

Online Appendix

Contents

Appendix A: The Participation Game with Preference Uncertainty **1**

A1. The game 1

A2. Bayesian-Nash equilibria in pure strategies; only floating voters 2

A3. Bayesian-Nash equilibria in mixed strategies; only floating voters 12

A4. Bayesian-Nash equilibria in pure strategies; allied and floating voters 14

A5. Bayesian-Nash equilibria in mixed strategies; allied and floating voters 19

A6. Quantal Response Equilibria 21

Appendix B: Experimental Instructions **23**

Appendix C: Procedures **28**

Appendix D: Equilibrium Predictions **29**

Appendix E: Electoral Efficiency **30**

Appendix A - The Participation Game with Preference Uncertainty

This appendix provides a description and Bayesian-Nash equilibria of the participation games with and without allied voters when polls are prohibited (meaning the electorate's preferences are uncertain). We derive pure strategy equilibria (propositions A1 and A2) and present numerical estimations of totally quasi-symmetric mixed strategy equilibria for varying choices of parameters. We also indicate how one can derive the logit equilibria for this game. We begin with an outline of the group structure that we use to introduce allied and floating voters, followed by a description of the PU-participation game. Our presentation continues with Bayesian-Nash equilibria for the PU-participation game without allied voters and then proceeds to the case with allied voters.

Group structure

Consider a democracy in which elections are decided by simple majority rule and ties are broken by a coin-toss. The electorate consists of two groups, each supporting one of two exogenous candidates. We will distinguish between two scenarios. In the first, there are no alliances between voters and candidates. Each voter can belong to one supporter group in one election and to the other group in the next. The second scenario is where there is a (same) number of fixed voters in each group. This number is commonly known. One interpretation is that these are 'allied' voters, whereas the others are 'floating' voters. Because of this asymmetry within a group, our model will allow allied and floating voters to follow distinct decision rules.

A1. The game

The players in the participation game (PR83) are the E (risk neutral) voters in an electorate, each seeking to maximize the own payoff. Each voter belongs to one of two supporter groups $i = A, B$. Let integer $N_i \geq 1$, $i = A, B$, be the number of voters in i, with $N_A + N_B = E$. The notation j_i, $i = A, B$, will be used to represent voter $j \in i$. Each voter j_i faces a strategy set consisting of two pure strategies $v_{j_i} \in \{0,1\}$, where $v_{j_i} = 1$ denotes participation in favor of candidate i and $v_{j_i} = 0$ denotes abstention. A mixed strategy profile for j_i is given by the probabilities of participation q_{j_i} and abstention $1 - q_{j_i}$, where $0 \leq q_{j_i} \leq 1$. All voters in the electorate make their decisions simultaneously. Aggregate participation in $i = A, B$ is

$$V_i \equiv \sum_{j_i} v_{j_i} \tag{A1}$$

and, for later use, aggregate participation by other voters in i than j_i is denoted by

$$V_i^{-j_i} \equiv V_i - v_{j_i} . \tag{A2}$$

Payoffs are determined by the outcome of the election and the cost of voting. Normalizing the individual benefit from having one's preferred candidate win (lose) the election to 1 (0), the election outcome determines an (expected) benefit for voter j_i, $i = A, B$, given by

$$w_{j_i}(V_i, V_{-i}) = \begin{cases} 0 & if \quad V_i < V_{-i} \\ ½ & if \quad V_i = V_{-i} \\ 1 & if \quad V_i > V_{-i}, \end{cases} \qquad (A3)$$

where V_{-i} denotes aggregate participation in the group j_i does not belong to. We also define total aggregate participation by $V \equiv V_i + V_{-i}$. Note that $w_{j_i}(V_i, V_{-i})$ is non-decreasing (non-increasing) in its first (second) argument and symmetric to the groups, $w_{j_{-i}}(.,.) = 1 - w_{j_i}(.,.)$. Furthermore, assume identical participation costs to all voters within the range

$$c \in (0,1), \qquad (A4)$$

$\forall j_i$, $i = A, B$. The payoff for any voter j_i, $i = A, B$, is then given by

$$\pi_{j_i} = w_{j_i}(V_i, V_{-i}) - v_{j_i} c. \qquad (A5)$$

We now introduce preference uncertainty (PU) to the participation game, thus creating a Bayesian game. PU is formally characterized by common knowledge about:

(*i*) an (equal) minimal group size, $\underline{N}_i \geq 1$, for each group $i = A, B$, implying a maximal group size $E - \underline{N}_i$, and;

(*ii*) a discrete probability distribution over all possible electoral compositions (N_i, N_{-i}), $i \neq -i$, from the set $\{(\underline{N}_i, E - \underline{N}_i), (\underline{N}_i + 1, E - \underline{N}_i - 1), ..., (E - \underline{N}_i, \underline{N}_i)\}$, with $prob(\,.\,,\,.\,) > 0$ for each element in the set.

Throughout, we will only consider the symmetric case, where $\underline{N}_i = \underline{N}_{-i} \geq 1$.

A2. Bayesian-Nash equilibria in pure strategies; only floating voters

The derivation of equilibria for the PU-participation game is a straightforward modification of that in Palfrey and Rosenthal (1983, 1985). Throughout, we assume risk neutrality.

In the equilibrium analysis, the central condition is that a player will vote with certainty if the expected payoff is higher than the expected payoff of abstaining. Formally, voter j_i, $i = A, B$, will vote with probability 1 iff

$$Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i}=1\right]\right] \;>\; Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i}=0\right]\right], \qquad (A6)$$

and will abstain with probability 1 if the reverse is true. Expectation operators refer to strategic uncertainty ($strat$) and PU ($size$).[1] Elaboration gives

$$\sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x)\left[prob(V_i^{-j_i}=V_{-i}|x)+prob(V_i^{-j_i}+1=V_{-i}|x)\right] \;>\; 2c\,, \qquad (A7)$$

where $prob(x)$ is the probability that electoral composition $(x, E-x)$ occurs, and $prob(V_i^{-j_i}=V_{-i}|x)+prob(V_i^{-j_i}+1=V_{-i}|x)$ gives voter j_i's probability of being pivotal, given *i*'s own group size x. The first of these terms gives the probability that j_i can turn a tie into a victory, and the second the probability that she can turn a defeat into a tie. For fixed *x*, $(A7)$ simply reduces to the equilibrium conditions for the 'standard' participation game (cf. Palfrey and Rosenthal 1983). Note that the expected payoff from voting is always negative for $c>1/2$, implying that a risk neutral voter will abstain in this case. Hence, for high costs, the only Bayesian-Nash equilibrium is for every voter to abstain.

Condition $(A7)$ can be used to determine pure strategy Bayesian-Nash equilibria for the PU-participation game with $c<1/2$. The following proposition will give a comprehensive overview of these equilibria for various values of *c*.

PROPOSITION A1 (pure strategy Bayesian-Nash equilibria in the PU-participation game without allied voters):[2]

(*i*) If $c>1/2$, the only Bayesian-Nash equilibrium in pure strategies is $v_{j_i}=0$, $\forall j_i$, $i=A,B$ (nobody participates).

(*ii*) If $c \le prob(N_i = N_{-i})/2$ $\{c \le prob(N_i = N_{-i}+1)/2\}$ for E even {odd}, there is a Bayesian-Nash equilibrium in pure strategies with $v_{j_i}=1$, $\forall j_i$, $i=A,B$ (everybody participates).

(*iii*) If $c = prob(x=V/2)/2$ $\{c = prob(x=\lceil V/2 \rceil)/2 \wedge c \le prob(x=\lfloor V/2 \rfloor)/2\}$ for E even {odd}, there are Bayesian-Nash equilibria in pure strategies with $v_{j_i}=1$, $\forall j_i$, and $v_{j_{-i}}=1$ for some or none of the voters in $-i$, $i \ne -i$ (everybody in *i* participates and possibly some in $-i$).

(*iv*) If $c<1/2$, then for E even {odd} and any symmetric group size distribution, any turnout level V, $0<V<E$ $\{0<V\le E/2\}$, is an outcome of a Bayesian-Nash equilibrium in pure strategies if $c \in [c(V)_{\min}, c(V)_{\max}]$ with $c(V)_{\min}$ ($c(V)_{\max}$) is $1/2$ times the probability that an abstainer (a

[1] Palfrey and Rosenthal (1985) refer to the uncertainty about group sizes (preferences) as 'strategic', because it enters the participation decision of voters. We agree and only use a different terminology for notational clarity.

[2] The trivial but laborious cases with $c=1/2$ are not discussed in this appendix.

participant) can change the election outcome by participating (abstaining) {for turnout levels $E/2 < V < E$ such equilibria exist for some specification of symmetric group size distributions}.

(v) No other Bayesian-Nash equilibria in pure strategies exist.

***Proof*:**

The proof is a straightforward probabilistic extension of Palfrey and Rosenthal (1983). Note that in our case we assume $N_i \geq 1$, $i = A, B$.

(i): It is easy to see [cf. condition (A7)] that if $c > 1/2$, participation is too costly for any voter and full abstention is the only equilibrium.

(ii) to (iv): If $c < 1/2$, in order for turnout $V = 0,1,\ldots,E$, $V = V_i + V_{-i}$, to be a pure strategy Bayesian-Nash equilibrium outcome, no (non-)participant may receive a strictly higher expected payoff by deviating to abstention (participation).

For V *even*, every decision is pivotal when $V_i = V_{-i}$. In all other cases, nobody is pivotal because $|V_i - V_{-i}| \geq 2$. This implies that for $V_i = V_{-i}$ changing one's decision affects revenues and for $V_i \neq V_{-i}$ it does not. Using this, we can derive necessary and sufficient conditions for pure strategy Bayesian-Nash equilibria with even turnout V to exist.

