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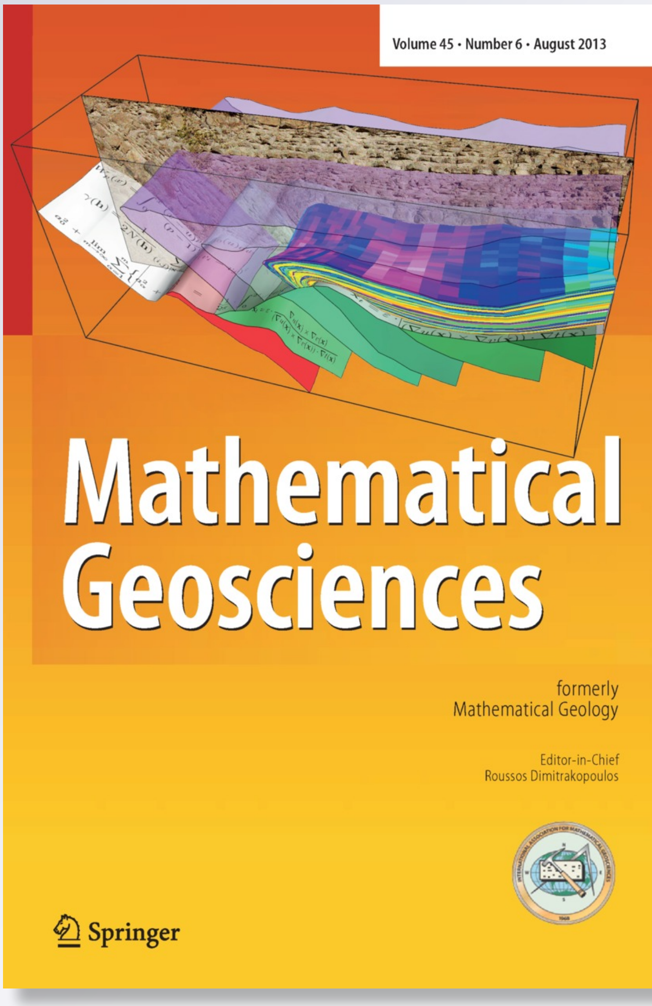

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A Spatial Point Process Model for Violent Tornado Occurrence in the US Great Plains

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Abstract The authors illustrate a statistical point process model that uses the spatial occurrence of nonviolent tornadoes to predict the distribution of the rare, violent tornadoes that occur during springtime across the US central Great Plains. The average rate of nonviolent tornadoes is 55 per 10^4 km^2 per 62 years which compares with an average rate of only 1.5 violent tornadoes per 10^4 km^2 over the same period (less than 3 %). Violent tornado report density peaks at 2.6 per 10^4 km^2 (62 yr) in the city but is only 0.7 per 10^4 km^2 in the countryside. The risk of a violent tornado is higher by a factor of 1.5, on average, in the vicinity of less violent tornadoes after accounting for the population bias. The model for the occurrence rate of violent tornadoes indicates that rates are lower by 10.3 (3.6, 16.5) % (95 % CI) for every 1 km increase in distance from the nearest nonviolent tornado, controlling for distance from the nearest city. Model significance and the distance-from-nearest nonviolent tornado parameter are not sensitive to population threshold or the definition of a violent tornado. The authors show that the model is useful for generating a catalogue of touchdown points that can be used as a component to a tornado catastrophe model.

Keywords Tornado · Spatial point process model · Spatial density · Report bias

1 Introduction

Reliable and stable estimates of tornado rates are critical for hazard assessment. Tornadoes occur throughout the world but are most common within the United States

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east of the Rocky Mountains. The south-central Great Plains in particular experience several dozen tornadoes each year on average, with a concentration during spring. But the vast majority of these tornadoes have wind speeds less than about 70 $\mathrm{m\,s^{-1}}$. A violent tornado with winds exceeding 90 $\mathrm{m\,s^{-1}}$ is very rare. Most of these potentially destructive and deadly tornadoes occur from rotating thunderstorms called supercells, with formation contingent on local (storm-scale) meteorological conditions. The long-term risk of a tornado at a given location is assessed using historical records, however, the rarity of the most violent tornadoes make these rate estimates unstable. Here, we propose to use stable rate estimates from the larger set of less violent tornadoes in order to create more stable rate estimates for violent tornadoes.

