

**ECO 4400: Games and Decisions**  
**Supplemental Handout**  
Spring 2007  
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## 1 Milgrom and Roberts Limit Pricing Model

This is a game that involves two players, a monopolist and a potential entrant. It is also a game that involves two periods:

- In period 1, the monopolist gets to be a monopolist with no one competing against him. He will choose a price, denoted as  $p(1)$ .
- In period 2, the entrant gets to decide if he wants to enter and they play a Cournot quantity competition game. If not the monopolist gets to be a monopolist again. In this case, they will be choosing quantities.

The key to this game is that the entrant is not going to know exactly what the cost structure is of the monopolist he is facing. If the monopolist has a high cost, then the entrant would like to enter as he can compete with such a monopolist. If the monopolist has a low cost of producing, the entrant will not be able to compete as well and would prefer not to enter the market. What we are going to assume is that our entrant can not observe this cost structure until after they have decided and committed to enter. The idea behind this model is that a high cost monopolist might want to try to signal to the entrant that he has low cost to keep the entrant from entering. So if he is high cost, he might produce at a point that does not maximize his short term profit if it helps to discourage the entrant from coming in.

What we want to do now, is try to figure out whether or not it is worthwhile and what the strategies would look like. To do that we need some specifics:

Assume that everyone knows that the base probability that the monopolist is high cost is  $p_H = .1$  and therefore  $p_L = (1 - p_H) = .9$ .

If the monopolist is a low cost type then his cost function is  $C_L^m = 5q^m$  and if he is high cost his cost function is  $C_H^m = 7q^m$ . There is only one type of entrant and his cost function is  $C^e = 600 + 7q^e$  if he enters, 0 if he does not.

Further the demand curve faced by the industry is  $P = 16 - .01Q$

We need to find a Bayes-Nash equilibria of this game. This means finding a set of strategies that are best responses to each other as well as the beliefs that support them.

To solve the game, we start at the end and work backward. We therefore need to work out the possible profits in period 2 each firm would get if they played the quantity competition game and possible profits if the entrant decides not to enter for both possible cost structures.

Start with the simple cases. Let us assume that the entrant has not entered. In that case the monopolist will simply choose his quantity to maximize his profit, or assuming low cost

$$\max_{q^m} ((16 - .01q^m)q^m - 5q^m)$$

This is a standard monopoly problem and can be solved as we did earlier in the semester. We can take the derivative of this with respect to  $q^m$  or we can also use our standard principles rule that

the firm will select the quantity at which MR=MC. MC is easy, that is 5. MR is  $16 - .02q^m$ , so  $5 = 16 - .02q_L^m \implies q_L^m = 11/.02 = 550.0$ . Price and profit are:

$$16 - .01 * 550 = 10.5 = p_L$$

$$(16 - .01 * 550)550 - 5 * 550 = 3025.0 = \pi_L$$

We can do the exact same thing for the case in which he is high cost as the only thing in that equation we need to change is the MC, so

$$\max_{q^m} ((16 - .01q^m)q^m - 7q^m)$$

This leads to:

$$q_H^m = 450.0$$

$$p_H = 11.5$$

$$\pi_H = 2025$$

That was the easy case, now for the slightly more difficult cases. What if the entrant decides to enter? In that case the two firms have to play a quantity competition game. This takes a bit more work, but I will give you the general formula for finding the quantities in a two firm Cournot game with different costs