First, the expected increase in revenue if a non-participant decides to vote must be equal to or smaller than the costs:

$$\Phi(V, v_{j_i} = 0)\frac{1}{2} \leq c, \qquad (\text{A8})$$

where $\Phi(V, v_{j_i} = 0)$ is the probability that the vote will affect the outcome, which (because V is even) only occurs if there is a tie:

$$\Phi(V, v_{j_i} = 0) = prob(V_i = V_{-i} = V/2 \,|\, V, v_{j_i} = 0)$$

$$= \sum_{x=\max[\underline{N}_i, V/2+1]}^{\min[\overline{N}_i, E-V/2]} prob(x)\, prob(V_i = V_{-i} = V/2 \,|\, V, v_{j_i} = 0, x).$$

Second, the expected decrease in revenue if a participant decides to abstain must be equal to or larger than the costs saved:

$$\Phi(V, v_{j_i} = 1)\frac{1}{2} \geq c, \qquad (\text{A9})$$

where $\Phi(V, v_{j_i} = 1)$ denotes the probability that the switch will affect the outcome, which (because V is even) only occurs if there is a tie:

$$\Phi(V, v_{j_i}=1) = prob(V_i = V_{-i} = V/2 \,|\, V, v_{j_i}=1)$$

$$= \sum_{x=\max[\underline{N}_i, V/2]}^{\min[\overline{N}_i, E-V/2]} prob(x)\, prob(V_i = V_{-i} = V/2 \,|\, V, v_{j_i}=1, x) \,.$$

For V *odd*, we first establish the cases where an abstainer is pivotal. This occurs when $V_i = V_{-i} - 1$, implying that j_i can force a tie. If, however, j_i is one of the V participants, (s)he is pivotal if $V_i = V_{-i} + 1$, because a switch to abstention would reduce a victory to a tie. In all other cases $|V_i - V_{-i}| \geq 3$ and changing one's decision has no effect on revenues. Using this, we can derive necessary and sufficient conditions for pure strategy Bayesian-Nash equilibria with (odd) turnout V to exist.

First, for a non-participant, (A8) must hold. Now, $\Phi(V, v_{j_i}=0)$ is given by the probability that j_i is in a group with a one-vote defeat to the other group:

$$\Phi(V, v_{j_i}=0) = prob(V_i = V_{-i} - 1 = \lfloor V/2 \rfloor \,|\, V, v_{j_i}=0)$$

$$= \sum_{x=\max[\underline{N}_i, \lceil V/2 \rceil]}^{\min[\overline{N}_i, E-\lceil V/2 \rceil]} prob(x)\, prob(V_i = V_{-i} - 1 = \lfloor V/2 \rfloor \,|\, V, v_{j_i}=0, x) \,.$$

Second, for a participant, (A9) must hold. Here, $\Phi(V, v_{j_i}=1)$ is the probability that j_i is in a group with a one-vote victory over the other group:

$$\Phi(V, v_{j_i}=1) = prob(V_i = V_{-i} + 1 = \lceil V/2 \rceil \,|\, V, v_{j_i}=1)$$

$$= \sum_{x=\max[\underline{N}_i, \lceil V/2 \rceil]}^{\min[\overline{N}_i, E-\lfloor V/2 \rfloor]} prob(x)\, prob(V_i = V_{-i} + 1 = \lceil V/2 \rceil \,|\, V, v_{j_i}=1, x) \,.$$

Next, we investigate whether and which pure strategy Bayesian-Nash equilibria exist that fulfill (A8) and (A9). We consider all possible cases $0 \leq V \leq E$.

Full abstention ($V = 0$):

Full abstention cannot be an equilibrium. We only need to consider (A8), because $v_{j_i} = 0$, $\forall j_i$, $i = A, B$. This reduces to

$$prob(V_i = V_{-i} = 0 \,|\, V = 0, v_{j_i} = 0) = \sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x) \cdot 1 = 1 \leq 2c \,, \qquad (\text{A10})$$

since $prob(V_i = V_{-i} = 0 \,|\, V = 0, v_{j_i} = 0, x) = 1$, $\forall x$, which contradicts our assumption that $c < 1/2$.

Full abstention in one group and positive participation in the other group ($V_i = 0; V_{-i} > 0$ *)*:

Full abstention $V_i = 0$ in i and positive participation $V_{-i} > 0$ in $-i$ cannot be an equilibrium. This is easy to see. Given the pure strategy of abstention followed by everyone in *i*, for any $V_{-i} > 1$, every participant in $-i$ has an incentive to abstain until $V_{-i} = 1$, which suffices to win the election. But if $V_{-i} = 1$, it is advantageous for every abstainer in i to participate, because the value from turnout is $1/2$, which exceeds $c < 1/2$.

Full participation ($V = E$ *)*:

For some $c < 1/2$, equilibria with full participation exist. We only need to consider $(\mathrm{A9})$, because $v_{j_i} = 1$, $\forall j_i$, $i = A, B$. For E even {odd} this reduces to

$$prob(V_i = V_{-i} = E/2 | V = E, v_{j_i} = 1) = prob(x = E/2) \cdot 1 \geq 2c \qquad (\mathrm{A11})$$

$$\{\ prob(V_i = V_{-i} + 1 = \lceil E/2 \rceil | V = E, v_{j_i} = 1) = prob(\ x = \lceil E/2 \rceil) \cdot 1 \geq 2c\ \}.$$

Hence, $(\mathrm{A9})$ is satisfied for $c \leq prob(x = E/2)/2 = prob(N_i = N_{-i})/2$ $\{ c \leq prob(x = \lceil E/2 \rceil)/2 = prob(N_i = N_{-i} + 1)/2 \}$, which proves (*ii*) of the proposition.

Full participation in one group and possibly some in the other group ($V_i = x,\ V_{-i} < E - x$ *)*:

$V = V_i = x = 1$ cannot be an equilibrium because voters in $-i$ have an incentive to switch to voting. For $1 < V < E$, there exist equilibria for some $c < 1/2$ with full participation $V_i = x$ in i and possibly some participation $V_{-i} < E - x$ in $-i$. For V even {odd}, $(\mathrm{A8})$ applied to $-i$ gives

$$prob(V_i = V_{-i} = x | V = 2x, v_{j_{-i}} = 0) = prob(x = V/2) \cdot 1 \leq 2c$$

$$\{\ prob(V_i = V_{-i} + 1 = x | V = 2x - 1, v_{j_{-i}} = 0) = prob(x = \lceil V/2 \rceil) \cdot 1 \leq 2c\ \}$$

and $(\mathrm{A9})$ applied to i gives

$$prob(V_i = V_{-i} = x | V = 2x, v_{j_i} = 1) = prob(x = V/2) \cdot 1 \geq 2c$$

$$\{\ prob(V_i = V_{-i} + 1 = x | V = 2x - 1, v_{j_i} = 1) = prob(\ x = \lceil V/2 \rceil) \cdot 1 \geq 2c\ \}$$

and to $-i$

$$prob(V_i = V_{-i} = x | V = 2x, v_{j_{-i}} = 1) = prob(x = V/2) \cdot 1 \geq 2c$$

$$\{\ prob(V_i = V_{-i} - 1 = x | V = 2x + 1, v_{j_{-i}} = 1) = prob(\ x = \lfloor V/2 \rfloor) \cdot 1 \geq 2c\ \}. \qquad (\mathrm{A12})$$

Hence, $c = prob(x = V/2)/2$ $\{c = prob(x = \lceil V/2 \rceil)/2 \wedge c \le prob(x = \lfloor V/2 \rfloor)/2\}$ are the only cases where (A8) and (A9) can be jointly fulfilled, which proves (*iii*) of the proposition.

Other equilibria with $0 < V < E$:

Note that by assuming symmetrically distributed group sizes, we can restrict our analysis to voters in i.

For such equilibria to exist, we need to show that there is some c that jointly fulfills (A8) and (A9):

$$\Phi(V, v_{j_i} = 0)\frac{1}{2} \le c \le \Phi(V, v_{j_i} = 1)\frac{1}{2}. \qquad (A13)$$

We give examples for $V = 1,2$ before providing a general proof that c exist that fulfill (A13).

Example $V = 1$:

For a given $x \in [\underline{N}_i, \overline{N}_i]$, the probability that the only vote cast is in the other group, as perceived by an abstainer in i, is $\frac{E-x}{E-1} \le 1$. Then, $\Phi(1, v_{j_i} = 0) = \sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x)\frac{E-x}{E-1} < \sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x) = 1$. Furthermore, for the only participant, the probability that the own group has one more vote than the other is 1. Hence, $\Phi(1, v_{j_i} = 1) = 1$. Therefore we have $\Phi(1, v_{j_i} = 1)\frac{1}{2} = \frac{1}{2} > \Phi(1, v_{j_i} = 0)\frac{1}{2}$ and for any $c \in \left[\Phi(1, v_{j_i} = 0)\frac{1}{2}, \frac{1}{2}\right)$ (A13) holds and E pure strategy Bayesian-Nash equilibria exist with exactly one voter turning out to vote.

Example $V = 2$:

From (A13) it follows that $V = 2$ is an equilibrium outcome for any $2c \in [\Phi(2, v_{j_i} = 0), \Phi(2, v_{j_i} = 1)]$. What needs to be shown is that this set is non-empty. For a given $x \in [\underline{N}_i, \overline{N}_i]$, the probability that the two votes are divided equally across the two groups, given that j_i abstains, is $\Phi(2, v_{j_i} = 0 | x) = prob\{1 \text{ vote in } -i,\ 1 \text{ vote in } i \,|\, v_{j_i} = 0, x\} = 2\frac{(E-x)(x-1)}{(E-1)(E-2)}$. Similarly, $\Phi(2, v_{j_i} = 1 | x) = \frac{E-x}{E-1}$. The set is non-empty, iff $\Delta\Phi \equiv \Phi(2, v_{j_i} = 1) - \Phi(2, v_{j_i} = 0) \ge 0$. Then, assuming for the moment $\underline{N}_i = 1$, we have $\Delta\Phi = \sum_{x=1}^{E-1} prob(x)\frac{E-x}{E-1} - 2\sum_{x=2}^{E-1} prob(x)\frac{(E-x)(x-1)}{(E-1)(E-2)}$. Since the probability distribution of x is symmetric around $x = E/2$, this gives

$$\Delta\Phi = \sum_{x=1}^{\lfloor (E-1)/2 \rfloor} prob(x)\left\{\frac{E-x}{E-1} + \frac{x}{E-1} - 2\frac{(E-x)(x-1)}{(E-1)(E-2)} - 2\frac{x(E-x-1)}{(E-1)(E-2)}\right\}$$

$$+\ prob(x=1)\left\{2\frac{(E-1)(1-1)}{(E-1)(E-2)}\right\}+prob(x=E/2)\left\{\frac{E-E/2}{E-1}-2\frac{(E-E/2)(E/2-1)}{(E-1)(E-2)}\right\},$$

or, after some rearrangements and because the last two terms of the sum are equal to zero,

$$\Delta\Phi=\sum_{x=1}^{\lfloor (E-1)/2\rfloor} prob(x)\frac{(E-2x)^2}{(E-1)(E-2)}.$$

Next, note that the fraction is positive for $x<E/2$. Hence, $\Delta\Phi>0$. It is easy to see that this still holds when we give up our assumption that $\underline{N}_i=1$. $\Delta\Phi>0$ shows that the range $[\Phi(2,v_{j_i}=0),\Phi(2,v_{j_i}=1)]$ is non-empty, so all combinations of strategies yielding $V=2$ constitute pure strategy Bayesian-Nash equilibria for $2c$ in this range.