Local rate estimation is hampered by a reporting bias. A tornado record depends on an observer making a report and on official documentation being entered into the database. Over the long run, this introduces a bias whereby reports outside towns and cities tend to be less numerous (population bias). An additional complication arises because of low public awareness before 1990 (Doswell et al. 1999). Here, we propose to use our model for the population bias (Elsner et al. 2013) to improve rate estimates for violent tornadoes. In short, this study illustrates a statistical model that uses the spatial occurrence of nonviolent tornadoes together with a component for the population bias to predict the distribution of the rare, violent tornadoes. The approach uses modern tools for analyzing and modeling spatial point pattern data. The primary goal is to demonstrate a statistically significant model for local violent tornado rates that can be used across the central Plains where tornadoes are frequent and possibly elsewhere. The code used for this study is available at http://rpubs.com/jelsner/4205.

The paper is outlined as follows. In Sect. 2, we describe the data and our study domain. The focus here is on springtime tornadoes in the central US Plains. In Sect. 3, touchdown points as spatial point pattern data with a planar projection are considered. This allows the spatial density of report occurrences to be defined regionally and locally. In Sect. 4, the relationship between the rates of violent tornado reports and the distance from nearest city center and nearest nonviolent tornado are examined. All cities in the study area with 1990 population exceeding 2000 residents are examined. Both relationships are found to be statistically significant. In Sect. 5, the violent tornado rates are modeled using these two distances as covariates. Section 6 examines the model fit and how the model is used to make a prediction and to generate sixteen 62-year data sets of synthetic violent activity. Section 7 provides a summary and list of conclusions.

2 Tornado Data and Study Area

The Storm Prediction Center (SPC) maintains a dataset of all reported tornadoes in the United States from January 1, 1950, to the present. Earlier records exist, but there has not been a consistent effort to investigate, document, or maintain a record of these earlier occurrences (Galway 1977). The SPC dataset is the most reliable archive available for tornado studies. The dataset used here has been downloaded from http://www.spc.noaa.gov/gis/svrgis/. At the time the data was downloaded in December 2012; the number of tornado reports was 56,221. The principal aim of this

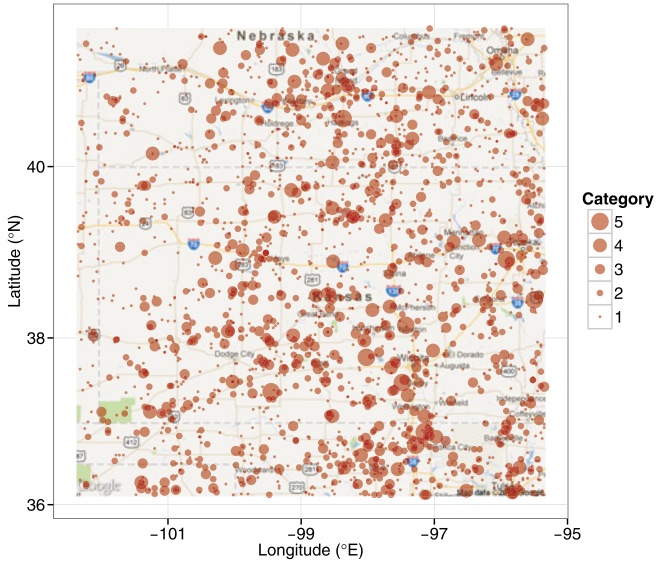


Fig. 1 Tornado reports (F1–F5) over the study region centered on Russell, KS during March–June for the period 1950–2011. The point size is proportion to the F scale

paper is to demonstrate a significant statistical model for local violent tornado rates. Thus, we are motivated to use a portion of the data where reports are numerous and spatially homogeneous. Here, the same region defined in Elsner et al. (2013) centered on Russell, Kansas, and bounded by 36.10° and 41.57° N latitudes and 102.37° and 95.34° W longitudes is used. The region is the central Plains from northern Texas to central Nebraska. This is an area with a high concentration of tornadoes and where there are no large spatial gradients in occurrence rates. It also corresponds to an area favored by storm chasers. The focus here is further restricted to the months of March through June when the primary tornado-producing severe convective storm is the supercell. Figure 1 shows a road map of the study domain and the 2116 touchdown points in the region during March–June by Fujita damage scale over the period 1950–2011. The touchdown points are provided on a Lambert conformal conic (LCC) projection with reference parallels of 33 and 45° N latitudes. The native spatial unit is meters.