$$q_1 = \frac{a + c_2 - 2c_1}{3b}$$

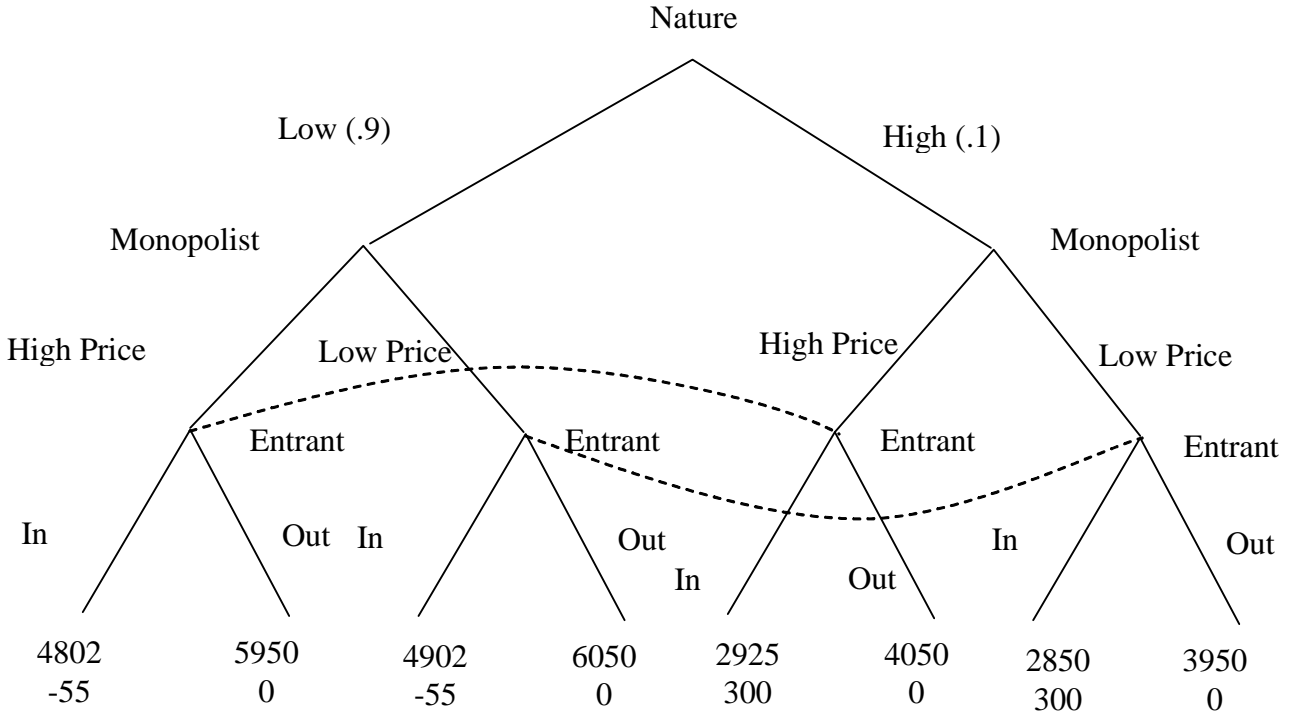
$$q_2 = \frac{a + c_1 - 2c_2}{3b}$$

This lets us construct the following table for payoffs from possible actions in period 2. The key bits are the profits.

		High cost M	Low Cost M
<b>If Entrant Enters:</b>	<b>Monop output</b>	300	433.33
	<b>Entrant Output</b>	300	233.33
	<b>Price</b>	\$10	\$9.33
	<b>Monop Profit</b>	\$900	\$1877.78
	<b>Entrant Profit</b>	\$300	-\$55.56
<b>No Entry</b>	<b>Monop Profit</b>	\$2025	\$3025
	<b>Entrant Profit</b>	0	0

To finish off our specification of payoffs we need to also calculate possible payoffs from period 1 for the monopolist. For simplicity we will assume that our monopolists can choose to either choose the price associated with the low cost monopolist's profit maximizing choice, \$9.33, or the high cost monopolist's profit maximizing price, \$10.00. We have actually already calculated the profits each type would earn pricing according to type but we have not calculated what profits they would earn if they priced as if they had the alternative cost structure. That is easy to do as we can simply use the price and quantity numbers from the table above to calculate:

$$\text{If high cost and price low then } (16 - .01 * 550)550 - 7 * 550 = 1925.0$$



If low cost and price high then  $(16 - .01 * 450)450 - 5 * 450 = 2925.0$ .

We can then represent all of these decisions in an extensive form as below. Note that nature moves first and assigns types to the Monopolist. The Monopolist is allowed to choose between the low price and high price, knowing his type, and then the entrant observes the pricing decision, not the type and chooses whether or not to enter. The payoffs can be calculated from the numbers worked out above.

Once we have the game fully specified we can begin looking for an equilibrium. There are two types we could find, either a separating equilibrium or a pooling equilibrium. The difference between the two in all Bayesian games is that in a separating equilibrium the player who can be of multiple types will engage in different actions depending on the type (so the types will “separate” out) while in a pooling equilibrium all the different types of agents will choose the same action (so they will “pool” on the same action). In principal either or both sort of equilibrium could exist in this game. We have to see which we can construct.