We now turn to the general case of turnout V being an equilibrium outcome.

1) $0<V<E$ *and* V *even*:

Similar to the argument in our examples, (A13) is used to determine a range $[\Phi(V,v_{j_i}=0),\Phi(V,v_{j_i}=1)]$ in which $2c$ should lie to make V an equilibrium outcome. We then proceed to show this range is non-empty. Once again, consider group sizes x and $E-x$, $x\in\left[\max[\underline{N}_i,V/2],\min[\overline{N}_i,E-V/2]\right]$. For x outside of this range, the probability of a tie at $V/2$ is 0, and drops out of the calculation of $\Phi(\cdot)$. For given x in this range, the probability that the V votes are split equally, given that j_i abstains, is

$$\Phi(V,v_{j_i}=0|x)=prob\{V/2\ \text{votes in}-i,V/2\ \text{votes in}\ i\,|v_{j_i}=0,x\}$$

$$=\binom{V}{V/2}\cdot\left\{\frac{E-x}{E-1}\cdot\frac{E-x-1}{E-2}\cdot\ldots\cdot\frac{E-x-V/2+1}{E-V/2}\right\}\left\{\frac{x-1}{E-V/2-1}\cdot\frac{x-2}{E-V/2-2}\cdot\ldots\cdot\frac{x-V/2}{E-V}\right\}$$

$$=\binom{V}{V/2}\cdot\frac{(E-x)\prod_{g=1}^{V/2-1}(E-x-g)(x-g)}{\prod_{h=1}^{V-1}(E-h)}\cdot\frac{x-V/2}{E-V}. \qquad (A14)$$

Similarly, $\Phi(V,v_{j_i}=1|x)=prob\{V/2\ \text{votes in}-i,V/2\ \text{votes in}\ i\,|v_{j_i}=1,x\}$

$$=\binom{V-1}{V/2-1}\cdot\left\{\frac{E-x}{E-1}\cdot\frac{E-x-1}{E-2}\cdot\ldots\cdot\frac{E-x-V/2+1}{E-V/2}\right\}\left\{\frac{x-1}{E-V/2-1}\cdot\frac{x-2}{E-V/2-2}\cdot\ldots\cdot\frac{x-V/2+1}{E-V+1}\right\}$$

$$=\binom{V-1}{V/2-1}\cdot\frac{(E-x)\prod_{g=1}^{V/2-1}(E-x-g)(x-g)}{\prod_{h=1}^{V-1}(E-h)}. \qquad (A15)$$

Defining $\phi(V,x) \equiv \dfrac{(E-x)\prod_{g=1}^{V/2-1}(E-x-g)(x-g)}{\prod_{h=1}^{V-1}(E-h)} \geq 0$, gives

$$\Phi(V, v_{j_i} = 0 | x) = \binom{V}{V/2}\phi(V,x)\cdot\frac{x-V/2}{E-V} \quad \text{and} \quad \Phi(V, v_{j_i} = 1 | x) = \binom{V-1}{V/2-1}\phi(V,x).$$

V-equilibria exist for some c iff $\Delta\Phi \equiv \Phi(V, v_{j_i} = 1) - \Phi(V, v_{j_i} = 0) \geq 0$. Then, assuming $\underline{N}_i < V/2+1$ for the moment, we have

$$\Delta\Phi = \sum_{x=V/2}^{E-V/2} prob(x)\binom{V-1}{V/2-1}\phi(V,x) - \sum_{x=V/2+1}^{E-V/2} prob(x)\binom{V}{V/2}\phi(V,x)\cdot\frac{x-V/2}{E-V}$$

$$= \binom{V}{V/2}\cdot\left\{\sum_{x=V/2}^{E-V/2} prob(x)\phi(V,x)\left[\frac{1}{2}-\frac{x-V/2}{E-V}\right]\right\}$$

$$+ prob(x=V/2)\binom{V}{V/2}\phi(V,V/2)\cdot\frac{V/2-V/2}{E-V}, \tag{A16}$$

where we use $\binom{V-1}{V/2-1} = \frac{1}{2}\binom{V}{V/2}$. Obviously, the second term disappears since $V/2 - V/2 = 0$. This shows that assuming $\underline{N}_i < V/2+1$ so far is innocent. Now using $\max[\underline{N}_i, V/2+1]$ instead and because the distribution of x is symmetric around $x = E/2$, we have:

$$\Delta\Phi = \binom{V}{V/2}\sum_{x=\max[\underline{N}_i,V/2]}^{\lfloor (E-1)/2 \rfloor} prob(x)\left\{\phi(V,x)\left[\frac{1}{2}-\frac{x-V/2}{E-V}\right] + \phi(V,E-x)\left[\frac{1}{2}-\frac{E-x-V/2}{E-V}\right]\right\}$$

$$+ prob(x=E/2)\binom{V}{V/2}\phi(V,E/2)\left[\frac{1}{2}-\frac{E/2-V/2}{E-V}\right]$$

$$= \binom{V}{V/2}\sum_{x=\max[\underline{N}_i,V/2]}^{\lfloor (E-1)/2 \rfloor} prob(x)\left\{\phi(V,x)\cdot\frac{E-2x}{2(E-V)} + \phi(V,E-x)\cdot\frac{2x-E}{2(E-V)}\right\}$$

$$= \binom{V}{V/2}\sum_{x=\max[\underline{N}_i,V/2]}^{\lfloor (E-1)/2 \rfloor} prob(x)\left\{\frac{[\phi(V,x)-\phi(V,E-x)](E-2x)}{2(E-V)}\right\} > 0, \tag{A17}$$

because $\phi(V,x) > \phi(V,E-x)$ for $x < E/2$.

Hence, for every PU-participation game with symmetrically distributed group sizes, we can find some c such that a pure strategy Bayesian-Nash equilibrium exists in which an even number V of voters from either group participates and all others abstain, with $0 < V < E$.

2) $0 < V < E$ *and* V *odd*:

Similarly, we use (A15) to determine $[\Phi(V, v_{j_i} = 0), \Phi(V, v_{j_i} = 1)]$ as a range in which $2c$ should lie to make V an equilibrium outcome, and proceed to show that this range is non-empty. Consider group sizes

x and $E-x$, $x \in \left[\max[\underline{N}_i, \lceil V/2 \rceil], \min[\overline{N}_i, E - \lfloor V/2 \rfloor]\right]$. Once again, if x is outside of the range, the probability of a tie is 0.

For given x, the probability that the V votes are split such that there is one vote less in i, given that j_i abstains, is

$$\Phi(V, v_{j_i} = 0 | x) = prob\{\lceil V/2 \rceil \text{ votes in } -i, \lfloor V/2 \rfloor \text{ votes in } i \,|\, v_{j_i} = 0, x\}$$

$$= \binom{V}{\lfloor V/2 \rfloor} \cdot \left\{ \frac{E-x}{E-1} \cdot \frac{E-x-1}{E-2} \cdot \ldots \cdot \frac{E-x-\lfloor V/2 \rfloor}{E - \lceil V/2 \rceil} \right\} \left\{ \frac{x-1}{E-\lceil V/2 \rceil - 1} \cdot \frac{x-2}{E-\lceil V/2 \rceil - 2} \cdot \ldots \cdot \frac{x - \lfloor V/2 \rfloor}{E-V} \right\}$$

$$= \binom{V}{\lfloor V/2 \rfloor} \cdot \frac{\prod_{g=0}^{\lfloor V/2 \rfloor - 1} (E-x-g)(x-g-1)}{\prod_{h=1}^{V-1} (E-h)} \cdot \frac{E-x-\lfloor V/2 \rfloor}{E-V}. \qquad (A18)$$

And, given x and that j_i participates, the probability that the V votes are split such that there is one vote more in i is

$$\Phi(V, v_{j_i} = 1 | x) = prob\{\lfloor V/2 \rfloor \text{ votes in } -i, \lceil V/2 \rceil \text{ votes in } i \,|\, v_{j_i} = 1, x\}$$

$$= \binom{V-1}{\lfloor V/2 \rfloor} \cdot \left\{ \frac{E-x}{E-1} \cdot \frac{E-x-1}{E-2} \cdot \ldots \cdot \frac{E-x-\lfloor V/2 \rfloor + 1}{E - \lceil V/2 \rceil + 1} \right\} \left\{ \frac{x-1}{E-\lceil V/2 \rceil} \cdot \frac{x-2}{E-\lceil V/2 \rceil - 1} \cdot \ldots \cdot \frac{x - \lfloor V/2 \rfloor}{E-V+1} \right\}$$

$$= \binom{V-1}{\lfloor V/2 \rfloor} \cdot \frac{\prod_{g=0}^{\lfloor V/2 \rfloor - 1} (E-x-g)(x-g-1)}{\prod_{h=1}^{V-1} (E-h)}. \qquad (A19)$$

Defining $\varphi(V,x) \equiv \dfrac{\prod_{g=0}^{\lfloor V/2 \rfloor - 1} (E-x-g)(x-g-1)}{\prod_{h=1}^{V-1} (E-h)} \geq 0$, gives

$$\Phi(V, v_{j_i} = 0 | x) = \binom{V}{\lfloor V/2 \rfloor} \varphi(V,x) \cdot \frac{E-x-\lfloor V/2 \rfloor}{E-V} \quad \text{and} \quad \Phi(V, v_{j_i} = 1 | x) = \binom{V-1}{\lfloor V/2 \rfloor} \varphi(V,x).$$