The Fujita scale (F scale) introduced in the 1970s is the standard measure of tornado intensity. It is based on the maximum damage caused along the tornado path and ranges from F0 (for minimum damage) to F5 (for total destruction). It was replaced by the enhanced Fujita scale during early 2007, using slightly different and more specific criteria for assessment (Potter 2007). The Fujita scale and the enhanced Fujita scale are considered equivalent for climatological applications. In this study, only tornadoes with an F scale rating of one or higher are considered, because the F0 level was historically used as default when the amount of damage was unknown (Doswell et al. 2009). For reference, the minimum velocity for a F1 tornado is 33 $\mathrm{m\,s^{-1}}$. The total number of F4 and F5 tornadoes over the study do-

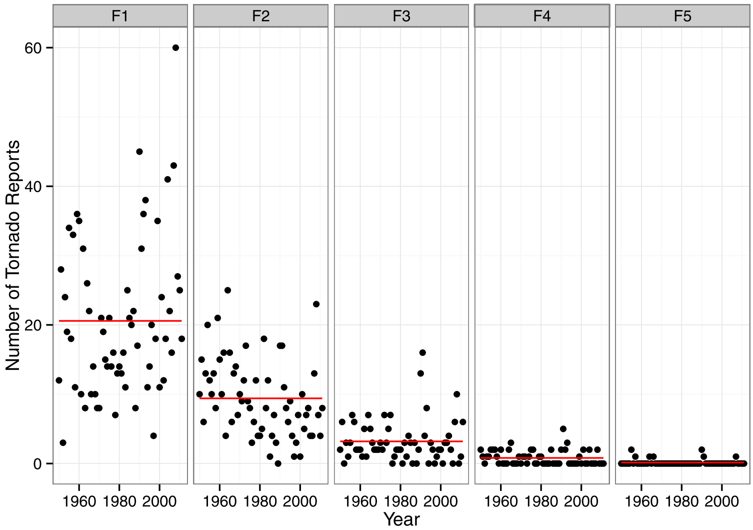


Fig. 2 Time series' of springtime (March–June) tornado reports by F scale over the study region centered on Russell, KS. The *horizontal red lines* indicate the average count over the period

main is 59, which represents 2.87 % of the total number of F1 through F5 tornadoes over the 62-year period. Figure 2 shows time-series plots of the annual number of tornadoes by F scale. The horizontal bar is the annual rate over the study domain. Occurrence rates decrease significantly with increasing tornado ferocity. The annual rate is 20.6 tornadoes per year for F1 tornadoes, which compares to 0.145 tornadoes per year for F5 tornadoes. The corresponding annual variance is much larger than the annual rate for F1, F2, and F3 tornadoes. This is because tornadoes frequently occur in clusters—defined as an outbreak—on days when weather conditions are particularly favorable for severe convective storms. Statistically, the occurrence of a tornado on a given day increases the chance of another one on the same day. Violent tornadoes (F4 and F5) are less obviously clustered. Of the 59 violent tornadoes, 35 days had a single event and another 8 days had 2 events. The largest outbreak over this region occurred on April 26, 1991, with 5 violent tornadoes.

Figure 3 shows the distribution of annual violent tornado counts in the region. The red line indicates the overall mean. More than half the years (32) are without a violent tornado. The expected number of years without a violent tornado is 24 assuming a Poisson distribution with an annual rate equal to the average count (Table 1). Of the remaining years, 43 % have one and 30 % have two violent tornadoes. The year with the single largest outbreak (1991) had a total of six violent tornadoes. A goodness-of-fit test for a Poisson distribution using the likelihood ratio gives a p-value of 0.0057 providing evidence that violent tornadoes tend to come in clustered outbreaks. This clustering means there will be more years without violent tornadoes and more years with four or more than would be expected under the assumption of a Poisson distribution (Thom 1963).

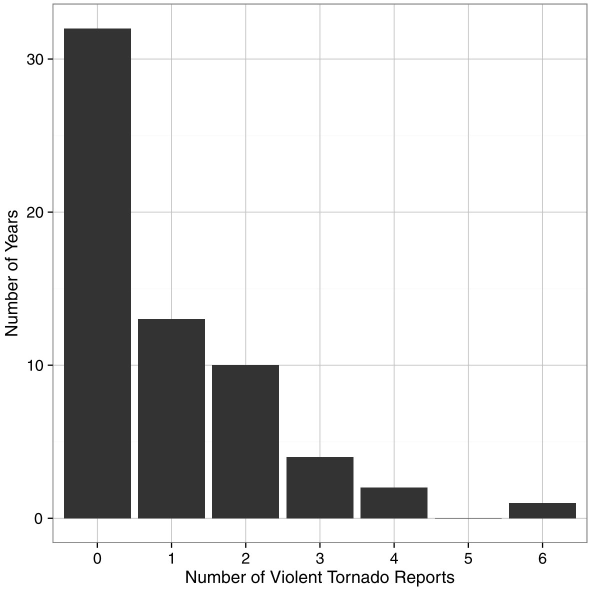


Fig. 3 Histogram of annual violent tornado counts (1950–2011) over the study region centered on Russell, KS

Table 1 Observed vs expected violent tornado counts

Count	Observed	Expected
0	32	23.9
1	13	22.8
2	10	10.8
3	4	3.44
4	2	0.82
5	0	0.16
6	1	0.02

3 Regional and Local Tornado Reporting Rates

The regional tornado rate is defined as the number of tornado reports per area. There are 2116 March–June (1950–2011) F1–F5 tornadoes in our central Plains region with an area of 3.84×10^5 km^2. This amounts to an average rate of 55 tornadoes per 10^4 km^2 per 62 years. The rate excludes F0 tornadoes and tornadoes with no F scale rating. However, there are only 59 violent tornado touchdowns over the same period for a regional rate of 1.5 per 10^4 km^2.

