RELATIVE SATURATION OF INTERVAL AND SET CLASSES: A NEW MODEL FOR UNDERSTANDING PCSET COMPLEMENTATION AND RESEMBLANCE

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An Introduction to Saturation Vectors and Associated Similarity Indices

In the well-established field of pitch-class set (pcset) theory, scholars have expressed rather different opinions regarding what exactly should constitute an equivalence and what sorts of materials and methodologies should be used in determining similarity. As Morris (1982) has amply demonstrated, there are essentially two ways that theorists have determined equivalence among pcsets: either operationally or through a comparison of common elements. The first method relies upon a canon of operators, such as transposition, inversion, or multiplication by 5 or 7;

when two pcsets can be related using a canonical operator, they are said to be equivalent. Vector equivalence, on the other hand, is based upon the comparison of two tallies of interval- or subset-classes (e.g., the interval-class vector); when two pcsets share a vector class, they too can be declared equivalent.[1]

Pcset resemblance has also been evaluated using these two types of data, though methods based upon vector relationships predominate. That a large number of similarity relations have arisen, reflecting a variety of methodologies, seems extremely healthy for our discipline, and I find it enormously helpful to have multiple means for comparing pcsets (or any musical entities).[2] Although there are many features that distinguish similarity indices such as Morris's ASIM, Rahn's ATMEMB, Lewin's REL, Isaacson's IcVSIM and Castrén's RECREL, all such methods for determining pcset similarity have examined raw counts of interval- or subset-classes and, through their mechanisms, attempted to form cardinality-neutral comparisons. My aim here is not to evaluate these indices, but to suggest that one way to facilitate such cardinality-neutral comparisons is through a contextualization of the vector data itself before any external comparisons are initiated.

Before discussing vectors, however, pcset embedding (particularly as defined by David Lewin) will be reviewed, and I will suggest some elaborations on Lewin's tools that take account of what is minimally and maximally possible in a given cardinality. This relative value will be called the "degree of saturation" of element or subset x in set X. Strings of saturation values will form "saturation vectors" that compare the quantity and types of embedded interval- or subset-classes to what is possible in any pcset of a given cardinality. Saturation vectors, in turn, will be used to illustrate some interesting relationships among complementary pcsets and, more generally, they will serve as data for comparing pcsets with a similarity index.

Relative Abstract Pcset Inclusion

As a point of departure, it will be useful to create a function that formally defines abstract pcset inclusion. This function will be called AS (for "abstract subset"). Given two set classes (scs) $/X/$ and $/Y/$,[3] where $/X/$ has the same number or fewer elements than $/Y/$ ($\#/X/ \leq \#/Y/$),[4] AS($/X/$, $/Y/$) is a Boolean function (relation) which returns "true" if at least one X of $/X/$ is embedded in some Y of $/Y/$.[5] AS($/X/$, $/Y/$) returns "false" if no form of $/X/$ is embedded in some Y of $/Y/$. If AS($/X/$, $/Y/$) = true, then $/X/$ is said to be an abstract subset of $/Y/$.[6] Consider, for example, the following three set classes: $/A/$ = [014], $/B/$ = [0145], $/C/$ = [0167]. AS($/A/$, $/B/$) = true; AS($/A/$, $/C/$) = false.

David Lewin's EMB($/X/$, $/Y/$) function provides more specific information than does my AS(/X/, /Y/).[7] EMB(/X/, /Y/) returns the number of distinct forms of /X/ that are embedded in some Y of /Y/. If, for example, /X/ = sc(3-1) [012] and Y = {1,2,3,6,7,8,a}, EMB(/X/, /Y/) = 2 because both {1,2,3} and {6,7,8} are forms of [012] and are included in {1,2,3,6,7,8,a}. Given that {1,2,3} (or any form of [012]) is an inversionally symmetrical set, it can map onto itself through two distinct operations: the identity operation (T_0{1,2,3} = {1,2,3}); and under transposition and inversion (T_4I{1,2,3} = {1,2,3}). These two transformations yield the same pcset and will be considered a single *distinct* form of 3-1.

Abstract inclusion vectors, including n-class vectors,[8] such as the #2 subset-class vector (which is equivalent in appearance to the ICV) can be derived by performing EMB(/X/, /Y/) for each distinct /X/ where $\#X = n$.[9] When $\#X = 2$, EMB(/X/, /Y/) returns an argument of the #2 subset-class vector of /Y/ (ICV(Y)). Formally, each argument (i) in the 2CV can be defined as follows: $2CV(X)_i$ = EMB(i, /X/).[10]

Later in this article, I will introduce a new inclusion function, SATEMB(/X/, /Y/), which returns two arguments that reflect a comparison between EMB(/X/, /Y/) and the largest and smallest EMB(/X/, /Y/) values for all sets of #Y. We will call SATEMB's output the degree to which /Y/ is saturated with /X/.[11] Just as abstract inclusion (n-class) vectors such as the interval-class vector can be derived by performing EMB(/X/, /Y/) for each distinct /X/ where $\#X = n$, we will similarly create n-class "saturation vectors" by concatenating SATEMB(/X/, /Y/) values for each /X/ where $\#X = n$. Wherever the places in a vector V(X) show the number of scs A_1 through A_z that are embedded in pcset X, a saturation vector SATV(X) can be constructed to display how the quantity of subsets A_1 through A_z fall into the range of what is possible given any pcset of #X.[12] Several n-class (largely interval-class) saturation vectors will be introduced.

One can construct saturation vectors that measure the degree to which different collections of scs ($/X/_1 \ldots /X/_z$) are maximally or minimally embedded in pcset Y (or sc /Y/) in a variety of musical spaces, and under different means of defining equivalence. An sc collection might contain all scs of a particular cardinality (e.g., interval classes, trichord classes) or might (or might not) share some other property (e.g., all transpositionally-symmetrical scs). Certain scs might even be weighted in the vector. Initially, the principal focus will be on interval-class vectors, but it should be evident that the same saturation functions used to determine relative ic embedding can also determine relative embedding of any other size subset class. Additionally, while T_x/T_xI-based n-class vectors (nCVs) will form the basis for the vectors demonstrated herein, one could certainly achieve similar goals using any other well-defined standard for equivalence.[13]

Minimal and Maximal Saturation of Interval Classes

The next section of this article will introduce a variation on the interval-class vector that compares the arguments of an ICV with the minimum and maximum such values found in *any* pcset of the same cardinality. Before discussing this vector, it will be useful to examine how one calculates the minimal and maximal amounts of each ic in any pcset of a particular cardinality. One could, of course, simply examine the ICVs of all sets of a particular cardinality and, by inspection, keep track of the values at each extreme. But one can derive this information more directly, and with a greater appreciation of the inherent boundaries of pitch-class space, through an examination of the properties of interval cycles. The relationship between the following systematic discussion of interval cycles in pc space and minimal and maximal saturation of interval classes shall become apparent as we progress.

In standard 12-pc space, where inverted set forms are considered equivalent, there are only six distinct interval cycles. These regular cyclic forms, listed in Figure 1 below, have been discussed by other authors.[14] I have previously defined an *i*-cycle (where *i* represents any interval class in pc space) as "a closed and finite ordered collection of pcs where one element maps onto the next (and the last onto the first) under transposition at a constant interval *i*."[15] The periodicity (p) of an *i*-cycle is its length, or the number of distinct elements before a pc is repeated (e.g., the periodicity of a 3-cycle is 4).[16] It therefore follows that the number of distinct *i*-cycles for any ic *i* (I will call this m) can be derived by dividing twelve by p (or $m = \frac{12}{p}$).