V-equilibria exist for some c iff $\Delta\Phi \equiv \Phi(V, v_{j_i} = 1) - \Phi(V, v_{j_i} = 0) \geq 0$. Then, assuming $\underline{N}_i < \lfloor V/2 \rfloor$ for the moment, we have

$$\Delta\Phi = \sum_{x=\lceil V/2 \rceil}^{E-\lfloor V/2 \rfloor} prob(x) \binom{V-1}{\lfloor V/2 \rfloor} \varphi(V,x) - \sum_{x=\lceil V/2 \rceil}^{E-\lceil V/2 \rceil} prob(x) \binom{V}{\lfloor V/2 \rfloor} \varphi(V,x) \cdot \frac{E-x-\lfloor V/2 \rfloor}{E-V}$$

$$= \binom{V}{\lfloor V/2 \rfloor} \left\{ \sum_{x=\lceil V/2 \rceil}^{E-\lceil V/2 \rceil} prob(x) \varphi(V,x) \left(\frac{1}{2} + \frac{1}{2V} - \frac{E - x - \lfloor V/2 \rfloor}{E-V} \right) \right\}$$

$$+ \; prob(x = E - \lfloor V/2 \rfloor) \binom{V-1}{\lfloor V/2 \rfloor} \varphi(V, E - \lfloor V/2 \rfloor) \,,$$

$$= \binom{V}{\lfloor V/2 \rfloor} \left\{ \sum_{x=\lceil V/2 \rceil}^{E-\lceil V/2 \rceil} prob(x) \varphi(V,x) \left(\frac{1}{2} - \frac{E - x - V/2}{E-V} + \frac{1}{2V} - \frac{1}{2(E-V)} \right) \right\}$$

$$+ \; prob(x = E - \lfloor V/2 \rfloor) \binom{V-1}{\lfloor V/2 \rfloor} \varphi(V, E - \lfloor V/2 \rfloor) \,, \quad (\mathrm{A}20)$$

where we use $\binom{V-1}{\lfloor V/2 \rfloor} \Big/ \binom{V}{\lfloor V/2 \rfloor} = \frac{1}{2} + \frac{1}{2V}$. Note that the second term of $(\mathrm{A}20)$ is positive and disappears if we drop the assumption $\underline{N}_i < \lfloor V/2 \rfloor$. Because the distribution of x is symmetric around $x = E/2$, we have:

$$\Delta\Phi \geq \binom{V}{\lfloor V/2 \rfloor} \sum_{x=\max[\underline{N}_i, \lceil V/2 \rceil]}^{\lfloor (E-1)/2 \rfloor} prob(x) \left\{ \varphi(V,x) \left[\frac{1}{2} - \frac{E - x - V/2}{E-V} \right] + \varphi(V, E-x) \left[\frac{1}{2} - \frac{x - V/2}{E-V} \right] \right\}$$

$$+ \; prob(x = E/2) \binom{V}{\lfloor V/2 \rfloor} \varphi(V, E/2) \left[\frac{1}{2} - \frac{E - E/2 - V/2}{E-V} \right]$$

$$+ \binom{V}{\lfloor V/2 \rfloor} \left\{ \sum_{x=\lceil V/2 \rceil}^{E-\lceil V/2 \rceil} prob(x) \varphi(V,x) \left(\frac{1}{2V} - \frac{1}{2(E-V)} \right) \right\}$$

$$= \binom{V}{\lfloor V/2 \rfloor} \sum_{x=\max[\underline{N}_i, \lceil V/2 \rceil]}^{\lfloor (E-1)/2 \rfloor} prob(x) \left\{ \frac{[\varphi(V, E-x) - \varphi(V,x)](E-2x)}{2(E-V)} \right\}$$

$$+ \binom{V}{\lfloor V/2 \rfloor} \left\{ \sum_{x=\lceil V/2 \rceil}^{E-\lceil V/2 \rceil} prob(x) \varphi(V,x) \left(\frac{1}{2V} - \frac{1}{2(E-V)} \right) \right\}. \quad (\mathrm{A}21)$$

Note that the first term in $(\mathrm{A}21)$ is positive, since $\varphi(V, E-x) > \varphi(V,x)$ for $x < E/2$. But the second term is positive or zero only if $V \leq E/2$. If this is the case, then $\Delta\Phi > 0$. However, if $V > E/2$, both terms in $(\mathrm{A}21)$ have to be evaluated, inclusive the possible second term in $(\mathrm{A}20)$. Whether $\Delta\Phi$ is positive or zero depends on the symmetric probability distribution at hand, which we have not specified further.

Hence, for every PU-participation game with any symmetrically distributed group sizes and $0 < V \leq E/2$, there is some range for c such that a pure strategy Bayesian-Nash equilibrium exists, in which an odd number V of voters from either group participates and all others abstain. For $E > V > E/2$, these equilibria may exist, depending on the specification of the symmetric group size distribution.

Together 1) and 2) prove (*iv*) of the proposition.

An interesting property of the V-equilibria just described is that the ranges for adjacent turnouts V are adjacent too and the c-values for which *V*-equilibria exist are non-increasing in V. To see this, look at $c(V)_{\min} \equiv \Phi(V, v_{j_i} = 0)/2$ and $c(V+1)_{\max} \equiv \Phi(V+1, v_{j_i} = 1)/2$. Obviously, $c < c(V=1)_{\max} = 1/2$ gives the upper value. It is readily verified that $c(V|x)_{\min} = c(V+1|x)_{\max}$ holds for V even, for which we have

$$c(V|x)_{\min} = \frac{1}{2}\binom{V}{V/2}\phi(V,x)\cdot\frac{x-V/2}{E-V} = \frac{1}{2}\binom{(V+1)-1}{\lfloor (V+1)/2 \rfloor}\varphi(V+1,x) = c(V+1|x)_{\max},$$

and for V odd, for which we have (A22)

$$c(V|x)_{\min} = \frac{1}{2}\binom{V}{\lfloor V/2 \rfloor}\varphi(V,x)\cdot\frac{E-x-\lfloor V/2 \rfloor}{E-V} = \frac{1}{2}\binom{(V+1)-1}{(V+1)/2-1}\phi(V+1,x) = c(V+1|x)_{\max}.$$

Using this, it is easy to see that the difference $c(V)_{\min} - c(V+1)_{\max}$ is also equal to zero, since for V even we have

$$\sum_{x=V/2+1}^{E-V/2} prob(x)c(V|x)_{\min} - \sum_{x=\lceil (V+1)/2 \rceil}^{E-\lfloor (V+1)/2 \rfloor} prob(x)c(V+1|x)_{\max} = 0$$

and for V odd we have (A23)

$$\sum_{x=\lceil V/2 \rceil}^{E-\lceil V/2 \rceil} prob(x)c(V|x)_{\min} - \sum_{x=(V+1)/2}^{E-(V+1)/2} prob(x)c(V+1|x)_{\max} = 0.$$

Hence, we established that the ranges of possible equilibrium costs c for adjacent V are adjacent as well and that the costs are non-increasing in V.

To (*v*): Conditions (A8) are (A9) are necessary and sufficient for the existence of pure strategy Bayesian-Nash equilibria.

Q.E.D.

A3. Bayesian-Nash equilibria in mixed strategies; only floating voters

Next, consider equilibria in totally quasi-symmetric mixed strategies, where voter *i* (–*i*) participates with probability $q_i \in (0,1)$ ($q_{-i} \in (0,1)$). A necessary and sufficient condition for Bayesian-Nash equilibria in such strategies to exist is that each voter j_i, $i = A, B$, is indifferent between participation and abstention (i.e., condition (A7) holds as an equality). Since in the experiment we use a symmetric group size distribution we focus on symmetric (q,q)-equilibria, where $q \equiv q_i$ and $q \equiv q_{-i}$. Then, elaboration and specification of (A7) implicitly defines best response q:

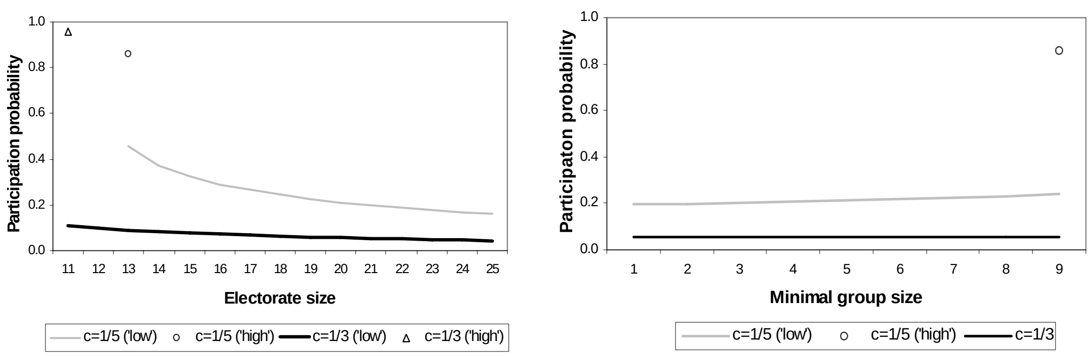
$$\sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x)\left[\sum_{k=0}^{\min[x-1,E-x]}\binom{x-1}{k}\binom{E-x}{k}q^{2k}(1-q)^{E-1-2k} + \sum_{k=0}^{\min[x-1,E-x-1]}\binom{x-1}{k}\binom{E-x}{k+1}q^{2k+1}(1-q)^{E-2-2k}\right] = 2c. \qquad (\text{A24})$$

The first term in the large square brackets gives the (binomial) probability that there is a tie of k votes between the $E-x$ members in the other group and the $x-1$ other members of j_i's own group (j_i can turn a tie into a victory). The second term gives the (binomial) probability that the other group outvotes j_i's co-members by one vote (j_i can turn a defeat into a tie). Below, numerical calculations will show that these equilibria exist for a variety of parameter values.

Totally quasi-symmetric mixed strategy equilibria ((q,q)-equilibria) can be derived numerically using eq. (A24). Figure A1 depicts examples of such equilibria for the PU-participation game without allied voters. The left panel varies the electorate size $E \in \{11,12,...,25\}$, using fixed and equal minimal group sizes of 5 voters in each group ($\underline{N}_A = \underline{N}_B = 5$) and an equal probability of being in either group for each voter (binomial distribution with p = 0.5).

FIGURE A1: (q,q)-EQUILIBRIA IN THE PU-PARTICIPATION GAME WITHOUT ALLIED VOTERS AND WITH BINOMIAL GROUP SIZE DISTRIBUTION

VARYING ELECTORATE SIZES AND $\underline{N}_A = \underline{N}_B = 5$ | VARYING MINIMAL GROUP SIZES AND $E = 20$



Participation probabilities are shown for voting costs $c = 1/5$ and $c = 1/3$. Participation is quite low for $c = 1/3$ and slightly decreasing in E. For $c = 1/5$, we find (q,q)-equilibria only for $E \geq 13$. With lower voting cost, participation is always higher as compared to higher costs and also decreasing in E. Aside from these equilibria with low levels of participation, we find two 'high'-participation (q,q)-equilibria: one for $E = 11$ when $c = 1/3$ and one for $E = 13$ when $c = 1/5$. The right panel of figure A1 shows equilibrium participation probabilities when the electorate size is kept constant ($E = 20$), but the minimal group sizes are varied and equal across groups ($\underline{N}_A = \underline{N}_B = 1,2,...,9$). As in the left panel, we use p = 0.5 and show participation for $c = 1/5$ and $c = 1/3$. We observe higher participation for the lower voting costs. Moreover, participation appears quite constant across different minimal group sizes. Again, we find a 'high-participation' (q,q)-equilibrium, in this case for $\underline{N}_A = \underline{N}_B = 9$, when $c = 1/5$.