The regional rate, based on the number of tornadoes over the entire area, might not represent rates locally. Figure 4 shows the local rates of nonviolent tornado reports using a Gaussian smoother with edge correction (Diggle 1985). The local rates are computed on a 128 by 128 grid of pixels by a convolution of the isotropic Gaussian kernel having a standard deviation of 77 km with point masses at each of the nonviolent tornado locations. Although the overall rate is 55 reports per 10^4 km^2, locally the values range between about 20 and 90 nonviolent tornadoes per 10^4 km^2 depending on the pixel.

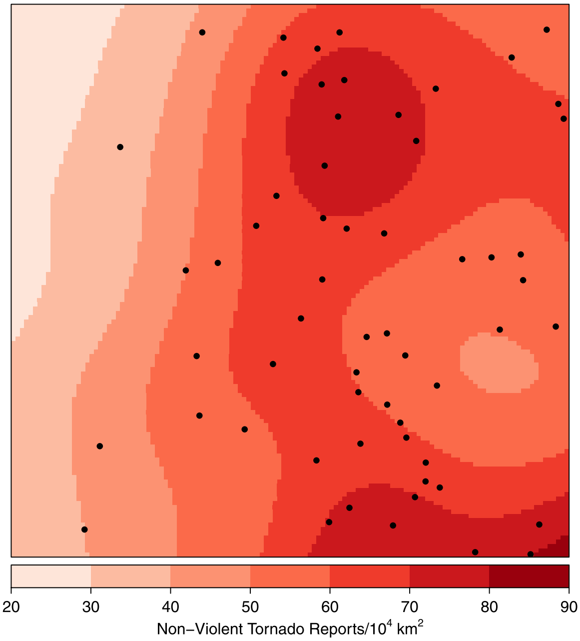


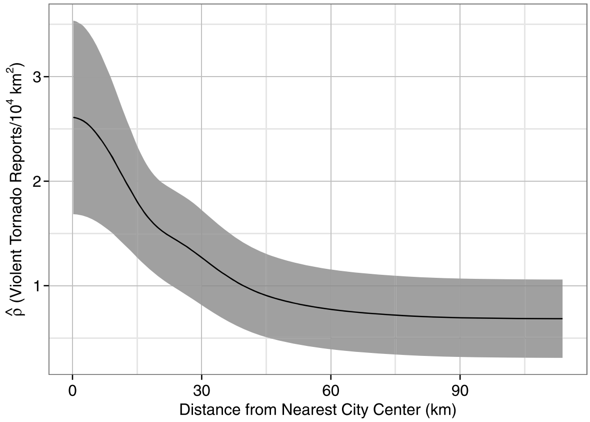
Fig. 4 Local rates of nonviolent (F1–F3) tornado reports (March–June) over the period 1950–2011. The rates are computed on a 128 × 128 grid of pixels using a kernel smoothing with a fixed bandwidth of 77 km. The points are the locations of all F4–F5 tornadoes over the same period

There appears to be some correspondence between nonviolent tornado rates and violent tornado occurrence, which are shown as points in Fig. 4. Regions with relatively low rates of nonviolent tornado activity tend to be regions without violent tornadoes. A kernel smoothing of the violent tornado occurrences on the same set of pixels results in a correlation of 0.84 between nonviolent and violent rates across the set of pixels. This correspondence seems to indicate a relationship between nonviolent and violent tornado occurrences, however, the observational bias must first be removed before testing this hypothesis.

4 Tornado Rates and Distance from Nearest City

As noted above the estimated tornado rates are affected by a population bias described by fewer reports outside towns and cities. Here, this bias is modeled on the rates of violent tornadoes using the method of Elsner et al. (2013). Centroid locations are obtained for all U.S. cities from http://www.nws.noaa.gov/geodata/catalog/national/html/cities.htm. Cities with a 1990 population of less than 2000 and those outside our study region are removed. City centers are specified with longitudes and latitudes. The centers are projected using the same LCC as for the tornado touchdown reports. This results in 177 cities with populations ranging from 2007 to 367,302 (Tulsa, OK) residents. At each pixel, where the local tornado rates are estimated in the previous section, we determine the distance the pixel center is from the nearest city center. Distances across the domain range from a minimum of 0.2 km to a maximum of 114 km with a median of 24 km. Fifty percent of all pixels are between 15 and 35 km from the nearest city. Figure 5 illustrates the population bias as a plot of the estimated