### 1.1 Separating Equilibrium

Let us start by looking for a separating equilibrium. We should start by being clear about the structure of what we have to fill out. To have a fully specified equilibrium we can always consult the definition and find out what is needed. In this case, we are looking for a Bayes Nash equilibrium<sup>1</sup> which requires finding a set of strategies that are mutual best responses given a set of sensibly derived beliefs. So we need to have choices defined for our monopolist contingent upon type and choices for the entrant conditional on price observations along with the supporting beliefs.

<sup>1</sup>Technically we will be solving for a weak Perfect Bayes Nash Equilibrium but I won't go into the distinction.

Since we are looking for a separating equilibrium, we know that there are only two possible strategies for our monopolist; either both types of monopolists price according to type or they price against type. While either could be possible, the more reasonable approach is to try to solve for the equilibrium in which the high cost monopolist chooses the high price and the low cost monopolist chooses the low price.

Given this proposal for the monopolist, the next step is to try to determine what beliefs the entrant should have after observing the price choice of the monopolist and then given those beliefs what choices he should make as a best response. The key point is that we need to derive the beliefs such that they are consistent with the strategy of the monopolist. In the separating case, this makes deriving the beliefs quite straightforward.

To find the belief of the entrant regarding the probability that the monopolist they are facing is high cost given that he has observed a high price we can technically apply something call Bayes' Rule. What we want to do is calculate the probability that the monopolist is high cost given that I have observed them choosing a high price. This is done by reading from the proposed strategy by the monopolist the probability that a high cost monopolist chooses a high price (which is 1), multiple that by the raw probability that a monopolist is high cost (which is .1) and then divide this by the percentage of all monopolists who choose the high price (this is the percentage of all high cost monopolists who price high times the percentage of monopolists who are high cost,  $1 * .1$ , plus the percentage of all low cost monopolists which price high times the percentage of monopolists who are low cost ( $0 * .9$ ). So in the end we get  $.1 * 1 / (.1 * 1 + 0 * .9) = 1$ . The intuitive way to get this is to just think through what is the percentage of monopolists choosing the high price who are high cost? According to the equilibrium strategy only high cost monopolists choose the high price so the probability that a monopolist is high cost given that the player has been observed choosing high price is equal to 1. We can similarly find the proper belief for the likelihood that a monopolist is high cost given that the monopolist has been observed choosing the low price. Since the equilibrium strategy tells us that no high cost firms choose the low price, this belief is 0. Notice that in a separating equilibrium, the monopolist chooses different actions depending on their type and this allows the entrant to perfectly infer the type of the monopolist from their action.

The next thing we need to do is figure out what the entrant wants to do given his beliefs. If the entrant observes someone choosing a high price, the entrant assumes the first player was high cost and compares getting 300 for entering to 0 for staying out and obviously prefers to enter. If the entrant observes someone choosing the low price then the entrant believes the monopolist is low cost with certainty and compares entering and getting -55 to staying out and getting 0. The choice is obvious. This leads to a proposed strategy and accompanying set of beliefs for our equilibrium as follows:

$$\begin{aligned} \text{Monopolist} & \begin{cases} \text{Price } H & \text{if } \textit{High Cost} \\ \text{Price } L & \text{if } \textit{Low Cost} \end{cases} \\ \text{Entrant} & \begin{cases} \textit{Stay Out} & \text{if } \textit{Observe Low Price} \\ \textit{Enter} & \text{if } \textit{Observe High Price} \end{cases} \\ \text{with beliefs} & \begin{cases} b_H = 0 & \text{if } \textit{Observe Low Price} \\ b_H = 1 & \text{if } \textit{Observe High Price} \end{cases} \end{aligned}$$

This does not yet mean we have found an equilibrium. We have constructed this set of strategies so that the entrant is best responding to the monopolist given the set of beliefs but for our monopolist we just proposed his strategy as a possibility. We now need to verify if the monopolist is best responding to what the entrant is doing. It is easy to see that the low cost monopolist is

choosing the best option, but the high cost monopolist has a choice between pricing high, seeing the entrant enter and receiving 2925 or pricing low, seeing the entrant stay out, and earning a total of 3950. The high cost monopolist would clearly prefer to deviate from the proposed strategy.