A4. Bayesian-Nash equilibria in pure strategies; allied and floating voters

Until now, we have allowed for PU without distinguishing between allied and floating voters. All voters were assumed to choose a candidate on election eve, under the condition that at least $\underline{N}_i$ voters would choose *i*. This describes a world with only floating voters. To allow for allied voters, we assume that the minimal group of $\underline{N}_i$ voters has determined their choice beforehand. Hence, a situation of PU appears, with the minimal group representing the number of allied voters in i. There is a subtle, but important difference in the information set of the two voter types: each floating voter has private information about the candidate she supports, which she can use to (subjectively) update the probability distribution of the electorate's composition. Allied voters, on the other hand, must rely on the common prior distribution. Because of this difference, we consider mixed strategy equilibria with distinct voting probabilities for the two voter types.

In our analysis (and experiments) we use a binomial distribution in which a priori each floating voter belongs to $i = A, B$ with equal probability $p = 0.5$. We restrict our analysis to the symmetric case with an equal number of allied voters in each group: $\underline{N}_A = \underline{N}_B \geq 1$.[3]

Next, we consider pure and totally quasi-symmetric mixed strategy equilibria for the PU-participation game with allied voters. Let F denote the number of floating voters: $F \equiv E - 2\underline{N}_i$. Proposition A2 gives pure strategy equilibria using a binomial group size distribution (p = .5). It is a straightforward extension of proposition A1. We only need to account for the differences in beliefs that allied and floating voters have about group sizes.

[3] The generalization to asymmetric cases through unequal minimal group sizes or $p \neq .5$ is straightforward, however, more laborious best response conditions and notations are needed.

PROPOSITION A2 (pure strategy Bayesian-Nash equilibria in the PU-participation game with allied voters):

Assume an equal number of allied voters $\underline{N}_A = \underline{N}_B \geq 1$, and binomially distributed floating voters with $p = .5$.

(i) If $c > 1/2$, the only Bayesian-Nash equilibrium in pure strategies is $v_{j_{i,a}} = v_{j_{i,f}} = 0$, $\forall j_{i,a}$, $\forall j_{i,f}$, $i = A, B$ (nobody participates).

(ii) If $c \leq \binom{F}{\lfloor F/2 \rfloor}(1/2)^{F+1}$, the only Bayesian-Nash equilibrium in pure strategies is $v_{j_i,a} = v_{j_{i,f}} = 1$, $\forall j_{i,a}$, $\forall j_{i,f}$, $i = A, B$ (everybody participates).

(iii) Other, more specific Bayesian-Nash equilibria in pure strategies exist.

***Proof*:**

The proof is a straightforward extension of the proof of proposition A1. Because a floating voter knows her preference (group), she can update the probability distribution of x (group sizes), so generally: $prob(x | j_{i,a}) \neq prob(x | j_{i,f})$, except for the case E even with $prob(x = E/2 | j_{i,a}) = prob(x = E/2 | j_{i,f})$. Due to symmetry $prob(x | j_{i,a}) = prob(x | j_{-i,a})$ and $prob(x | j_{i,f}) = prob(x | j_{-i,f})$, hence, all allied (floating) voters have the same posterior probability distribution of group sizes. Contrary to that of floating voters, the preferences (group memberships) of allied voters are 'identifiable'. Define total aggregate participation of allied (floating) voters by $V_a \equiv V_{i,a} + V_{-i,a}$ ($V_f \equiv V_{i,f} + V_{-i,f}$), and the difference in participation between both allied groups by $\Delta V_a \equiv V_{i,a} - V_{-i,a}$.

To (i): See proof of proposition A1(i).

To (ii) and (iii): If $c < 1/2$, in order for $V = 0,1,\ldots,E$, $V = V_a + V_f$, votes to be a pure strategy Bayesian-Nash equilibrium, no participant (non-participant) may receive a strictly higher expected payoff by deviating to abstention (participation). Then, it is necessary and sufficient for equilibria with V turnouts to exist that the following conditions hold for all allied and floating (non-) participants:

No non-participant $j_{i,a}$ and $j_{i,f}$, $i = A, B$, will change her decision if

$$\Phi(V, v_{j_{i,a}} = 0)\frac{1}{2} \leq c, \qquad \Phi(V, v_{j_{i,f}} = 0)\frac{1}{2} \leq c, \tag{A25}$$

and no participant $j_{i,a}$ and $j_{i,f}$, $i = A, B$, will change her decision if

$$\Phi(V, v_{j_{i,a}} = 1)\frac{1}{2} \geq c, \qquad \Phi(V, v_{j_{i,f}} = 1)\frac{1}{2} \geq c, \tag{A26}$$

where the probabilities $\Phi(\cdot)$ of being pivotal for allied and floating voters are similar to those in (A8) and (A9), except that we now use updated probabilities of group sizes only for floating voters (cf. proof of proposition A1).

Next, we establish whether and which pure strategy Bayesian-Nash equilibria exist that fulfill (A25) and (A26). We consider all possible cases $0 \le V \le E$.

Due to the binomial distribution of group sizes with $p = 0.5$ the probability of being pivotal of a floating (allied) non-participant $j_{i,f}$ ($j_{i,a}$) and an allied participant $j_{i,a}$ for $V_f - \Delta V_a$ even is given by

$$\Phi(V, v_{j_{i,f}} = 0) = \Phi(V, v_{j_{i,a}} = 0) = \Phi(V, v_{j_{i,a}} = 1) = \begin{cases} \binom{V_f}{(V_f - \Delta V_a)/2} .5^{V_f} & if \quad V_f \ge |\Delta V_a| \\ 0 & otherwise, \end{cases} \qquad (A27)$$

where ΔV_a corrects for the 'identifiable' participations of allied voters. Obviously, the strict inequalities in (A25) and (A26) cannot be fulfilled simultaneously, because floating and allied non-participants as well as allied participants have the same probability of being pivotal.

Hence, for an equilibrium to exist, it must hold that $\Phi(V, v_{j_{i,f}} = 0) = \Phi(V, v_{j_{i,a}} = 0) = \Phi(V, v_{j_{i,a}} = 1) = 2c$. We investigate whether these equalities can be fulfilled jointly with $\Phi(V, v_{j_{i,f}} = 1) \ge 2c$ for floating participants. A floating participant $j_{i,f}$'s probability of being pivotal is given by

$$\Phi(V, v_{j_{i,f}} = 1) = \begin{cases} \binom{V_f - 1}{(V_f - \Delta V_a)/2 - 1} .5^{V_f - 1} & if \quad V_f \ge |\Delta V_a| + 2 \\ 0 & otherwise, \end{cases} \qquad (A28)$$

which is, given that the probability of all other voters is equal to $2c$, larger than (smaller than; equal to) $2c$ for floating participants in the group with $\Delta V_a < 0$ ($\Delta V_a > 0$; $\Delta V_a = 0$). It follows that for $V_f - \Delta V_a$ even, (A25) and (A26) being fulfilled jointly can only occur if $\Phi(V, v_{j_{i,f}} = 0) = \Phi(V, v_{j_{i,a}} = 0) = \Phi(V, v_{j_{i,a}} = 1) = 2c$ and all floating participants are members of a group with $\Delta V_a \le 0$. For $\Delta V_a < 0$, however, since it is known that only floating voters in *i* participate, they are only pivotal if $V_f = \Delta V_a$. But then allied non-participants in *i* would prefer to vote (yielding $V_f - \Delta V_a$ odd, which is discussed below). Hence, these cases cannot be equilibria. Note further that for $\Delta V_a = 0$, the trivial cases with all voters having probabilities of being pivotal equal to 2*c* occur. These equilibria are not further discussed here.

No other pure strategy Bayesian-Nash equilibria exist for $V_f - \Delta V_a$ even, in which abstainers and participants coexist.

For $V_f - \Delta V_a$ odd, we can write the probability for an allied non-participant $j_{i,a}$ as

$$\Phi(V, v_{j_{i,a}} = 0) = \begin{cases} \binom{V_f}{\lfloor (V_f - \Delta V_a)/2 \rfloor} .5^{V_f} & if \quad V_f \geq |\Delta V_a| \\ 0 & otherwise, \end{cases} \tag{A29}$$

and that for an allied participant $j_{i,a}$ as

$$\Phi(V, v_{j_{i,a}} = 1) = \begin{cases} \binom{V_f}{\lceil (V_f - \Delta V_a)/2 \rceil} .5^{V_f} & if \quad V_f \geq |\Delta V_a| \\ 0 & otherwise. \end{cases} \tag{A30}$$

It is readily verified that (A30) and (A31) are equal for allied non-participants $j_{i,a}$ and allied participants $j_{-i,a}$. But the probability of allied participants $j_{i,a}$ being pivotal is larger than (smaller than; equal to) that of allied non-participants $j_{i,a}$ if $\Delta V_a > 0$ ($\Delta V_a < 0$, $\Delta V_a = 0$). It follows that allied participants and allied abstainers cannot coexist in the same group, respectively (A29) and (A30) cannot be fulfilled jointly, unless $\Delta V_a = 0$ or every allied voter in i participates and none in $-i$. Discussing $\Delta V_a = 0$ first, it is easy to see that (A29), (A30), and the probability of floating non-participants $j_{i,f}$ being pivotal are the same, or

$$\Phi(V, v_{j_{i,f}} = 0) = \Phi(V, v_{j_{i,a}} = 0) = \Phi(V, v_{j_{i,a}} = 1) = \binom{V_f}{\lfloor V_f / 2 \rfloor} .5^{V_f}, \tag{A31}$$

which is similar to (A27) for $V_f - \Delta V_a$ even. As before, we investigate next whether $\Phi(V, v_{j_{i,f}} = 0) = \Phi(V, v_{j_{i,a}} = 0) = \Phi(V, v_{j_{i,a}} = 1) = 2c$ and $\Phi(V, v_{j_{i,f}} = 1) \geq 2c$ for floating participants can be fulfilled jointly for $\Delta V_a = 0$, where the probability of being pivotal of a floating participant $j_{i,f}$ is given by

$$\Phi(V, v_{j_{i,f}} = 1) = \binom{V_f - 1}{\lceil V_f / 2 \rceil - 1} .5^{V_f - 1}, \ \Delta V_a = 0 . \tag{A32}$$

The expression in (A32) is always larger than that in (A31). Hence, for $V_f - \Delta V_a$ odd, pure strategy Bayesian-Nash equilibria with abstainers and participants together indeed exist for $\Phi(V, v_{j_{i,f}} = 1) > \Phi(V, v_{j_{i,f}} = 0) = \Phi(V, v_{j_{i,a}} = 0) = \Phi(V, v_{j_{i,a}} = 1) = 2c$.