Elsewhere I have formally defined an *i*-set X as a pcset that is 1) either a completed *i*-cycle or the union of two or more completed *i*-cycles (of the same type); 2) a contiguous *i*-cycle fragment with no extraneous notes; or 3) the combination of any number of completed *i*-cycles and at most a single contiguous fragment.[17] The group of *i*-sets also forms Tore Ericksson's maxpoint series.[18] Since all instances of ic *i* are found within the *i*-cycles, it stands to reason that any *i*-set X contains as many instances of

1 cycle:	(C, C♯, D, D♯, E, F, F♯, G, G♯, A, A♯, B)	(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b)
2 cycles:	(C, D, E, F♯, G♯, A♯), (D♭, E♭, F , G, A, B)	(0, 2, 4, 6, 8, a), (1, 3, 5, 7, 9, b)
3 cycles:	(C, D♯, F♯, A), (C♯, E, G, A♯), (D, F, G♯, B)	(0, 3, 6, 9), (1, 4, 7, a), (2, 5, 8, b)
4 cycles:	(C, E, G♯), (C♯, F, A), (D, F♯, A♯), (E♭, G, B)	(0, 4, 8), (1, 5, 9), (2, 6, a), (3, 7, b)
5 cycle:	(C, F, B♭, E ♭, A ♭, D ♭, F♯, B, E, A, D, G)	(0, 5, a, 3, 8, 1, 6, b, 4, 9, 2, 7)
6 cycles:	(C, F♯), (D♭, G), (D, G♯), (E♭, A), (E, B♭), (F, B)	(0, 6), (1, 7), (2, 8), (3, 9), (4, a), (5, b)

Figure 1. Interval-class cycles in twelve-pc space.

ic i as possible for a set of $\#X$ (i.e., it is *maximally saturated* with ic i).[19]

The maximum quantity of ic i in any pcset of cardinality c shall be called max(c, i). The algorithm for deriving this value can now be formally expressed (for ease of reading, it will also be *less* formally expressed below). Since we are working in the realm of integers, all division will be integer division, which truncates any remainder (e.g., $\frac{7}{2} = 3$). "Mod" is a binary operation which returns the remainder of integer division (e.g., 7 mod 2 = 1). Variables are defined as follows:

Let c = the cardinality of pcset X.

Let p = the periodicity of interval cycle i ($p = \frac{12}{m}$).
Let m = the number of distinct cycles of i in U ($m = \frac{12}{p}$).
Let s be a temporary variable that we shall use *en route* to deriving min(c, i) and max(c, i).

Derivation of max(c, i):[20]

$$(c \bmod p = 0) \Rightarrow s = \frac{c}{p} \cdot p$$
$$\sim (c \bmod p = 0) \Rightarrow s = \left(\frac{c}{p} \cdot p\right) + (c \bmod p) - 1$$
$$(i = 6) \Rightarrow s = \frac{s}{2}$$
$$\max(c,i) = s$$

The first statement (which might be read: if the periodicity (p) of an i-cycle divides evenly into c, then s equals the product of p and the integer quotient of c and p) is a test of the first i-set condition. If the left side of the statement is true, then the completed i-cycle or combination of completed i-cycles will yield the maximum number of ic i in any pcset of cardinality c. That number will equal the product of the i-cycle periodicity and the number of completed i-cycles that fit into c (this is represented by $\frac{c}{p}$), except in cases where $i = 6$. In the case of the tritone, s must be divided in half to account for the fact that completed 6-cycles yield only a single ic6. This "fix" is provided on the third line of the above equation. The second line of the equation describes the second and third I-set conditions where completed i-cycles cannot be evenly partitioned into a pcset of c. In such cases, we take the number of ic is from any potentially-completed cycles $\left(\frac{c}{p} \cdot p\right)$ and add 'to it the number of ic is found in a fragment the size of the remainder of $\frac{c}{p}$. The number of ic is in any i-cycle fragment equals its cardinality minus 1 (since it is not a closed cycle, the "wraparound" interval must be subtracted).

The minimal values of interval-class i in any pcset of cardinality c (denoted min(c, i)) can also be described formally using these parameters. Min(c, i) is not always 0 but is variable, dependent on the periodicity of the i-cycle and the cardinality of the pcset. Before presenting the entire equation for min(c, i), let us state the conditions where min(c, i) = 0:

$$\left(c \le \frac{p}{2} \cdot m\right) \Rightarrow \min(c,i) = 0$$

Earlier, I remarked that "all instances of ic i are found within the i-cycles." By extension, there are no i-cyclic non-adjacencies or elements in different i-cycles that are separated by ic i. This corollary motivates the first half of the above formula. If we remove every other element of a particular i-cycle (thus removing all instances of the generating ic), the length of what remains is, quite obviously, $\frac{p}{2}$. We then multiply the length of our half-cycle by the number of i-cycles (m) to determine the maximum cardinality with no embedded ic is. The values of $\frac{p}{2} \cdot m$ for all p and m are given in Figure 2.

With this condition in place, we can now compose an algorithm to determine min(c, i) values for all c and i:

Derivation of min(c, i):

$$s = c - \frac{p}{2} \cdot m$$

$$\left(\frac{p}{2} > 1\right) \vee \left(\left(\left(\frac{p}{2} \cdot m\right) \cdot 2\right) < c\right) \Rightarrow s = (s \cdot 2) - \left(12 - \frac{p}{2} \cdot m\right) \cdot 2$$

$$(s < 0) \Rightarrow s = 0$$

$$\min(c,i) = s$$

In most cases where $c = \frac{p}{2} \cdot m$, a pcset X which has the minimum number of i will be optimally fragmented with respect to the i-cycle. This means that it is impossible to add a single pc to pcset X without adding two ic is. {0, 2, 4, 6, 8, a} might be regarded as an optimally fragmented 1-cycle. Any (non-redundant) pc that we add to that pcset will add two

i	m	p	$\frac{p}{2} \cdot m$
1	1	12	6
2	2	6	6
3	3	4	6
4	4	3	4
5	1	12	6
6	6	2	6

Figure 2. Values of $\frac{p}{2} \cdot m$ for all m and p.

ic1s yet the resultant seven-note pcset will still include the fewest ic1s in any septachord. Therefore, min(c, i) is incremented by 2 for every c higher than $\frac{p}{2} \cdot m$.

The only exceptions arise when deriving the minimal amounts of ic6 and ic4 where min(c, i) $\neq$ 0. Because it is impossible to add one note to a set and consequently add two ic6s, we only increment min(c, 6) by 1 for every c higher than $\frac{p}{2} \cdot m$. The case of min(c, 4) is more complex. Since the periodicity of completed 4-cycles is 3, it is impossible to have two non-adjacent elements. Thus, a pcset can only exclude ic4 when all its elements are members of different 4-cycles. Unlike maximally fragmented 1-, 2-, 3-, and 5-cycles, any pc added to a maximally fragmented 4-cycle will add only one ic4 to the set's ic content. This condition covers five- through eight-note pcsets. An eight-note pcset that minimally includes ic4 will have two pcs from each of the four 4-cycles. Any note added to it will complete a 4-cycle and therefore add two more ic4s. Thus, min(c, i) is incremented by two when $c = 9 \ldots 12$. The special conditions to handle ic4 and ic6 are found in the second line of the above formula.

In some cases $c - \frac{p}{2} \cdot m$ will yield a negative number. Because it is impossible for members of any pcset X to be separated by a negative number of i, the third line of the formula maps all negative values to 0.[21] Figure 3 lists all min(c, i) and max(c, i) values. For example, min(6,4)—the minimal amount of ic4 in any pcset of #6—carries the value 2 and max(6, 4) carries the value 6. Such values will be used frequently in this article.

More than one i-set (for a given i) sometimes exists within a particular cardinality. This is, in part, because we have not specified an intervallic distance that must separate the multiple completed cycles (under condition 2) or completed cycle(s) combined with a cyclic fragment (under condition 3). The following hexachords are all 6-cycle sets:

{0, 1, 2, 6, 7, 8}
{0, 1, 3, 6, 7, 9}
{0, 2, 4, 6, 8, a}

All three contain three instances of ic6, but in other respects they possess different properties. The first and third are all-combinatorial sets; the second is not. The third pcset is also a completed 2-cycle and the combination of two completed 4-cycles. Thus, we see that it is possible for a pcset to satisfy a condition of cyclic set membership for more than a single interval class. Such pcsets (and their scs) will be called "multiple-cyclic sets." There are only three varieties of multiple-cyclic sets: 1) those which are products of all six i-cycles; 2) those which are products of i-cycles 2,

Cardinality	Minimum ic counts						Maximum ic counts					
	1	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**	**5**	**6**
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	1	1	1	1	1
3	0	0	0	0	0	0	2	2	2	3	2	1
4	0	0	0	0	0	0	3	3	4	3	3	2
5	0	0	0	1	0	0	4	4	4	4	4	2
6	0	0	0	2	0	0	5	6	5	6	5	3
7	2	2	2	3	2	1	6	6	6	6	6	3
8	4	4	4	4	4	2	7	7	8	7	7	4
9	6	6	6	6	6	3	8	8	8	9	8	4
10	8	8	8	8	8	4	9	9	9	9	9	5
11	10	10	10	10	10	5	10	10	10	10	10	5
12	12	12	12	12	12	6	12	12	12	12	12	6

Figure 3. Min(c, i) and max(c, i) values for all *c*ardinalities (rows) and *i*nterval classes (columns).