You will also find that the alternative strategy for our monopolists of pricing against type in the first round can not be supported as an equilibrium either. So in this version of the game, we can not construct a separating equilibrium.

## 1.2 Pooling Equilibrium

Let us now look for a pooling equilibrium instead. If I know I am looking for a pooling equilibrium, the key thing I know is that it means that the person who could be of multiple types must pool together and do the same thing regardless of type. So one of our types of incumbents is going to have to price against type in period 1. Who might do it? If you are a low cost guy, would you ever want to price as if you were high cost? That seems like it would only make the entrant more likely to enter. What if you are high cost, why might you want to price as if you were low cost? To keep the entrant from entering as we saw above. So let us try that. We will propose that the first part of our equilibrium is

$$\text{Monopolist} \begin{cases} \text{Price } L & \text{if } \textit{High Cost} \\ \text{Price } L & \text{if } \textit{Low Cost} \end{cases}$$

To see if this will work, I need to see what our entrant's best response would be to this. The first step is in deriving the beliefs of the entrant. Let us say that the entrant sees the monopolist price Low, what should be their most reasonable belief about the probability that the monopolist is high cost? Again we have to do something similar to what we did before by deriving this belief to be consistent with the proposed strategy of the monopolist. Now, all monopolists choose to price Low so this actually conveys no useful information to the entrant about the type of monopolist. You can either go through the formula explained before or just realize that if 10% of all monopolists are high cost and all monopolists price low then 10% of monopolists who price low must be high cost. So  $b_H = .1$  if the entrant observes a low price.

Given this belief the entrant can calculate their expected utility of entering and compare this to their expected utility of staying out:

$$E\pi(\textit{enter}) = .1 * 300 + .9 * -55.56 = -20.004$$

$$E\pi(\textit{exit}) = .1 * 0 + .9 * 0 = 0$$

So they should stay out if they observe a low price in period 1.

The more difficult part is to derive the proper beliefs and choice if the entrant observes a high price in period 1. The problem is that our prior formula for deriving beliefs breaks down because according to the equilibrium strategy there are no monopolists who should be choosing the high price. Consequently it is impossible to derive a belief here that is consistent with the proposed strategy. Technically our equilibrium definition allows us to choose beliefs any way we want to in this situation, but there are a few approaches to doing it that will work for us. One approach involves logically thinking through the fact that any monopolist choosing the high price is one who made a mistake relative to the equilibrium strategy. Which type of monopolist is most likely to make that mistake? High cost type. So one reasonable belief is that  $b_H = 1$  given an observation of a high price. We'll use that one below. The other two most likely alternatives involve simply using the original base rates (essentially assumes either monopolist is equally likely to have made a mistake) or trying to set the belief in such a way that the equilibrium will hold.

If we go with the first approach and set  $b_H = 1$  given an observation of a high price then we can clearly see that the entrant would prefer to enter. This leads to a full proposal for our equilibrium of

$$\text{Monopolist} \begin{cases} \text{Price } L & \text{if } \textit{High Cost} \\ \text{Price } L & \text{if } \textit{Low Cost} \end{cases}$$

$$\text{Entrant} \begin{cases} \textit{Stay Out} & \text{if } \text{Observe } \textit{Low Price} \\ \textit{Enter} & \text{if } \text{Observe } \textit{High Price} \end{cases}$$

$$\text{with beliefs} \begin{cases} b_H = .1 & \text{if } \text{Observe } \textit{Low Price} \\ b_H = 1 & \text{if } \text{Observe } \textit{High Price} \end{cases}$$

We still do not know for certain that this is an equilibrium because the monopolist's strategy was simply proposed as a hypothetical possibility. We have to determine whether the monopolist's proposed strategy turns out to be a best response for the strategy we derived for the entrant. It is quite easy to verify that it will be. So this is a valid pooling equilibrium of our limit price game.

Just as a note, I'll mention that it is possible to derive a separating equilibrium of the general limit pricing game defined above. It will involve the low cost monopolist pricing below their profit maximizing level in the first period to separate away from the high costs monopolist. It is not clear what the low cost monopolist would gain from doing this, but we could technically solve for this equilibrium. It would involve using a continuous price space for the first period rather than restricting it to just the two options left in for the extensive form game represented above.