With respect to the second case, it is readily verified that it cannot be an equilibrium with $\Delta V_a > 0$ and all allied voters in *i* participating and all allied voters in $-i$ abstaining. This is because the probability of being pivotal of required floating participants $j_{-i,f}$ is given by

$$\Phi(V, v_{j_{-i,f}} = 1) = \begin{cases} \binom{V_f - 1}{\lceil (V_f - \Delta V_a)/2 \rceil - 1} .5^{V_f - 1} & \text{if} \quad V_f \geq |\Delta V_a| + 2 \\ 0 & \text{otherwise}, \end{cases} \qquad (\text{A33})$$

which is smaller than that of allied abstainers in $-i$, $\Phi(V, v_{j_{-i,a}} = 0)$. Hence, such equilibria cannot exist. No other pure strategy Bayesian-Nash equilibria exist in which abstainers and participants coexist.

This leads us to the analysis of the two possible equilibria left:

Full abstention ($V = 0$):

Full abstention cannot be an equilibrium, because any single voter can raise payoff by $1/2$ by turning out, which is larger than the participation costs ($c < 1/2$).

Full participation ($V = E$):

Equilibria with full participation exist for $V_f = F$ even {*odd*}, with $F \equiv E - 2\underline{N}_i$, as long as

$$\Phi(V = E, v_{j_{i,f}} = 1) = \binom{F-1}{F/2-1} .5^{F-1} = \Phi(V = E, v_{j_{i,a}} = 1) = \binom{F}{F/2} .5^{F} \geq 2c$$

$$\{\ \Phi(V = E, v_{j_{i,f}} = 1) = \binom{F-1}{\lfloor F/2 \rfloor} .5^{F-1} > \Phi(V = E, v_{j_{i,a}} = 1) = \binom{F}{\lfloor F/2 \rfloor} .5^{F} \geq 2c\ \}. \qquad (\text{A34})$$

To (*iii*): Conditions (A25) to (A26) are necessary and sufficient for the existence of pure strategy Bayesian-Nash equilibria. ***Q.E.D.***

Similar to proposition A1(*i*), proposition A2(*i*) states that all voters abstain if the voting costs are too high. As proposition A1(*ii*), A2(*ii*) is an intuitive extension of the full participation equilibrium for $N_i = N_{-i}$ analyzed in Palfrey and Rosenthal (1983). If all voters have high enough (compared to costs) expectations that both groups are of equal size respectively that there is one voter more in the own group, a full participation equilibrium exists.

A5. Bayesian-Nash equilibria in mixed strategies; allied and floating voters

For mixed strategy equilibria, we focus on totally quasi-symmetric cases were all allied voters participate with the same probability $q_a \in (0,1)$ and all floating voters with the same probability $q_f \in (0,1)$. A necessary and sufficient condition for Bayesian-Nash equilibria in such strategies to exist is that each allied voter and each floating voter is indifferent between participation and abstention. Elaboration and specification of $(\text{A}7)$ as an equality gives implicit functions for the best responses q_a (A35a) and q_f (A35b):

$$\begin{aligned}
&\sum_{y=0}^{F}\binom{F}{y}(.5)^{y}(.5)^{F-y} \\
&\quad\times\Bigg[\sum_{k=0}^{\min[\underline{N}_i+y-1,\underline{N}_i+F-y]}\ \sum_{k_i=\max[0,k-\underline{N}_i+1]}^{\min[y,k]}\ \sum_{k_{-i}=\max[0,k-\underline{N}_i]}^{\min[F-y,k]}\binom{\underline{N}_i-1}{k-k_i}\binom{y}{k_i}\binom{\underline{N}_i}{k-k_{-i}}\binom{F-y}{k_{-i}} \\
&\qquad\times\ q_a^{\,2k-k_i-k_{-i}}(1-q_a)^{2\underline{N}_i-1-2k+k_i+k_{-i}}\,q_f^{\,k_i+k_{-i}}(1-q_f)^{F-k_i-k_{-i}} \\
&\quad+\sum_{k=0}^{\min[\underline{N}_i+y-1,\underline{N}_i+F-y-1]}\ \sum_{k_i=\max[0,k-\underline{N}_i+1]}^{\min[y,k]}\ \sum_{k_{-i}=\max[0,k+1-\underline{N}_i]}^{\min[F-y,k+1]}\binom{\underline{N}_i-1}{k-k_i}\binom{y}{k_i}\binom{\underline{N}_i}{k+1-k_{-i}}\binom{F-y}{k_{-i}} \\
&\qquad\times\ q_a^{\,2k+1-k_i-k_{-i}}(1-q_a)^{2\underline{N}_i-2-2k+k_i+k_{-i}}\,q_f^{\,k_i+k_{-i}}(1-q_f)^{F-k_i-k_{-i}}\Bigg] = 2c \qquad (\text{A35a})
\end{aligned}$$

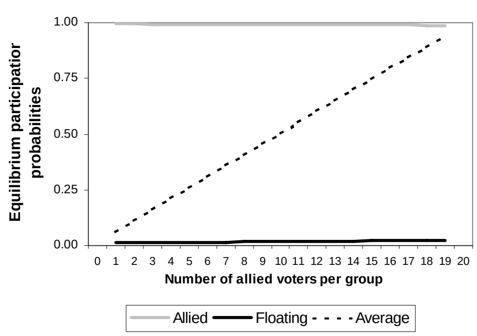
$$\begin{aligned}
&\sum_{y=1}^{F}\binom{F-1}{y-1}(.5)^{y-1}(.5)^{F-y} \\
&\quad\times\Bigg[\sum_{k=0}^{\min[\underline{N}_i+y-1,\underline{N}_i+F-y]}\ \sum_{k_i=\max[0,k-\underline{N}_i]}^{\min[y-1,k]}\ \sum_{k_{-i}=\max[0,k-\underline{N}_i]}^{\min[F-y,k]}\binom{\underline{N}_i}{k-k_i}\binom{y-1}{k_i}\binom{\underline{N}_i}{k-k_{-i}}\binom{F-y}{k_{-i}} \\
&\qquad\times\ q_a^{\,2k-k_i-k_{-i}}(1-q_a)^{2\underline{N}_i-2k+k_i+k_{-i}}\,q_f^{\,k_i+k_{-i}}(1-q_f)^{F-1-k_i-k_{-i}} \\
&\quad+\sum_{k=0}^{\max[\underline{N}_i+y-1,\underline{N}_i+F-y-1]}\ \sum_{k_i=\max[0,k-\underline{N}_i]}^{\min[y-1,k]}\ \sum_{k_{-i}=\max[0,k+1-\underline{N}_i]}^{\min[F-y,k+1]}\binom{\underline{N}_i}{k-k_i}\binom{y-1}{k_i}\binom{\underline{N}_i}{k+1-k_{-i}}\binom{F-y}{k_{-i}} \\
&\qquad\times\ q_a^{\,2k+1-k_i-k_{-i}}(1-q_a)^{2\underline{N}_i-2k-1+k_i+k_{-i}}\,q_f^{\,k_i+k_{-i}}(1-q_f)^{F-1-k_i-k_{-i}}\Bigg] = 2c\,. \qquad (\text{A35b})
\end{aligned}$$

To understand theses conditions, consider (A35a) ((A35b) is a similar application to floating voters). The equation elaborates the condition that the probability of being pivotal is equal to $2c$ for a mixed strategy to be a best response. The left-hand side of (A35a) shows this probability for an allied voter. The term outside of the square brackets gives the probabilities of y $(F-y)$ floating voters being 'allocated' to the same (other) group as this allied voter. For each y, the first (second) term inside the square brackets gives the probability that this voter can break (create) a tie by participating. In the first term, all ties with k votes are considered. In the allied voter's own group, the k votes consist of k_i votes by floating voters and $k-k_i$ by the other allied voters. In the other group k_{-i} ($k-k_{-i}$) of the floating (allied) voters turn out. The first term gives the probability for each event (k, k_i, k_{-i}), given the best responses. In a similar way, the second term inside the square brackets represents the probabilities of all events where k other voters in the allied voter's own group vote and $k+1$ in the other group.

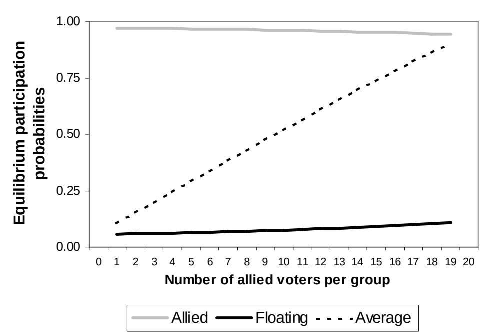
Once again, these equilibria cannot be derived analytically. Numerical estimations show that they do exist for a wide range of parameter values, however. Figure A2 shows numerical examples of such '(q_a, q_f, q_a, q_f)-equilibria' for a fixed electorate size $E = 40$ and varying numbers of allied and floating voters. The number of allied voters per group is from the set $\underline{N}_i \in \{1,,2,...,19\}$ and equal across groups ($\underline{N}_A = \underline{N}_B$). We present participation probabilities for allied and floating voters for voting costs $c = .40$ (upper left panel), $c = .25$ (upper right panel), and $c = .10$ (lower panel). The figure indicates very high (low) participation for allied (floating) voters for both higher costs cases. Only for $c = .10$, equilibrium participation is in the middle range and similar for the two types. We find no equilibrium for these costs for $\underline{N}_i > 8$.