4, and 6; and 3) those which are products of *i*-cycles 3 and 6. All multiple-cyclic sets are therefore 6-cycle sets (the converse is obviously not true).

Figure 4 lists all *i*-sets in 12-pc space. The first column provides the cardinality of the cyclic set class, the second column lists the Forte number for each set class, the third column lists the prime form of each set class (under T_n/T_nI equivalence), the fourth column, labeled "type," states the condition under which each pcset qualifies as a cyclic set ("C" = completed cycle or concatenation of completed cycles; "F" = cyclic fragment; "CF" = one or more completed cycles plus one cyclic fragment), the fifth column identifies mutliple-cyclic scs ("1" indicates a cyclic set for all interval-class cycles; "2" indicates an ic2, ic4, and ic6 cyclic set; and "3" indicates an ic3 and ic6 cyclic set), and the remaining columns show the interval-class content (interval-class vector).[22]

It has already been demonstrated that every *i*-set *X* embeds the maximal number of ic *i* in any pcset of #*X*. An interesting by-product of this maximal saturation is that *most* cyclic sets also include the minimal amount of some other ic or ics (that is to say maximal *i*-cycle adjacencies usually necessitate that a different *i*-cycle be maximally fragmented). The italicized "most" becomes a more definitive "all" if a single condition is added (I will call this the *maximal cyclic fragmentation condition*): an *i*-set will not only feature maximal values for ic *i*, but also minimal values for some other ic *j* if, after one *i*-cycle has been completed in the generation of the pcset, a new cycle is begun at T_1, T_{11} (T_{-1}), T_5, or T_7 (T_{-5}) of some pc in the completed cycle. All 1-, 2-, 3-, and 5-cycle sets

Ic1 cyclic set classes			Type	MC	**ic1**	ic2	ic3	ic4	ic5	ic6
#0	0-1	[ ]	F	1	**0**	0	0	0	0	0
#1	1-1	[0]	F	1	**0**	0	0	0	0	0
#2	2-1	[01]	F		**1**	0	0	0	0	0
#3	3-1	[012]	F		**2**	1	0	0	0	0
#4	4-1	[0123]	F		**3**	2	1	0	0	0
#5	5-1	[01234]	F		**4**	3	2	1	0	0
#6	6-1	[012345]	F		**5**	4	3	2	1	0
#7	7-1	[0123456]	F		**6**	5	4	3	2	1
#8	8-1	[01234567]	F		**7**	6	5	4	4	2
#9	9-1	[012345678]	F		**8**	7	6	6	6	3
#10	10-1	[0123456789]	F		**9**	8	7	8	8	4
#11	11-1	[0123456789a]	F	1	**10**	10	10	10	10	5
#12	12-1	[0123456789ab]	**C**	1	**12**	12	12	12	12	6

Ic2 cyclic set classes			Type	MC	ic1	**ic2**	ic3	ic4	ic5	ic6
#0	0-1	[ ]	F	1	0	**0**	0	0	0	0
#1	1-1	[0]	F	1	0	**0**	0	0	0	0
#2	2-2	[02]	F		0	**1**	0	0	0	0
#3	3-6	[024]	F		0	**2**	0	1	0	0
#4	4-21	[0246]	F		0	**3**	0	2	0	1
#5	5-33	[02468]	F	2	0	**4**	0	4	0	2
#6	6-35	[02468a]	**C**	2	0	**6**	0	6	0	3
#7	7-33	[012468a]	CF	2	2	**6**	2	6	2	3
#8	8-21	[0123468a]	CF		4	**7**	4	6	4	3
#9	9-6	[01234568a]	CF		6	**8**	6	7	6	3
#10	10-5	[012345678a]	CF		8	**9**	8	8	8	4
#11	11-1	[0123456789a]	CF	1	10	**10**	10	10	10	5
#12	12-1	[0123456789ab]	**C**	1	12	**12**	12	12	12	6

Ic3 cyclic set classes			Type	MC	ic1	ic2	**ic3**	ic4	ic5	ic6
#0	0-1	[ ]	F	1	0	0	**0**	0	0	0
#1	1-1	[0]	F	1	0	0	**0**	0	0	0
#2	2-3	[03]	F		0	0	**1**	0	0	0
#3	3-10	[036]	F	3	0	0	**2**	0	0	1
#4	4-28	[0369]	**C**	3	0	0	**4**	0	0	2
#5	5-31	[01369]	CF	3	1	1	**4**	1	1	2
#6	6-27	[013469]	CF		2	2	**5**	2	2	2
#7	7-31	[0134679]	CF	3	3	3	**6**	3	3	3
#8	8-28	[0134679a]	**C**	3	4	4	**8**	4	4	4
#9	9-10	[01234679a]	CF	3	6	6	**8**	6	6	4
#10	10-3	[012345679a]	CF		8	8	**9**	8	8	4

Figure 4. Cyclic sets in 12 pc space

			Type	MC	ic1	ic2	ic3	ic4	ic5	ic6
#11	11-1	[0123456789a]	CF	1	10	10	**10**	10	10	5
#12	12-1	[0123456789ab]	**C**	1	12	12	**12**	12	12	6
Ic4 cyclic set classes			Type	MC	ic1	ic2	ic3	**ic4**	ic5	ic6
#0	0-1	[ ]	F	1	0	0	0	**0**	0	0
#1	1-1	[0]	F	1	0	0	0	**0**	0	0
#2	2-4	[04]	F		0	0	0	**1**	0	0
#3	3-12	[048]	**C**		0	0	0	**3**	0	0
#4	4-19	[0148]	CF		1	0	1	**3**	1	0
	4-24	[0248]	CF		0	2	0	**3**	0	1
#5	5-21	[01458]	CF		2	0	2	**4**	2	0
	5-33	[02468]	CF	2	0	4	0	**4**	0	2
#6	6-20	[014589]	**C**		3	0	3	**6**	3	0
	6-35	[02468a]	**C**	2	0	6	0	**6**	0	3
#7	7-21	[0124589]	CF		4	2	4	**6**	4	1
	7-33	[012468a]	CF	2	2	6	2	**6**	2	3
#8	8-19	[01245689]	CF		5	4	5	**7**	5	2
	8-24	[0124568a]	CF		4	6	4	**7**	4	3
#9	9-12	[01245689a]	**C**		6	6	6	**9**	6	3
#10	10-4	[012345689a]	CF		8	8	8	**9**	8	4
#11	11-1	[0123456789a]	CF	1	10	10	10	**10**	10	5
#12	12-1	[0123456789ab]	**C**	1	12	12	12	**12**	12	6
Ic5 cyclic set classes			Type	MC	ic1	ic2	ic3	ic4	**ic5**	ic6
#0	0-1	[ ]	F	1	0	0	0	0	**0**	0
#1	1-1	[0]	F	1	0	0	0	0	**0**	0
#2	2-5	[05]	F		0	0	0	0	**1**	0
#3	3-9	[027]	F		0	1	0	0	**2**	0
#4	4-23	[0257]	F		0	2	1	0	**3**	0
#5	5-35	[02479]	F		0	3	2	1	**4**	0
#6	6-32	[024579]	F		1	4	3	2	**5**	0
#7	7-35	[013568a]	F		2	5	4	3	**6**	1
#8	8-23	[0123578a]	F		4	6	5	4	**7**	2
#9	9-9	[01235678a]	F		6	7	6	6	**8**	3
#10	10-5	[012345789a]	F		8	8	8	8	**9**	4
#11	11-1	[0123456789a]	F	1	10	10	10	10	**10**	5
#12	12-1	[0123456789ab]	**C**	1	12	12	12	12	**12**	6
Ic6 cyclic set classes			Type	MC	ic1	ic2	ic3	ic4	ic5	**ic6**
#0	0-1	[ ]	F	1	0	0	0	0	0	**0**
#1	1-1	[0]	F	1	0	0	0	0	0	**0**
#2	2-6	[06]	**C**		0	0	0	0	0	**1**

Figure 4. (continued)

