The (Student) t distribution.

Let $Z \sim N(0,1)$ and $V \sim \chi_r^2$. If $Z$ and $V$ are statistically independent, then

$$\frac{Z}{\sqrt{V/r}} \sim t_r$$

That is, the $t_r$ is defined as the ratio of a standard normal to the square root of a statistically independent $\chi^2_r$ divided by its degrees of freedom. The $t_r$ is a continuous random variable with sample space $\mathbb{R}$, and a single parameter, $r$, called the "degrees of freedom." The $t_r$ has a symmetric density function. It has mean zero for $r>1$, and variance $r/(r-2)$ for $r>2$. If we replace the assumption that $Z \sim N(0,1)$ with $Z \sim N(0,I_q)$, then each element of the $q$-vector $(V/r)^{1/2}Z \sim t_r$.

Tables for the $t_r$ distribution function are provided in the appendices of most statistics and econometrics textbooks.

Statistical Independence

Assume that $Z \sim N(0,I_n)$. Let $A$ denote an $rXn$ matrix and $B$ an $qXn$ matrix. Then $AZ \sim N(0,AA')$ and $BZ \sim N(0,BB')$. The covariance matrix of

$$\begin{bmatrix} AZ \\ BZ \end{bmatrix} \quad \text{is} \quad \begin{bmatrix} AA' & AB' \\ BA' & BB' \end{bmatrix}$$
Since the "stacked" vector is multi-normal, AZ and BZ are statistically independent if the off-diagonal covariance block BA'=0.

**Application for the t Distribution**

Assume that $Z \sim N(0, I_n)$. Let $A$ denote an $n \times n$ idempotent symmetric matrix of rank $r$, and $B$ a $q \times n$ matrix of rank $q$. We have seen that $BZ \sim N(0, BB')$ and $Z'AZ \sim \chi_i^2$. But $Z'AZ$ is a function of $AZ$. Specifically, $Z'AZ = Z'A'AZ = (AZ)'(AZ) = \gamma(AZ)$. We have seen, that if $AZ$ and $BZ$ are statistically independent, then arbitrary functions $\gamma(AZ)$ and $\eta(BZ)$ are also statistically independent. Thus, $Z'AZ$ and $BZ$ are statistically independent if $AZ$ and $BZ$ are statistically independent. Since $AZ$ and $BZ$ are multinormal, they are statistically independent if $BA'=BA=0$. Thus, a quadratic form in an idempotent symmetric matrix, $Z'AZ$, is statistically independent of a set of linear transformations, $BZ$, if $BA=0$. 