Fig. 5 Violent tornado report density as a function of distance from nearest city center



violent tornado density as a function of distance from nearest city center. Let $Z_c(u)$ be the distances on grid u, then the model for the estimated tornado rate in pixel ($\hat{\lambda}(u)$) is given by

$$\hat{\lambda}(u) = \hat{\rho}\big(Z_c(u)\big) \tag{1}$$

where $\hat{\rho}(z_c)$ is estimated using kernel smoothing, implemented by applying the probability integral transform to the distance-to-nearest city value, yielding values in the range 0 to 1, then applying edge-corrected density estimation on the interval [0, 1], and back-transforming (Baddeley and Turner 2005). The probability integral transform uses the empirical cumulative distribution function for the covariate Z_c ($P(Z(u) \leq z)$ for a random selection of pixels). The bandwidth is set to be 0.2 standard deviations of the kernel to ensure a smooth relationship.

The curve shows the average violent tornado report density as a function of distance from nearest city. Report density peaks at 2.6 violent tornadoes per 10^4 km^2 (over the 62-yr period) in the city to 0.7 per 10^4 km^2 in the countryside. The difference between report density at zero distance and the report density at maximum distance provides a description of the population bias in violent tornado rates (Elsner et al. 2013). For comparison, the nonviolent tornado report density peaks at 72.7 per 10^4 km^2 (over the 62-yr period) in the city to 37.2 per 10^4 km^2 in the countryside. The ratio of city to countryside report density is higher for the violent tornadoes, perhaps related to the nonzero probability of missing a weak tornado even in a city.

5 Spatial Models for Violent Tornado Occurrences

The previous analysis suggests that violent tornado rates across the study domain might be modeled successfully using nonviolent tornado rates after controlling for distance from nearest city. We begin by fitting a model to the violent tornado occurrences using distance from nearest city. The fit uses a maximum likelihood procedure with a Berman–Turner device (Berman and Turner 1992). The results are insensitive to moving the domain borders in by up to 100 km on each side. As expected, the model shows a decreasing trend with increasing distance from the city. The trend

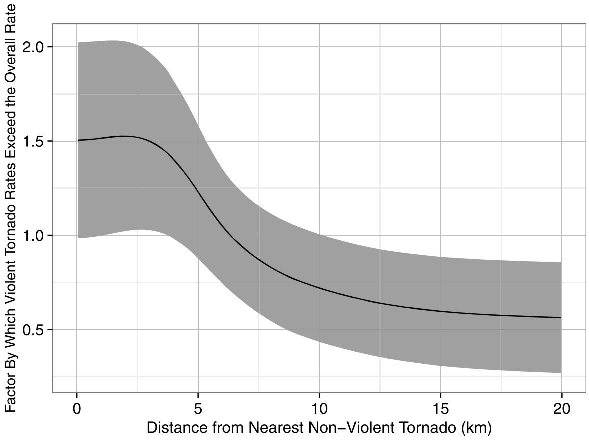


Fig. 6 Factor by which the violent tornado report rates exceed the overall rate as a function of distance to nearest nonviolent tornado. The model includes a distance-from-nearest-city term to account for the population bias

term amounts to a decrease of 3.9 % for every 1 km increase in distance. The 95 % confidence bound on the trend estimate is (1.8, 5.9 %) consistent with the notion that there is a significant population bias. We check to see if the model with the trend term is better than a model without it by comparing the AIC (Akaike Information Criterion) from the two alternatives. The AIC is smaller with the distance from city model so we choose it over the null model. In fact, two times the difference in log likelihood between the two models divided by the number of violent tornadoes is close to 1 at 0.996, indicating a huge improvement in modeling the data. Next, the violent tornado density relative to the nonviolent tornado density is examined. Let $\hat{\kappa}(u)$ be the violent tornado density on grid of pixels u conditional on the pixel distance from nearest city and let $Z_{nv}(u)$ be the distance from the nearest nonviolent tornado, then the model is given by

$$\hat{\lambda}(u) = \hat{\rho}\big(Z_{nv}(u)\big)\hat{\kappa}(u). \tag{2}$$

A smoothing estimator for $Z_{nv}(u)$ is computed that gives the factor by which the violent tornado rates exceed the overall violent tornado rate in the vicinity of nonviolent tornadoes. The curve in Fig. 6 shows this factor as a function of distance from nearest nonviolent tornado. The factor peaks at 1.5 in the immediate vicinity of a nonviolent tornado. This says that, on average, the risk of a violent tornado is higher by a factor of 1.5 in the vicinity of less violent tornadoes after accounting for the population bias. To check the possibility of an artifact, we randomize the nonviolent touchdowns locations and repeat the smoothing and find, as expected, the ratio does not depart significantly from a ratio of one at any distance.