#3	3-5	[016]	CF		1	0	0	0	1	**1**
	3-8	[026]	CF		0	1	0	1	0	**1**
	3-10	[036]	CF	3	0	0	2	0	0	**1**
#4	4-9	[0167]	**C**		2	0	0	0	2	**2**
	4-25	[0268]	**C**		0	2	0	2	0	**2**
	4-28	[0369]	**C**	3	0	0	4	0	0	**2**
#5	5-7	[01267]	CF		3	1	0	1	3	**2**
	5-19	[01367]	CF		2	1	2	1	2	**2**
	5-15	[01268]	CF		2	2	0	2	2	**2**
	5-28	[02368]	CF		1	2	2	2	1	**2**
	5-33	[02468]	CF	2	0	4	0	4	0	**2**
	5-31	[01369]	CF	3	1	1	4	1	1	**2**
#6	6-7	[012678]	**C**		4	2	0	2	4	**3**
	6-30	[013679]	**C**		2	2	4	2	2	**3**
	6-35	[02468a]	**C**	2	0	6	0	6	0	**3**
#7	7-7	[0123678]	CF		5	3	2	3	5	**3**
	7-15	[0124678]	CF		4	4	2	4	4	**3**
	7-19	[0123679]	CF		4	3	4	3	4	**3**
	7-31	[0134679]	CF	3	3	3	6	3	3	**3**
	7-28	[0135679]	CF		3	4	4	4	3	**3**
	7-33	[012468a]	CF	2	2	6	2	6	2	**3**
#8	8-9	[01236789]	**C**		6	4	4	4	6	**4**
	8-25	[0124678a]	**C**		4	6	4	6	4	**4**
	8-28	[0134679a]	**C**	3	4	4	8	4	4	**4**
#9	9-5	[012346789]	CF		7	6	6	6	7	**4**
	9-8	[01234678a]	CF		6	7	6	7	6	**4**
	9-10	[01234679a]	CF	3	6	6	8	6	6	**4**
#10	10-6	[012346789a]	**C**		8	8	8	8	8	**5**
#11	11-1	[0123456789a]	CF	1	10	10	10	10	10	**5**
#12	12-1	[0123456789ab]	**C**	1	12	12	12	12	12	**6**

Figure 4. (continued)

(those *i*-cycles where $\frac{p}{2} > 1$) will meet this condition. In the case of the 1- and 5-cycles, completion is only attained with the entire aggregate of pcs; in the case of 2- and 3-cycles, there are no pcs that do not lie within a semitone of some element of a completed 2- or 3-cycle. In effect, this condition sifts out type 2 and type 3 multiple-cyclic sets from the group of 4- and 6-cycle sets, assigning them sole status as either 2- or 3-cycle sets.

The above condition is true of all *i*-sets, regardless of their cardinality (e.g., any 2-cycle pcset *X* will have the minimal amount of ic1, ic3, and ic5 for any pcset of #*X*). With the maximal cyclic fragmentation condition in place, we can now chart those cases in which the saturation of

INTERVAL CLASSES

i-SETS

	1	2	3	4	5	6
1-sets	Max			Min		Min
2-sets	Min	Max	Min	Max	Min	Max
3-sets			Max	Min		Max
4-sets		(Min)		Max		(Min)
5-sets				Min	Max	Min
6-sets			(Min)	(Min)		Max

Figure 5. Minimal and maximal ic content generated in all cardinalities by each *i*-set (rows = *i*-set type; columns = interval class levels)

some ic *i* is inversely proportional to the saturation of another ic *j*. Figure 5 depicts these relationships. Where, for example, "min" is found in a location in the table, all *i*-sets (where *i* = row) will generate the min(c, j) amount of ic *j* (where *j* = column). Places where "min" is in parentheses are true only when the maximal cyclic fragmentation condition is satisfied. Similarly, where "max" appears in the table, all *i*-sets will generate the max(c, j) amount of ic *j*. For example {0, 1, 3, 4, 5, 8} and {0, 2, 3, 4, 5, 7} are both minimally saturated with ic6, yet neither is a cyclic set. In general, 2-sets only generate maximal saturation of ic4 and ic6 when they are comprised of one or more complete 2-cycles (type "c" in figure 4), give or take one note. 3-sets (trivially) only produce maximal ic6 saturation in trichrods and larger sets. It should be noted that while all sets that are *maximally* saturated with a particular interval class are *i*-sets generable from the same *i*-cycle, not all sets that are *minimally* saturated with a particular interval class are *i*-sets (e.g., {0, 1, 3, 4, 5, 8} and {0, 2, 3, 4, 5, 7} are both minimally saturated with ic6, yet neither is a cyclic set.)

Figure 5 shows some interesting patterns that will not only impact how we interpret data from the ic-based vector proposed in the next section, but should also influence the manner in which we interpret data from any ICV-based similarity relation. We can, for example, make the following observations about particular families of pcsets: any time that one finds a maximal amount of ic1 or ic5, a minimal amount of ic6 and ic4 is guaranteed as a by-product; it is impossible to maximize ic3 content without also maximizing ic6 content—but the inverse of this statement is not true; ic4 and ic6 are either maximized or minimized in every cyclic set, but, while the six ic cycles yield minimal ic6 content equally as often as maximal ic6 content, there is twice the likelihood of finding minimal ic4 content as maximal ic4 content.

As mentioned earlier, it might have been simpler to survey all set classes and make note of which in each cardinality had the largest and smallest numbers of each ic. I hope that this detailed approach has revealed more about the interplay among ics, the cardinalities of pcsets, and the properties associated with minimal and maximal interval-class saturation.

PSATV: The Proportional Saturation Vector

This section introduces a relatively simple saturation vector, containing six arguments that reflect the saturation level of each interval class as a percentage of what is maximally possible. Simply dividing each value in the interval-class vector by the maximum possible ic value for that cardinality (max(c, i)) would effectively eliminate any comparison with the min(c, i) values. To avoid this situation, we first create a "min-adjusted ic vector" (MAV) by subtracting the min(c, i) values from the ICV values. PSATV is then derived by dividing the MAV values from the respective "min-adjusted" max(c, i) values (i.e., max(c, i) – min(c, i)). PSATV is demonstrated in two steps in Figure 6.

The ranges of possible ICV values in each cardinality (i.e., the distance between min(c, i) and max(c, i) for all c and i) are shown in Figure 7. As one can see, complementary cardinalities yield for each ICV place ranges of possible values of the same size (bandwidths). It follows, therefore, that the "min-adjusted" max(c, i) values of complementary set class cardinalities are also the same.[23]

The above observation—combined with Ericksson's theorem that when a set class maximally embeds one or more interval class, its complement will also maximally embed the same interval class(es)—leads to a very interesting property of interval-class saturation vectors.[24] Because the PSATV places each interval-class vector argument into the context of its range of possible values, and because complementary set classes saturate each interval class to the same degree (within that range), PSATVs of complementary set classes yield the same values. For example, 3-1 [012] and 9-1 [012345678] both feature the maximal possible ic1 content, the max-1 possible ic2 content, and the minimum amount of all the other interval classes. In fact, complementary set classes also produce identical MAVs (the basis for PSATV). Figure 8 offers one illustration using complementary set classes 5-z36 [01247] and 7-z36 [0123568].

The saturation vector can contribute significantly to analyses of pieces in which complementary relationships play a structural role. However, the equivalence of complementary scs under PSATV should not limit its application *only* to such pieces since, fundamentally, all compositions written in twelve-tone equal temperament (and where it is reasonable to assert octave equivalence) operate to some degree within the boundaries of pc-space.[25] The PSATV simply represents one method of examining

Step 1. Create a min-adjusted ic vector (MAV):

$MAV(X)_i = ICV(X)_i - \min(c, i)$

for all $i \in I$.

Each ic i place in the MAV of X is derived by subtracting the minimal amount of ic content in any set of cardinality X (c) from the respective ICV(X) argument.

For example, one would perform the following steps to arrive at the MAV of set class [012678]:

ICV([012678])	=	<	4	2	0	2	4	3	>
min(6, i)	=		0	0	0	2	0	0	
MAV([012678])	=	<	4	2	0	0	4	3	>

Step 2. Divide the values in the min-adjusted vector (MAV) by the min-adjusted maximal values of any set of #X:

$$PSATV(X)_i = \frac{MAV(X)_i}{\max(c,i) - \min(c,i)}$$

for all $i \in I$.