FIGURE A2: (q_a, q_f, q_a, q_f)-EQUILIBRIA IN THE PU-PARTICIPATION GAME WITH ALLIED VOTERS, BINOMIAL GROUP SIZE DISTRIBUTION, $E = 40$, AND $\underline{N}_A = \underline{N}_B \in (1,2,...,19)$

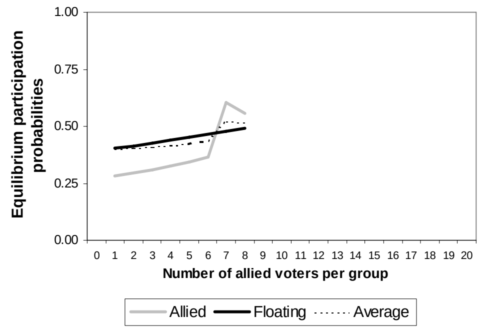
$c = .40$



$c = .25$



$c = .10$



A6. Quantal response equilibria

Goeree and Holt (2005) and Cason and Mui (2005) show that quantal response (logit) equilibria, predict behavior in experimental participation games better than (Bayesian) Nash equilibria do. Here, we show how such equilibria can be derived for the PU-participation game.

Starting point for the quantal response analysis is the comparison of expected payoffs for voting and abstaining described in condition (A6). A stochastic term $\mu\varepsilon_{j_i}$ is added to the expected payoff of each decision (vote or abstain) to allow for the possibility that voters perceive these payoffs subject to noise. It includes an error parameter $\mu \geq 0$ common to all and ε_{j_i} as a realization of j_i's individually specific random variable, which is identically and independently distributed per voter and decision (cf. Goeree and Holt 2005). Voter j_i will participate iff the expected payoff from voting is higher than that of abstaining:

$$Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 1\right]\right] + \mu\varepsilon^1_{j_i} \; > \; Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 0\right]\right] + \mu\varepsilon^0_{j_i}, \qquad (A36)$$

where ε's superscript '1' ('0') refers to the realization of the random variable in the stochastic term that is added to the payoff from voting (abstaining). In the absence of noise (μ = 0), (A36) reduces to condition (A6) for a Bayesian-Nash equilibrium. Hence, the equilibria described above are a limit case of the quantal response equilibria described here (McKelvey and Palfrey 1995; Goeree and Holt 2005).

For μ > 0 it follows from (A36) that voter j_i will vote iff

$$\varepsilon^0_{j_i} - \varepsilon^1_{j_i} \; < \; \frac{Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 1\right]\right] - Exp_{size}\left[Exp_{strat}\left[\pi_{j_i} \middle| v_{j_i} = 0\right]\right]}{\mu}. \qquad (A37)$$

Denoting the distribution function of the difference $\varepsilon^0_{j_i} - \varepsilon^1_{j_i}$ by *F*, this gives the probability *q* that voter j_i will vote:

$$q = F\left[\frac{Exp_{size}\left[Exp_{strat}\left[\pi_{j_i}\middle|v_{j_i} = 1\right]\right] - Exp_{size}\left[Exp_{strat}\left[\pi_{j_i}\middle|v_{j_i} = 0\right]\right]}{\mu}\right], \qquad (A38a)$$

or, after elaboration (cf. condition A7),

$$q = F\left[\sum_{x=\underline{N}_i}^{\overline{N}_i} prob(x)\left[\frac{prob(V_i^{-j_i} = V_{-i}|x)}{2} + \frac{prob(V_i^{-j_i} + 1 = V_{-i}|x)}{2}\right] - c\right] \Big/ \mu \,. \qquad (A38b)$$

This equation describes the voting probability q as a 'noisy best response' to the expected payoff difference between voting and abstaining. Assuming symmetry not only within but also between groups (because all voters face exactly the same decisions) and using the binomials in eq. (A24), the right hand side of (A38b) is a function of the probability, q, that a randomly drawn other voter will vote. A quantal response equilibrium (McKelvey and Palfrey 1995) for some specification of error distribution F occurs if the participation probability on the right hand side is equal to the q that shows up on the left hand side. This can be found numerically for specific values of the error parameter μ.

The quantal response equilibrium for the case with allied and floating voters can be derived in a similar way. Each type is symmetric across groups due to the symmetric group size distribution. Then, to calculate the noisy best responses for allied and floating voters, q_a respectively q_f, two equations similar to (A38b) have to be solved simultaneously.

Appendix B – Experimental Instructions

This appendix presents the read-aloud instructions for treatment UF. Variations for the other treatments are presented in square brackets [*UM*, *IF*, *IM*] These instructions have been translated from _______ .

Welcome to our experiment on decision-making. Depending on your *own choices* and the *choices of other participants*, you may earn money today. Your earnings in the experiment are expressed in *tokens*. <u>4 tokens are worth one Guilder</u>. At the end of the experiment your total earnings in tokens will be exchanged into Guilders and paid to you in cash. The payment will remain *anonymous*. No other participant will be informed about your payment.

Please remain quiet and do not communicate with other participants during the entire experiment. Raise your hand if you have any questions. One of us will come to you to answer them.

<u>*Rounds, 'your group' and the 'other group'*</u>

The experiment consists of <u>*100 rounds*</u>. At the <u>*beginning of the experiment*</u> the computer program will randomly split all participants into two different <u>*populations*</u> of 12 participants. In addition, at the <u>*beginning of each round*</u> the computer program will randomly divide the participants in each population into <u>*two groups*</u>. The group you are part of will be referred to as <u>*your group*</u> and the group in your population which you are not part of will be called the <u>*other group*</u>. You will not know which of the participants belongs to the other group and which to your group. You will have nothing to do with participants in the other population in this experiment.

<u>*Number of participants in 'your group' and the 'other group'*</u>

At the beginning of <u>*each round*</u> the computer program will randomly determine the *number of participants* in your group and the number of participants in the other group. **At no point in time will you or anybody else receive information about the number of participants in your group and the number of participants in the other group.** [*In IF and IM instead*: **You and all other**

participants in both groups will then receive information about the number of participants in your group and the number of participants in the other group.]

However, you and all other participants know that [*in IF and IM*: There is the following structure of group sizes]:

(1) Independent of the round, the sum of participants in *both* groups (your group and the other group) is always **12**.

(2) Both groups contain a *minimum* of **3** participants and a *maximum* of **9** participants.

Because the sum of participants in both groups is always twelve, there are the following **7** possible *combinations of group sizes*:

(3-9) (4-8) (5-7) (6-6) (7-5) (8-4) (9-3),

whereby the first number represents the group size of the first group and the second number the group size of the second group.

The arrangement of a population (12 participants) into two groups by the computer program proceeds in the following two steps:

(1) Both groups are randomly filled with **3** participants, the minimal number of participants per group (in total **6** participants). Each participant has the *same* chance of being selected.

(2) Each of the remaining **6** participants is randomly put into one of the two groups, with a chance of 50% for each group.

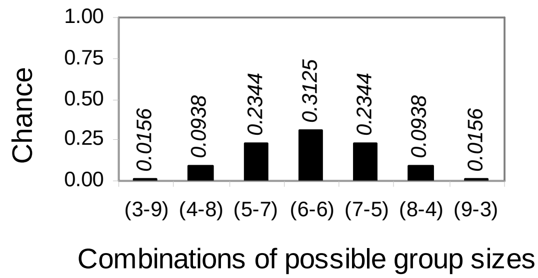
[*For UM and IM instead*:

(1) At the beginning of the *first round* both groups are randomly filled with **3** participants, the minimal number of participants per group (in total **6** participants). Each participant has the *same* chance of being selected. The chosen participants will be called '*FIX*'**-**participants, because they will not change groups during the whole experiment.

(2) At the beginning of *each round* each of the remaining **6** participants is randomly put into one of the two groups, with a chance of 50% for each group. These participants will be called '*VAR*'**-** (=variable) participants, because they will randomly change groups during the whole experiment.

At the beginning of the *first round* you will receive information about your own type *FIX* or *VAR*. Your own type will not change during the whole experiment.]

The following figure shows for all seven possible combinations of group sizes the chance that a particular combination occurs.



Note again that group sizes will be randomly determined at the beginning of *each* round. Hence, the group sizes may change from one round to another.

<u>*Choices and earnings*</u>

In <u>*each round*</u> you and all other participants will face an identical choice problem. You will be asked to make <u>*one choice*</u>. You can choose between the following two alternatives:

- '<u>*Choice A*</u>': no costs involved (**0 tokens**).
- '<u>*Choice B*</u>': costs are **1 token**.

When making your choice, nobody else in your group or in the other group will know this choice. After *all* participants have made their choices, the computer program will count the number of *B*-choices in your group and in the other group and will compare the numbers in both groups. There are **3** possible outcomes that are relevant for your *revenue* in the following way. You will receive the revenue irrespective of the choice you made.

(1) <u>The number of *B*-choices in your group **exceeds** the number of *B*-choices in the other group</u>. In this case *each* participant in your group (including yourself) will get revenue of **4 tokens**. *Each* participant in the other group will get **1 token**.

(2) <u>The number of *B*-choices in your group **is smaller than** the number of *B*-choices in the other group</u>. In this case *each* participant in your group (including yourself) will get revenue of **1 token**. *Each* participant in the other group will get **4 tokens**.

(3) <u>The number of *B*-choices in your group **is equal to** the number of *B*-choices in the other group</u>. In this case the computer program will randomly determine the group in which *each* participant gets revenue of **4 tokens** (each group has the same chance of 50% of being chosen). *Each* participant in the group that is not chosen will get **1 token**.

Your <u>*round earnings*</u> are calculated in the following way: *round earnings = round revenue – round costs*. Your <u>*total earnings*</u> are the sum of all of your round earnings.

The following table gives your possible round earnings:

<u>**Your possible round earnings:**</u>

Your choice	*Your group has <u>more</u> B-choices*	*Your group has <u>less</u> B-choices*	*<u>Equal</u> number of B-choices in both groups*
Choice *A*	**4 tokens**	**1 token**	**4** *or* **1 token** (50% chance each)
Choice *B*	**3 tokens**	**0 token**	**3** *or* **0 token** (50% chance each)

<u>*Computer screen*</u>

The computer screen has four main windows.

(1) The <u>*Status*</u> window shows [*for UM and IM*: your <u>*type*</u> (*FIX* or *VAR*),] the actual <u>*round number*</u> and the <u>*total earnings*</u> up to the previous round.

(2) The <u>*Previous round*</u> window depicts the following information about the previous round:

(a) The number of *B-choices in your group* [*in IF and IM*: and, in brackets, the size of your group].

(b) The number of *B-choices in the other group* [*in IF and IM*: and, in brackets, the size of the other group].

(c) Your *choice*.

(d) Your *revenue*.

(e) Your *costs*.

(f) Your *round earnings*.

Note that no information about the group sizes will be given [*this sentence not for IF and IM*].

(3) In the <u>*Choice*</u> window you will find two *buttons*. Press the button “Choice *A*” or the button “Choice *B*” with the mouse, or press the key “*A*” or “*B*”. When you have chosen you will have to wait until all participants have made their choices.