Next, the method of maximum likelihood is used to fit a Poisson point process model to the occurrence of violent tornadoes using distance from nearest city and distance from nearest nonviolent tornado as covariates. We assume the point process is inhomogeneous Poisson. Again the results are insensitive to moving the domain borders in by up to 100 km on each side. As expected, the model shows fewer violent tornadoes with increasing distance from the nearest city. The decrease amounts to 3.1 (1.0, 5.2) % (95 % confidence interval, CI) for every 1 km increase in distance. The model also shows fewer violent tornadoes with increasing distance from the nearest

Table 2 Sensitivity of trend terms to changes in population threshold. The trend terms are expressed in percent per km. The number in parentheses is the standard error

Population threshold	Number of cities	City trend (% per km)	Nonviolent tornado trend (% per km)
1000	320	−4.1 (1.6)	−11.1 (3.7)
1500	226	−3.0 (1.3)	−11.5 (3.6)
2000	177	−3.1 (1.1)	−10.8 (3.7)
2500	141	−2.2 (0.9)	−11.3 (3.7)
3000	124	−1.8 (0.8)	−11.5 (3.7)

nonviolent tornado. The decrease amounts to 10.3 (3.6, 16.5) % (95 % CI) for every 1 km increase in distance. Table 2 shows results from a check on the sensitivity of the model parameters to changes in population threshold. The magnitude of the distance-from-nearest-city trend decreases with increasing population threshold. With a minimum threshold of 1000 residents defining a city the decrease per kilometer is 4.1 %. With a minimum threshold of 3000 residents the decrease per kilometer is less than half of that at 1.8 %. The magnitude of the distance-from-nearest-nonviolent-tornado trend is insensitive to these changes. With a threshold defining a city less than 2000, the distance from nearest city parameter is larger along with a larger standard error. With a threshold greater than 2000, the city parameter is about the same.

To check the sensitivity of choosing F4 as our threshold for defining a violent tornado we decrease the threshold to F3 and call F1 and F2 nonviolent tornadoes. We then refit the model to the data and find the distance-from-nearest city parameter is smaller at −1 % per km and the distance-from-nearest nonviolent tornado parameter is about the same at −12.3 % per km. The two terms remain statistically significant with this new definition. The smaller population bias is consistent with Anderson et al. (2007) who show that, in contrast with expectations, weaker tornado reports in Oklahoma vary less with population density than do the stronger tornadoes. Including F0 tornadoes in the set of nonviolent tornadoes does not change the results.

6 Model Diagnostics, Prediction, and Simulation

The two-term final model appears to be a good spatial point process representation of violent tornado occurrences across the study domain. The AIC for the model with only the distance-from-nearest-city term is 2726 and the AIC for the two-term final model is 2718. The difference in log likelihoods between the two models divided by the number of violent tornadoes is 0.172 and suggests a large improvement. Adding a cluster process (Matèrn or Thomas) to the model changes the parameter values by less than 4 %. We divide the region into tiles (quadrats) and compare the expected value from our Poisson process model with the observed count in each tile. Figure 7 shows the actual violent tornado count versus the expected number from the final model using 36 tiles (6 by 6 quadrat). In each tile, the actual count is the integer on the top left and the expected number is the value on the top right. The value at the bottom is the standardized residual. Negative residuals indicate where the model

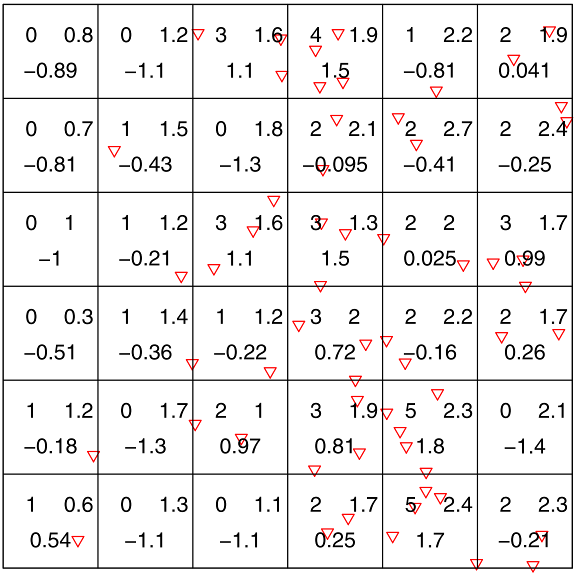


Fig. 7 Actual violent tornado count versus the expected number predicted by the model. The observed count is the number in the *upper left part of the box*. The model-predicted expected count is in the *upper right part of the box*. The standardized residual value is near the *bottom of the box*. The locations of the violent tornadoes are shown as *red triangles*

predicts a greater number of violent tornadoes than have occurred. The location of the violent tornadoes are shown as red triangles.