For example, one would perform the following steps to arrive at the PSATV of set class [012678]:

MAV[012678]	= <	**4**	**2**	**0**	**0**	**4**	**3**	>
max(6, i)	=	5	6	5	6	5	3	
min(6, i)	=	0	0	0	2	0	0	
max(6, i) - min(6, i)	=	**5**	**6**	**5**	**4**	**5**	**3**	
PSATV[012678]	= <	4/5	2/6	0/5	0/4	4/5	3/3	>
	<	0.80	0.33	0.00	0.00	0.80	1.00	>

Figure 6. Demonstration of PSATV of SC 6-7 [012678]

these boundaries. The musical logic supporting complementary PSATV equivalence may also reinforce complement-based relations such as Kh and ZC, even in pieces that do not actually exhibit pcset or sc complementation.[26]

The Interval-Class Saturation Vector (SATV)

We will now introduce a slightly more complex tool for measuring interval-class saturation with greater specificity. The interval-class saturation vector (SATV(X)) is a dual six-argument vector: each place in the upper (A) row of the vector carries the designation "min+n" or "max-n" (n represents the difference between the corresponding ic vector place i (that is, ic i) and the minima (min(c, i)) or maxima (max(c, i)) of its cardinality), depending upon which extreme is closer to the ic vector value. In the case of a tie, the ic vector place is (arbitrarily) compared to the maximal value. Each place in the lower (B) row of the vector shows the opposite (furthest) comparison (if the A row compares the ic vector place with min(c, i), the B row compares the ic vector place with max(c, i), and vice versa). For example, the SATV for the all-combinatorial hexachord 6-7 [012678] (ICV: <420243>) is shown below.

$SATV_A$[012678]: <max–1, min+2, min+0, min+0, max–1, max–0>

$SATV_B$[012678]: <min+4, max–4, max–5, max–4, min+4, min+3>

For the sake of concision, and in order to use vectors that are entirely numerical, I will henceforth omit the keywords "min" and "max" from the vectors and retain only the signs (+ or –) and the difference values. "Positive" values will imply a comparison to the appropriate min(c, i) values and "negative" values will imply a comparison to the appropriate max(c, i) values.[27] Thus the SATV for [012678] will appear as follows:

$SATV_A$[012678]: <-1, +2, +0, +0, -1, -0>

$SATV_B$[012678]: <+4, -4, -5, -4, +4, +3>

Both rows of the vector are necessary because the range of possible values for ic i in #X is not consistent for all i values. In other words, the difference between min(c, i) and max(c, i) is different in many cases

Interval-class vector range (max(c, i) - min(c, i))

Cardinality	**1**	**2**	**3**	**4**	**5**	**6**
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	1	1	1	1	1	1
3	2	2	2	3	2	1
4	3	3	4	3	3	2
5	4	4	4	3	4	2
6	5	6	5	4	5	3
7	4	4	4	3	4	2
8	3	3	4	3	3	2
9	2	2	2	3	2	1
10	1	1	1	1	1	1
11	0	0	0	0	0	0
12	0	0	0	0	0	0

Figure 7. Maximum min-adjusted ic values for all set-class cardinalities

5-z36 [01247]		7-z36 [0123568]	
ICV(5-z36)	< 2 2 2 1 2 1 >	ICV(7-z36)	< 4 4 4 3 4 2 >
min(5, *i*)	0 0 0 1 0 0	min(7, *i*)	2 2 2 3 2 1
MAV(5-z36)	< 2 2 2 0 2 1 > =	MAV(7-z36)	< 2 2 2 0 2 1 >

Figure 8. Min-adjusted ic vectors (MAV)
for two complementary set classes

(e.g., when $c = 4$ and $i = 2$, the distance between max(c, i) and min(c, i) is 3, but when $c = 4$ and $i = 3$, the distance between the two extremes is 4). The reason I scatter min- and max-related values among the two rows of SATV rather than segregating them into a row of min-related values and a row of max-related values will become apparent in the next section of this article, where SATV is used for relating set classes.

The Interval-Class Saturation Similarity Index—SATSIM(*X, Y*)

This section of the article describes one method of utilizing the SATV to examine how closely two set classes resemble each other. The ic saturation similarity index—SATSIM(*X*, *Y*)—is a function that compares ic saturation vectors of two sets, and returns a real number between 0 and 1 that serves as an indicator of the two sets' relative similarity. Since a high SATSIM(*X*, *Y*) value indicates a lack of similarity among pcsets *X* and *Y*, one might more properly call this a "dissimilarity index."[28] Architecturally, SATSIM(*X*, *Y*) resembles Morris's ASIM(*X*, *Y*); the principal difference between the two is that Morris's measure uses one-part ic vectors while SATSIM employs values in a two-part vector.[29]

I have elsewhere detailed how one might compare saturation vectors in a similarity measure.[30] I will avoid duplicating that work here, limiting my description to a few words and a demonstration. Readers who are familiar with the Cyclic Saturation Similarity Measure (CSATSIM) can skip past the similarity measure demonstration. While CSATSIM processes different data, it is structurally the same as the Saturation Similarity Measure introduced herein.

The basic idea of SATSIM(*X*, *Y*) is to compare SATVs of two set classes by adding together their differences and dividing that sum by the sum of all the numbers in both vectors. We thus divide the sum of the differences by the sum of the possible differences to calculate the degree to which both set classes are similar. The only formidable aspects of exacting this comparison come in dealing with two-part saturation vectors and comparing non-commensurable values.

Because the designations "min" and "max" do not refer to absolute values but are variable depending on interval class and set cardinality, one cannot calculate the difference between a min-related and a max-related value *a priori*. It is only possible to compare min-related values to other min-related values and max-related values to other max-related values. A new function, *row,* will determine which SATV value to use in each case when calculating SATSIM.

In a nutshell, SATSIM is calculated by adding together the differences between the two vectors being compared and dividing that sum by the total of all the arguments in both vectors. Totaling the arguments in a saturation vector is a bit less straightforward than when working with purely numerical vectors. Figure 9 provides a formal definition of SATV totals (this is sometimes called "vector cardinality"); it is derived by adding together the distances between the numerical values in the respective arguments of both lines of the vector. Figure 10 lists all SATV totals for all set-class cardinalities. Like ic vector totals, the combined values in saturation vectors will always total the same number for sets of the same cardinality. Unlike the ic vector, however, saturation vectors of complementary sets will also add up to the same combined total, since complementary sets yield precisely the same saturation vectors. Figure 11 provides a concise definition of SATSIM and function *row*, mentioned above.

Figure 12 presents a demonstration of how SATSIM values are attained. In it, we see that */X/*, [012678], has the value (max)-1 in the ic1 column. */Y/*, [0369], on the other hand, has +0, which means it is minimally saturated with ic1. Because *X* and *Y* are different cardinality scs (and because it will always be possible that $\#X \neq \#Y$ for any *X* and *Y*),

$$\#\mathrm{SATV}(X) = \sum_{n=1}^{6} \left(|\mathrm{SATV}_{\mathrm{A}}(X)_n - \mathrm{SATV}_{\mathrm{B}}(X)_n| \right)$$

Figure 9. Formal definition of SATV totals (ΣSATV)

c	ΣSATV(X)
0 or 12	0
1 or 11	0
2 or 10	6
3 or 9	12
4 or 8	18
5 or 7	21
6	28

Figure 10. SATV totals for all set-class cardinalities *c*

SATSIM(X, Y) =

$$\frac{\sum_{n=1}^{6}\left(|\mathrm{SATV_A}(X)_n - \mathrm{SATV}_{row}(X)_n| + |\mathrm{SATV_A}(Y)_n - \mathrm{SATV}_{row}(X)_n|\right)}{\sum_{n=1}^{6}\left(|\mathrm{SATV_A}(X)_n - \mathrm{SATV_B}(X)_n| + |\mathrm{SATV_A}(Y)_n - \mathrm{SATV_B}(Y)_n|\right)}$$

Where X_n and Y_n are the nth entries in the SATVs of pcsets X and Y respectively and *row* is a function that decides which row of the SATV to use.

Function *row*:
If $\mathrm{SATV_A}(X)_n$ is a max-related value and $\mathrm{SATV_A}(Y)_n$ is also a max-related value, then the function *row* returns row A ($\mathrm{SATV_A}(X)_n$ is compared to $\mathrm{SATV_A}(Y)_n$); otherwise, *row* returns row B (in that case, $\mathrm{SATV_A}(X)_n$ is compared to $\mathrm{SATV_B}(Y)_n$).