(4) The *Result* window shows the result of the *current* round, hence after each participant has made a choice. Each *yellow* rectangle shown represents one *B*-choice of your group and each *blue* rectangle represents one *B*-choice of the other group. After a few seconds the result will also appear in numbers.

At the top of the screen you will find a *Menu bar*. You can use this to access the *Calculator* and *History* functions. The calculator can be handled with the numerical pad on the right side of your keyboard or with the mouse buttons. The function 'history' shows all information of the last *sixteen* rounds as this had appeared in the window 'Previous round'. At the bottom of your screen the *Information bar* is located. There you are told the current status of the experiment.

Further procedures

Before the 100 rounds of the experiment start, we will ask you to participate in *three training-rounds*. You will have to answer questions in order to proceed further in these training-rounds. In the training-rounds you are not matched to other participants but to the computer program. **You cannot draw conclusions about choices of other participants based on the results in the training-rounds.** The training-rounds will not count for your payment.

We will now start with the three trainings-rounds. If you have any questions, please raise your hand. One of us will come to you to answer them.

Appendix C – Procedures

TABLE C: SEQUENCE OF ELECTORAL COMPOSITIONS

Round	Elect. comp.	Round	Elect. comp.	Round	Elect. comp.	Round	Elect. comp.	Round	Elect. comp.
1	*6-6*	21	*5-7*	41	*6-6*	61	*7-5*	81	*4-8*
2	*7-5*	22	*6-6*	42	*6-6*	62	*7-5*	82	*6-6*
3	*6-6*	23	*4-8*	43	*7-5*	63	*5-7*	83	*6-6*
4	*5-7*	24	*7-5*	44	*6-6*	64	*6-6*	84	*6-6*
5	*6-6*	25	*6-6*	45	*4-8*	65	*4-8*	85	*5-7*
6	*4-8*	26	*9-3*	46	*5-7*	66	*5-7*	86	*5-7*
7	*6-6*	27	*6-6*	47	*7-5*	67	*5-7*	87	*6-6*
8	*7-5*	28	*7-5*	48	*7-5*	68	*6-6*	88	*8-4*
9	*7-5*	29	*5-7*	49	*6-6*	69	*6-6*	89	*6-6*
10	*5-7*	30	*5-7*	50	*8-4*	70	*5-7*	90	*6-6*
11	*5-7*	31	*5-7*	51	*6-6*	71	*5-7*	91	*7-5*
12	*7-5*	32	*8-4*	52	*5-7*	72	*6-6*	92	*6-6*
13	*4-8*	33	*8-4*	53	*6-6*	73	*8-4*	93	*5-7*
14	*7-5*	34	*6-6*	54	*7-5*	74	*5-7*	94	*8-4*
15	*7-5*	35	*7-5*	55	*7-5*	75	*7-5*	95	*7-5*
16	*8-4*	36	*5-7*	56	*4-8*	76	*7-5*	96	*3-9*
17	*6-6*	37	*6-6*	57	*6-6*	77	*4-8*	97	*5-7*
18	*5-7*	38	*5-7*	58	*3-9*	78	*5-7*	98	*6-6*
19	*4-8*	39	*6-6*	59	*6-6*	79	*6-6*	99	*6-6*
20	*9-3*	40	*7-5*	60	*7-5*	80	*8-4*	100	*8-4*

Appendix D – Equilibrium Predictions

TABLE D: OVERVIEW OF EQUILIBRIUM PREDICTIONS, OBSERVED TURNOUT RATES FOR ROUNDS 21 TO 100, AND MAXIMUM LIKELIHOOD ESTIMATES OF THE NOISE PARAMETERS

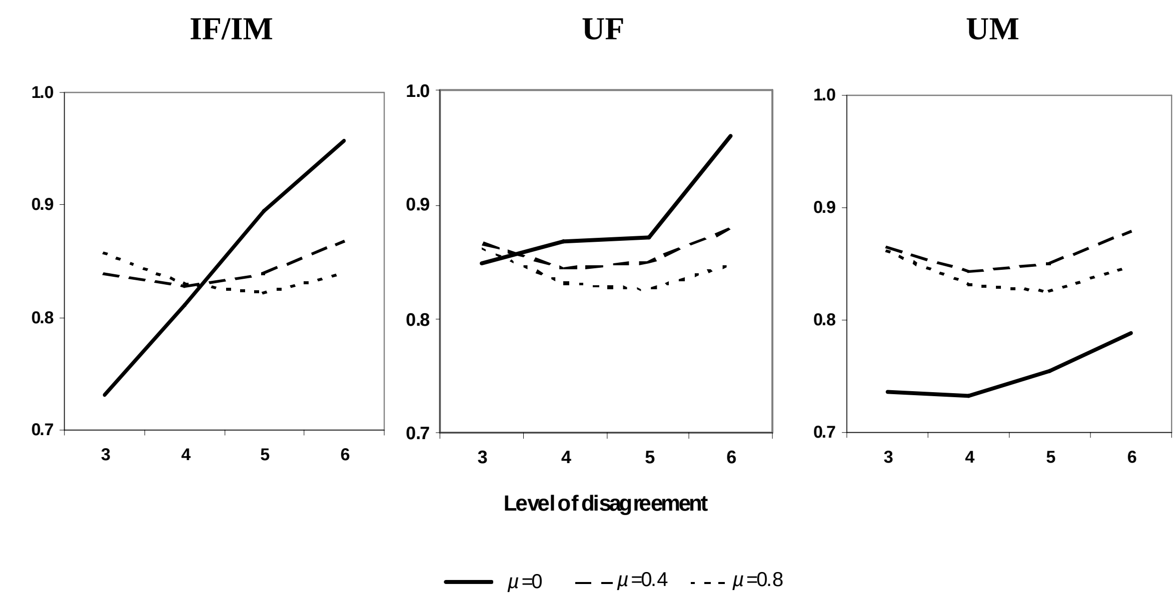
Treatment	Levels of support	Observed turnout rates	Predictions: (Bayesian) Nash	Predictions: QRE		Estimated μ	# observations	Log-likelihood		
			$\mu = 0$	$\mu = 0.4$	$\mu = 0.8$			(Bayesian) Nash	QRE	Random play
UF	-	.297	.098	.302	.381	.39	5616	-4286.62	-3418.01	-3892.71
UM	-	*Allied*: .426	.934	.306	.382	.05 or 1.38	2880	-4576.80	-1964.64	-1996.26
		Floating: .293	.123	.299	.380	.38	2880	-2033.92	-1741.13	-1996.26
	All	.359	.529**	.303**	.381**	.65	5760	-4094.34	-3761.61	-3992,53
IF	3	.296	.256	.306	.341	.08	54	-33.04	-32.82	-37.43
	4	.301	.173	.320	.369	.29	336	-221.73	-205.42	-232.90
	5	.329	.132	.331	.395	.39	1050	-798.61	-664.82	-727.80
	6	.481	.107*	.329	.404	4.28	2016	-2285.99	-1395.95	-1397.38
	7	.437	.091	.306	.384	1.58	1470	-1619.11	-1007.13	-1018.93
	8	.281	.080	.271	.349	.44	672	-517.75	-399.26	-465.79
	9	.241	.074	.234	.313	.43	162	-111.11	-89.41	-112.29
	All	.400	.109**	.313**	.385**	.95	5760	-5501.26	-3875.74	-3992.53
IM	3	.296	.256	.306	.341	.08	54	-33.04	-32.82	-37.43
	4	.286	.173	.320	.369	.22	336	-213.92	-201.02	-232.90
	5	.393	.132	.331	.395	.79	1050	-926.74	-703.73	-727.80
	6	.585	.107*	.329	.404	∞	2016	-2731.49	-1397.38	-1397.38
	7	.448	.091	.306	.384	1.96	1470	-1658.27	-1011.05	-1018.93
	8	.341	.080	.271	.349	.74	672	-615.48	-431.12	-465.79
	9	.302	.074	.234	.313	.73	162	-136.43	-99.30	-112.29
	All	.459	.109**	.313**	.385**	2.55	5760	-6215.60	-3972.84	-3992.53

* Or, .893; **weighted overall participation probability.

Appendix E – Electoral Efficiency

The voting and victory probabilities from appendix D can be used to calculate voters' expected payoffs, which determine the electoral efficiencies in equilibrium (Palfrey and Rosenthal 1983). Figure E shows these efficiencies per treatment, noise-level, and level of disagreement. Efficiency is calculated as the electorate's aggregate payoff in equilibrium, divided by its socially optimal (efficient) total payoff. For unequal group sizes, surplus is maximized when one voter in the majority participates and all other voters abstain. For example, for electorates with group sizes (3-9) or (9-3), the efficient aggregate payoff is $3\times 1+9\times 4-1=38$ [similarly, 35 for (4-8)/(8-4), and 32 for (5-7)/(7-5)]. For equal group sizes (6-6), it is efficient if nobody participates, in which case total payoff is $6\times 1+6\times 4=30$.

FIGURE D: EQUILIBRIUM EFFICIENCY



Note from figure E that once again the treatment effect is strongest in the Bayesian-Nash equilibrium ($\mu=0$). When voters are informed about group sizes, efficiency is lowest (73%) when the level of disagreement is lowest and monotonically increases with the size of the minority to 96% when there are 6 voters in each group. This pattern occurs for noise level $\mu=0$, because overall expected participation decreases and the majority's probability of winning increases in the level of disagreement. Compare this to the treatments with PU. In UF efficiency for $\mu = 0$ is very high (96%) for equal levels of support, whereas intermediate values of efficiency between 85% and 87% are observed for all other levels of disagreement. In UM, efficiency for $\mu = 0$ is low (between 73% and 79%) for all levels of disagreement. This is because allied voters participate extensively. When noise is introduced, the differences across treatments are minor. In all cases the efficiency curves are U-shaped and show intermediate values (83-88% for $\mu = 0.4$ and 82-86% for $\mu = 0.8$).

Table E gives realized efficiencies and their standard deviations. The data are pooled across the two voter alliance treatments, since virtually no differences in patterns are observed for this variable.

TABLE E: ELECTORAL EFFICIENCY

Treatment	**Without poll releases**				**With poll releases**			
Level of disagreement	3	4	5	6	3	4	5	6
Efficiency (standard dev.)	.854 (.081)	.842 (.051)	.831 (.046)	.867 (.042)	.887 (.066).	.861 (.042)	.835 (.045)	.786 (.059)
Weighted average	.846				.826			