The Pearson χ-squared statistic, where k is the number of tiles, is computed as

$$\chi^2 = \sum \frac{(O_k - E_k)^2}{E_k}. \tag{3}$$

We use a Monte Carlo test by simulating 999 point patterns from our fitted model and computing the χ^2 statistic for each simulation. The p-value is computed as the percent of the simulated point patterns the result in a χ^2 value that is more extreme (at either tail) than the χ^2 value computed from the observations. The procedure results in a p-value of 0.53 when using a 6 by 6 quadrat suggesting the model is adequate. Similar large p-values are obtained using other quadrats.

Figure 8 shows predicted violent tornado rates per 10^4 km^2 per century. Actual occurrences are shown as points. The model appears to do a good job; showing higher rates in regions where violent tornadoes have occurred and lower rates in regions without a tornado. Figure 9 maps the observed minus model predicted violent tornado rates. The observed rates are based on a kernel smoother. Areas shown in red indicate where the model predicts more violent tornadoes than have been observed. The correlation between observed and predicted rates over this 128 by 128 grid of pixels is 0.45.

Insured loss estimates from tornadoes require an estimate of the tornado rate. The rate together with exposure information, policy conditions, and vulnerability functions for construction and automobile types can be used to simulate events to produce a probabilistic description of potential losses. Here, we use our model to generate an event set containing sixteen 62-year (992 years) simulations of violent tornado touchdowns (Fig. 10). The method simulates realizations from our fitted inhomogeneous point process model using the Metropolis–Hastings algorithm. Simulations provide

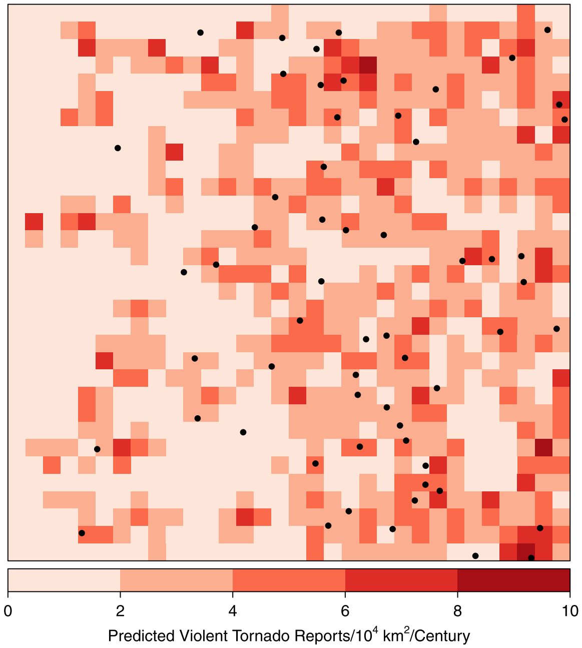


Fig. 8 Predicted violent tornado rates per 10^4 km^2/century. Location of the violent tornadoes are shown as *black dots*

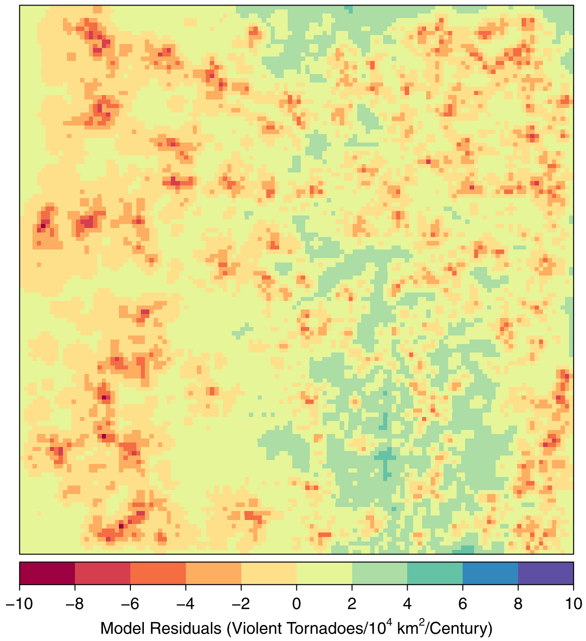


Fig. 9 Model residuals computed as the difference between the observed and model rates and expressed as the number per 10^4 km^2 per century. The observed rates are obtained using a kernel density smoother on the violent tornado points

a catalogue of synthetic touchdown locations that can be used as part of a larger catastrophe model that also includes information on track length, width, and heading.