Figure 11. Formal definition of function SATSIM(X, Y)

min(c, i) and max(c, i) will represent different extremes for each i. It is therefore impossible to compare a min-related value directly with a max-related value; in this case, we must look to line B of */Y/*'s ic saturation vector, which shows that [0369] contains the (max)-3 amount of ic1 (see function *row*, described formally in Figure 11). The absolute value of the difference between -1 and -3 (i.e., 2) is the value returned for the ic1 column. In the ic2 column, $\mathrm{SATV_A}$(*/X/*) carries the value (min)+2, while $\mathrm{SATV_A}$(*/Y/*) has the value (min)+0, also yielding a difference of 2. In this case, one need not check the value in $\mathrm{SATV_B}$(*/Y/*) since row A had the necessary min-related value. This procedure (step 1 in Figure 12) is repeated for each argument in $\mathrm{SATV_A}$(*/X/*).

One then compares each argument in $\mathrm{SATV_A}$(*/Y/*) to either row A or B of */X/*'s saturation vector, creating a two-part difference vector (after Isaacson, 1990). Because only the A row of one saturation vector is compared to whichever row has the matching argument in the other vector, not all the max- and min-related values are necessarily employed in the comparison. In fact, when both sets have, for example, a max-related value in some ic column of row A, the corresponding min-related values in the B rows are never actually compared. While an index that does not always consider all available arguments might be viewed as incomplete (or even inconsistent), the effect of cardinality is further reduced by comparing only the *closest* arguments in the SATVs. If, for example, a trichord and a hexachord both have the max(c, i) amount of ic1 (that is to say, the first argument in $\mathrm{SATV_A}$ of both sets contains the value -0), they would have $\mathrm{SATV_B}$ ic1 values of +2 and +5, respectively. If SATSIM employed all these values in its comparison, then we would see that the two sets are $\frac{|0-0|+|2-5|}{2+5} = \frac{3}{7} = 42.8\%$ different in their ic1 content (I have added together the difference between the min-related values and max-related values and divided that sum by the sum of the distance between SATV_A and SATV_B in the ic1 place for each pcset). Considering that these two sets are both maximally similar in ic1 content (for a trichord and hexachord), this difference seems extreme, and it occurs solely as a product of their difference in cardinality, a factor I have tried to temper in this index.

When the differences between row A of */X/*'s saturation vector and the corresponding min- or max-related values in either row of */Y/*'s vector are added together, they may not necessarily be the same as the equivalent comparison of */Y/* to */X/* (i.e., $\mathrm{SATV}_A(/X/) : \mathrm{SATV}_{row}(/Y/) \neq \mathrm{SATV}_A(/Y/) : \mathrm{SATV}_{row}(/X/)$ for all */X/* and */Y/*). This is the case in the two scs compared in Figure 12. Therefore, in order to obtain the same value from a comparison of */X/* to */Y/* and */Y/* to */X/*, it is necessary to add all the difference values together to obtain a composite that reflects both comparisons (step 2 in Figure 12).

We could stop here and have a perfectly acceptable, context-free similarity index, one that has, in large part, solved the problem of comparing sets with different cardinalities. However, an even more cardinality-neu-

X [012678]	<420243>	SATV_A:	<	-1	+2	+0	+0	-1	-0	>
		SATV_B:	<	+4	-4	-5	-4	+4	+3	>
Y [0369]	<004002>	SATV_A:	<	+0	+0	-0	+0	+0	-0	>
		SATV_B:	<	-3	-3	+4	-3	-3	+2	>

Step 1: Compare the vectors, creating a two-part difference vector:

$\mathrm{SATV}_A(X) : \mathrm{SATV}_{row}(Y) =$	2	2	4	0	2	0	= 10
$\mathrm{SATV}_A(Y) : \mathrm{SATV}_{row}(X) =$	4	2	5	0	4	0	= 15

Step 2: Add together the values in the difference vectors: = **25**

Step 3: Add together all the numerical distances between $SATV_A$ and $SATV_B$ for each set:

$\#\mathrm{SATV}_{A+B}(X) =$	5 + 6 +5 + 4 + 5 +3	= 28
$\#\mathrm{SATV}_{A+B}(Y) =$	3 + 3 +4 + 3 + 3 +2	= 18
		= **46**

Step 4: Divide the sum from step 2 by the sum from step 3 to complete the SATSIM function:

SATSIM(*X*, *Y*) = 25/46 = **0.54**

Figure 12. SATSIM(X, Y) comparison of [012678] and [0369]

tral index is attained by dividing the sum of the differences by the combined SATV totals (step 4 in Figure 12). This cardinality adjustment better allows us to compare SATSIM(X, Y) and SATSIM(S, T), where $\#S$ or $\#T$ is not necessarily equal to $\#X$ or $\#Y$.[31]

The comparison value 0.54 that SATSIM yields for [012678] and [0369] (step 4 in Figure 12) represents the very great differences in their ic1, ic2, ic3, and ic5 content. It also represents the congruence of values in the ic4 and ic6 columns, returning a value which indicates that more than fifty percent of the interval classes in the two sets are not mutually saturated. The larger the number, the more dissimilar two sets are said to be using this formula. The number 0 indicates an equivalence relation, while the number 1 indicates maximal dissimilarity. Maximal dissimilarity, however, is impossible to attain in pitch-class space. Even when two set classes share no common interval classes (e.g., [012] and [048]), they will necessarily share some mutual exclusions ([012] and [048] both minimally saturate ic3, ic5, and ic6).

By definition, any similarity index or equivalence relation that uses the interval-class vector as its sole datum will consider members of the same set class and Z-related sets as either maximally similar or equivalent.[32] Because of the way SATV interprets the data from the interval-class vector, maximal similarity is found in more places using this index than with others because complementary sets, like members of the same set class and Z-related sets, also return a value of 0 using SATSIM, reflecting the fact that they are equally saturated with the same interval classes.

Through the lens of SATSIM, sc 4-28 [0369] appears identical to sc 8-28 [0134679a]. Therefore, rather like Forte, I will consider a broader definition of set class that equates T_n/T_nI-related sets *and* their complements. More generally, any mention of the SATV or SATSIM properties of, for example, #4 scs will also be taken to include #8 scs.

To avoid confusion, I will adopt a standard of referring to SATSIM-equivalent sets together at all times. Scs 4-28 and 8-28 will be referred to singly as sc 4/8-28 or [0369] / [0134679a]. This coupling of complementary cardinalities will be called "cardinality pairs" (or c.p.s). For the sake of comparison, sets of c.p. #4/#8 will be considered larger than sets of c.p. #3/#9; or, more formally stated, the larger the difference in cardinality between a pcset and its complement, the smaller the combined set class size. When a single cardinality is required for the sake of a formalization, the lower number is chosen (thus, $\#W = 4$ where $W =$ sc[0369] / [0134679a]).

Z-related scs are also SATSIM-equivalents (as are, by transitivity, the complements of Z-related scs). Therefore, Z and ZC-related set classes will also be referred to jointly as, for example, 6-3/36 ([012356] / [012347]) or 5/7-18/38 ([01457] / [01258] / [0145679] / [0124578]). Let us define a SATSIM group as an equivalence class containing all scs that are SATSIM equivalent.[33] There is only one distinct sc in the 6-14 [013458] SATSIM group; there are two distinct scs in the 6-3/36 or 5/7-1 SATSIM groups; there are four distinct scs in the 5/7-18/38 SATSIM group.

In addition to the SATSIM equivalence of complementary and Z-related scs, there are also six SATSIM groups that include scs which are neither complements nor Z-relations (see Figure 13). We will consider these special equivalences to be SATV_A Z-related sets since each pair shares the same SATV_A. When $\text{SATV}_\text{A}(X)$ and $\text{SATV}_\text{A}(Y)$ contain the same pattern of min- and max-related ("positive" and "negative") values, SATSIM(X, Y) will yield the value 0.00 since it only evaluates line A. The SATV_B values for sets X and Y in the second through sixth groups in Figure 13 are, in fact, different, reflecting the differences in cardinality between the two scs and the resultant differences in ic ranges of possibility.[34] (Both SATV_A and SATV_B are given for each complementary sc pair in Figure 13.)