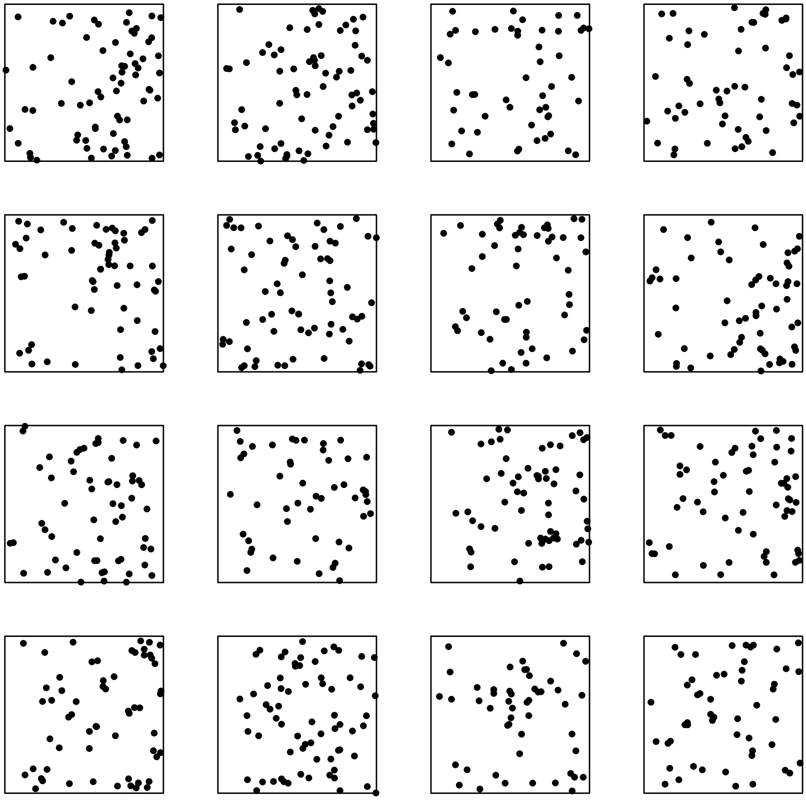


Fig. 10 Synthetic violent tornado touchdown points in sixteen independent 62-year periods

7 Summary and Conclusions

Reliable and stable estimates of tornado rates are critical for hazard assessment. The high level of tornado activity in 2008 and 2011 is a call to increase research efforts to better understand the relationship between tornadoes and climate change (Diffenbaugh et al. 2008). In this paper, a statistical point process model that uses the spatial occurrence of nonviolent tornadoes to predict the distribution of the rare, violent tornadoes during springtime across the US central Great Plains is presented. The model has a component that accounts for the population bias in the tornado record based on the distance to nearest city (Elsner et al. 2013). The model is based on modern tools for statistically analyzing spatial point data. It is used to make predictions of violent tornado rates. It is also used to simulate approximately 1000 years of touchdown points. The principal findings of the research in this paper include: the total number of F4 and F5 tornadoes over the central Great Plains is 59, which represents 2.87 % of the total number of F1 through F5 tornadoes over the 62-year period (1950–2011); the average rate of nonviolent tornadoes is 55 per 10^4 km^2 per 62 years; the average rate of violent tornadoes over the same period is 1.5 per 10^4 km^2; the correlation between nonviolent and violent rates using a 128 by 128 grid of pixels covering the study domain is 0.84; violent tornado report density peaks at 2.6 per 10^4 km^2 (62 yr) in the city to 0.7 per 10^4 km^2 in the countryside; and the risk of a violent tornado is higher by a factor of 1.5, on average, in the vicinity of less violent tornadoes after accounting for the population bias.

A model for the occurrence rate of violent tornadoes indicates that rates are lower by 10.3 (3.6, 16.5) % (95 % CI) for every 1 km increase in distance from nearest nonviolent tornado controlling for distance from nearest city. Model significance and distance-from-nearest nonviolent tornado parameter are not sensitive to population threshold or definition of violent tornado. The model is useful for generating a catalogue of touchdown points as part of a comprehensive catastrophe model. The model can be used to estimate local violent tornado rates. Regions where the estimated rates are higher than the smoothed estimates based on the historical occurrences indicate where insurance coverage might be too low. The model can be improved by including measurement error on the location estimates. Measurement error will attenuate the response of the violent tornado density to distance from nonviolent tornadoes. The model might be improved by considering the events as a marked point process where the marks are the estimated E(F) scale rating. Results from the model are strictly valid only for the study area. Evaluation of the approach for its general applicability is a future research direction. The code used in creating the analyzes and models in this paper is available at http://rpubs.com/jelsner/4205.

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