Each SATSIM group in Figure 13 shares some common traits. Unlike ICV Z-related sets, none of these special SATV_A Z-pairs are of the same cardinality. Each is a pair of i-sets, using the same i-cycle(s) as generators. Each X/Y or $\bar{X}/\bar{Y}$ pair differs in size by a single member (i.e., pcset X is either one note larger or smaller than pcset Y).[35] In each case, pcset Y (and $\bar{Y}$) is comprised of a completed cycle (or concatenation of completed cycles) and $\bar{X} \supseteq (\bar{Y} \supseteq)\ Y \supseteq X$. That is, the complement of X is a superset of Y, which is a superset of X (and, if the complement of Y is distinct from Y, then it is a superset of Y and a subset of the complement of X).

Since complementary set classes also yield identical PSATVs, that vector likewise produces a number of special Z-relations beyond those of complementary set classes and ic vector Z-relations. There are, however, only three special PSATSIM groups (shown in Figure 14), and these are a subset of the SATSIM groups (shown in Figure 13). The special PSATSIM groups share the same traits as the special SATSIM groups, and they are also bound by one additional constraint: all PSATV values must be either 0 or 1, reflecting minimal or maximal saturation of each ic. This is a necessary constraint for these special non-complementary PSATV Z-pairs of different cardinality, because the range of ic values in each cardinality is different and the real-number values of PSATV will reflect those (sometimes subtle) differences.

One might expect that SATSIM group #1 from Figure 13 (0/12-1 compared to 1/11-1) would also appear as a PSATSIM group. This would seem logical given that each ic is both minimally and maximally saturated in each sc. However, because the ICV content of these sets is, in each case, equal to the min(c, i) place, the MAV yields nothing but zeros. The min-adjusted max values are therefore also zeros. Since one cannot

Set Classes (Complementary set classes shown together)		Forte #	SATV
SATSIM group #1 (1/2/3/4/5/6-cyclic sets)			
$X/\overline{X}$	[ ] / [0123456789ab]	0/12-1	< -0 -0 -0 -0 -0 -0 > < +0 +0 +0 +0 +0 +0 >
$Y/\overline{Y}$	[0] / [0123456789a]	1/11-1	< -0 -0 -0 -0 -0 -0 > < +0 +0 +0 +0 +0 +0 >
SATSIM group #2 (4-cyclic sets)			
$X/\overline{X}$	[04] / [012345689a]	2/10-4	< +0 +0 +0 -0 +0 +0 > < -1 -1 -1 +1 -1 -1 >
$Y/\overline{Y}$	[048] / [01245689a]	3/9-12	< +0 +0 +0 -0 +0 +0 > < -2 -2 -2 +3, -2, -1 >
SATSIM group #3 (6-cyclic sets)			
$X/\overline{X}$	[016] / [012346789]	3/9-5	< -1 +0 +0 +0 -1 -0 > < +1 -2 -2 -3 +1 +1 >
$Y/\overline{Y}$	[0167] / [01236789]	4/8-9	< -1 +0 +0 +0 -1 -0 > < +2 -3 -4 -3 +2 +2 >
SATSIM group #4 (3/6-cyclic sets)			
$X/\overline{X}$	[036] / [01234679a]	3/9-10	< +0 +0 -0 +0 +0 -0 > < -2 -2 +2 -3 -2 +1 >
$Y/\overline{Y}$	[0369] / [0134679a]	4/8-28	< +0 +0 -0 +0 +0 -0 > < -3 -3 +4 -3 -3 +2 >
SATSIM group #5 (4-cyclic sets)			
$X/\overline{X}$	[01458] / [0124589]	5/7-21	< -2 +0 -2 -0 -2 +0 > < +2 -4 +2 +3 +2 -2 >
$Y/\overline{Y}$	[014589]	6-20	< -2 +0 -2 -0 -2 +0 > < +3 -6 +3 +4 +3 -3 >
SATSIM group #6 (2/4/6-cyclic sets)			
$X/\overline{X}$	[02468] / [012468a]	5/7-33	< +0 -0 +0 -0 +0 -0 > < -4 +4 -4 +3 -4 +2 >
$Y/\overline{Y}$	[02468a]	6-35	< +0 -0 +0 -0 +0 -0 > < -5 +6 -5 +4 -5 +3 >

Figure 13. Special SATSIM groups ($SATV_A$ Z-relations)

Set Classes		Forte #	PSATV
PSATSIM group #1 (4-cyclic sets)			
$X / \overline{X}$	[04] / [012345689a]	2/10-4	< 0.00 0.00 0.00 1.00 0.00 0.00 >
$Y / \overline{Y}$	[048] / [01245689a]	3/9-12	< 0.00 0.00 0.00 1.00 0.00 0.00 >
PSATSIM group #2 (3/6-cyclic sets)			
$X / \overline{X}$	[036] / [01234679a]	3/9-10	< 0.00 0.00 1.00 0.00 0.00 1.00 >
$Y / \overline{Y}$	[0369] / [0134679a]	4/8-28	< 0.00 0.00 1.00 0.00 0.00 1.00 >
PSATSIM group #3 (2/4/6-cyclic sets)			
$X / \overline{X}$	[02468] / [012468a]	5/7-35	< 0.00 1.00 0.00 1.00 0.00 1.00 >
$Y / \overline{Y}$	[02468a]	6-35	< 0.00 1.00 0.00 1.00 0.00 1.00 >

Figure 14. Special PSATSIM groups (PSATSIM Z-relations). Complementary set classes shown together

divide zero by zero, it is impossible to create PSATVs for these set classes. One could include them as PSATV Z-relations if one were willing to allow for PSATVs with undefined values. Perhaps PSATV is only of limited use because it cannot represent these two sc pairs in a manner consistent with the way it represents all the other scs. Yet "undefined" may be the best possible way to describe their intervallic content. After all, they each maximally *and* minimally saturate each ic. Therefore, neither the value 0.00 nor 1.00 is entirely representative—and certainly neither is anything in between.

While there are more occurrences of maximal similarity found using SATSIM(*X*, *Y*) than with other similarity indices, there are no instances of total dissimilarity (SATSIM value of 1).[36] The highest value returned by SATSIM in all comparisons of sets of cardinality 2/10 through 6 is 0.74. This value is found between scs 2/10-4 [04] / [012345689a] and either 6-27 [013469] or 6-30 [013679]. Figure 15 shows these comparisons. For the sake of space, demonstrations of SATSIM will herein be abbreviated from the illustration in Figure 12. Rather than showing each $\mathrm{SATV}(X)_{\mathrm{A}} : \mathrm{SATV}(Y)_{row}$ and $\mathrm{SATV}(Y)_{\mathrm{A}} : \mathrm{SATV}(X)_{row}$ comparison, I will use a difference vector (Diff SATV(*X*, *Y*)) that, in each place, has the former comparison followed by the latter (the two comparisons are separated by commas). The values in each difference vector are added together, and the total appears at the end of the line. For clarity, Figure 16 abbreviates the illustration in Figure 12.

There are two sc pairs that generate the smallest non-zero SATSIM value (0.02). The first pair is 5/7-4 [01236] / [0123467] and 6-2 [012346]; the second is 5/7-29 [01368] / [0124679] and 6-33 [023579].[37] These two comparisons are illustrated in Figure 17. Notice that the $\mathrm{SATV}_{\mathrm{A}}$ of each

SATV(2/10-4)	<	+0	+0	+0	-0	+0	+0	>		
	<	-1	-1	-1	+1	-1	-1	>		
SATV(6-27)	<	+2	+2	-0	+0	+2	-1	>		
	<	-3	-4	+5	-4	-3	+2	>		
SATV(6-30)	<	+2	+2	-1	+0	+2	-0	>		
	<	-3	-4	+4	-4	-3	+3	>		
Diff. SATV(2-4, 6-27)		2, 2	2, 2	5, 1	4, 1	2, 2	2, 0		=	25
Diff. SATV(2-4, 6-30)		2, 2	2, 2	4, 0	4, 1	2, 2	3, 1		=	25

SATSIM(2-4, 6-27) = 25 / 34 = **0.74**

SATSIM(2-4, 6-30) = 25 / 34 = **0.74**

Figure 15. Largest possible SATSIM relations

SATV(6-7)	<	-1	+2	+0	+0	-1	-0	>		
	<	+4	-4	-5	-4	+4	+3	>		
SATV(4-28)	<	+0	+0	-0	+0	+0	-0	>		
	<	-3	-3	+4	-3	-3	+2	>		
Diff. SATV(6-7, 4-28)		2, 4	2, 2	4, 5	0, 0	2,4	0, 0		=	25

SATSIM(6-7, 4-28) = 25 / 46 = **0.54**

Figure 16. SATSIM(X, Y) comparison of [012678] and [0369] using the difference vector

set pair is identical except in the ic6 columns, accounting for the very close relation.

A summary of the possible SATSIM values is given in Figure 18 in the form of a "value group matrix" (after Castrén 1994).[38] Each cell represents a statistical summary of the values possible using SATSIM(*X*, *Y*) where *X* is a pcset of the X-axis cardinality and *Y* is a pcset of the Y-axis cardinality (or vice versa). The upper left corner of each cell is the lowest SATSIM value possible in the value group; the upper right corner is the highest SATSIM value possible in the value group; the middle left number is the lowest non-zero SATSIM value; the lower left corner contains the average of all the values in the group; and the lower right corner contains the number of distinct SATSIM values in the value group.[39]

Some patterns become apparent when examining the SATSIM value group matrix. Almost without exception, the average SATSIM value increases with the difference in cardinality. (Again, for our purposes, sets larger than hexachords count as the cardinality of their complement;

therefore the comparison of a #4 pcset to a #9 pcset is equivalent to the comparison of a #4 pcset to a #3 pcset—a difference of one, not five.) This seems reasonable when one considers the number of variants possible in a #3/#9 pcset compared to the number possible in a #6 pcset. In the #3/#9 pcset, not only are there many fewer set classes (12 tri/nonachord classes compared to 50 hexachord classes), but there is a much smaller range of possible values in each ICV place. Naturally, the greater the range of ic values, the more potential there will be for relatively similar constructions. Therefore, it should not be surprising that, as a general trend, the larger the c.p. of the set classes in the value group, the smaller the SATSIM values will tend to be. Thus, the smallest average SATSIM values occur when comparing hexachords to hexachords. The largest average SATSIM values occur when comparing #2/#10 sets to #6; this is, not coincidentally, also the source of the largest cardinality difference in our value group.[40]

The above statistical summary should not imply that either SATV or SATSIM is weighted in such a way to reward sc comparisons which have a small difference in cardinality. Such mention of statistical likelihood is more a reflection of the range of possible ICV values in different cardinalities of pcsets. As we have seen when considering individual comparisons, SATSIM's primary determinant of sc similarity is the degree to which each sc is saturated with the same ics. The fact that SATSIM can

SATV(5/7-4)	<	-1	-2	-2	+0	+1	-1	>	
	<	+3	+2	+2	-3	-3	+1	>	
SATV(6-2)	<	-1	-2	-2	+0	+1	+1	>	
	<	+4	+4	+3	-4	-4	-2	>	
Diff. SATV(5-4, 6-2)		0, 0	0, 0	0, 0	0, 0	0, 0	1, 0	=	1
SATSIM(5-4, 6-2) = 1 / 49 = **0.02**									

SATV(5/7-29)	<	+1	-2	-2	+0	-1	-1	>	
	<	-3	+2	+2	-3	+3	+1	>	
SATV(6-33)	<	+1	-2	-2	+0	-1	+1	>	
	<	-4	+4	+3	-4	+4	-2	>	
Diff. SATV(5-29, 6-33)		0, 0	0, 0	0, 0	0, 0	0, 0	1, 0	=	1
SATSIM(5-29, 6-33) = 1 / 49 = **0.02**									

Figure 17. Smallest possible (non-zero) SATSIM relations

	#2/#10	**#3/#9**	**#4/#8**	**#5/#7**	**#6**
#2/#10	*0.000* 0.333 0.333 0.278 2				
#3/#9	0.000 0.500 0.056 0.318 10	*0.000* 0.500 0.167 0.312 4			
#4/#8	0.125 0.583 0.125 0.414 12	0.000 0.567 0.033 0.345 18	0.000 0.556 0.111 0.288 6		
#5/#7	0.222 0.704 0.222 0.504 14	0.061 0.636 0.061 0.363 18	0.051 0.590 0.051 0.299 22	0.000 0.571 0.095 0.250 7	
#6	0.265 0.735 0.265 0.575 15	0.100 0.700 0.100 0.421 23	0.087 0.609 0.087 0.339 24	0.000 0.653 0.020 0.253 32	0.000 0.643 0.071 0.214 10

Key to figures in value group matrix:

v	w
x	
y	z

v = smallest SATSIM number

w = largest SATSIM number

x = smallest non-zero SATSIM number

y = average SATSIM value

z = number of distinct SATSIM values

The value 0.000 is italicized when it only occurs trivially (when comparing a sc to itself or to its complement)

Figure 18. SATSIM value group matrix

return the value 0.000 when comparing scs of different size should dispel any notion that such sets are automatically considered less similar. Despite the statistical information presented above, one could actually make what might seem like a counterintuitive, or even contradictory, claim: that distinct scs of different sizes are potentially more similar than sets of the same size. Let us sort out the instances of maximal similarity yielded by SATSIM and examine the smallest *non-zero* values returned by SATSIM in each value group (the middle left value in each matrix cell). This removes comparisons of each sc to itself and to its SATV Z-relations, leaving the nearest relations possible where the vectors are not the same.

We can see from these values that given any SATSIM(X, Y) comparison, the closest possible non-equivalent relationship between any $\#X$ and $\#Y$ sets almost always occurs when $\#X \neq \#Y$.[41] That is to say, comparisons where $\#X = \#Y$ generally yield a higher minimal value than comparing pcset X to the "most similar" pcset Y where $\#Y \neq \#X$. Almost every sc is highly similar to at least another sc that is one pc larger or smaller (its superset(s) and subset(s), for instance). This is particularly true of cyclic set classes that are structurally unique in their cardinality. While there may be an sc of another size that shares the same structural features, there will not be any other sc of the same size that does so.[42] This is why many of the most similar (non-equivalent) sc pairs are of different sizes. Naturally, though, when $\#X = \#Y$, the nearer the cardinality is to 6, the larger the comparison group, and the greater the likelihood that some pcset Y will share many of pcset X's degrees of intervallic saturation. This is reflected in the gradually smaller numbers that occur from left to right along the top diagonal in the value group matrix (Figure 18).

Appendix A contains an sc-specific statistical summary of SATSIM values. It is a table that shows the average SATSIM(*/X/*, */Y/*) value for each */X/* compared to all */Y/* in the comparison group of all scs of c.p. 2/10 through 6. Additionally, it shows the lowest and highest possible SATSIM(*/X/*, */Y/*) values for each */X/* with respect to all */Y/* in the comparison group. Appendix A also contains the average, lowest, and highest values found when comparing each */X/* to the entire range of */Y/*s. Both Figure 18 and Appendix A contextualize the range of SATSIM values, allowing one to make more meaningful analytical statements. In particular, Appendix A illustrates that some scs are, on average, more distantly related to all other sets. For example, *i*-cyclic set classes and scs which are similar to *i*-cyclic scs are more distinctive—or *singular*—than average (i.e., their average SATSIM value is much higher than the average SATSIM values for other scs of the same cardinality).

PSATSIM: The PSATV Similarity Index

By and large, PSATV conveys the same information as SATV, but without the necessity of a two-part vector. This makes it a bit easier to compare PSATVs at a glance. However, one potential downside of using PSATV as the basis for a similarity index is that the vector does not produce a constant cardinality (vector total) for each X of $\#X$.[43] For example, the values in PSATV[036] add up to 2.00 while the values in PSATV[048] add up to only 1.00 (see Figure 19).

The problem with having no constant cardinality is that it is not immediately obvious how one might use this vector in an ASIM-style index; that is, an index that compares the arguments in two vectors and then divides the sum of the difference-vector values by the combined cardinality of the saturation vectors. Arguably, one does not need this type of index where the differences in cardinality have, in large part, been adjusted for in the vector. This is a valid premise—even more so than in the case of comparing SATVs—but it is useful to bring the measure's values within the constant range of 0 and 1 for the sake of comparison.

The largest value in any argument of a difference vector (diffV) that compares two PSATVs is 1.00. This is the case when one sc has the maximal amount (1.00) of some ic and the other has the minimal amount (0.00) of the same ic. Given that we are currently working with a six-place vector, maximal dissimilarity is represented by the number 6.00. This, then, is the number by which we will divide the sum of the PSATV diffV values to create an index that is comparable to SATSIM. Figure 20 contains a step-by-step demonstration of PSATSIM; Figure 21 provides a formalization of the index.

Appendix B is an sc-specific statistical summary of PSATSIM values. A value group matrix providing a more general summary of the possible PSATSIM values is given in Figure 22. As before, each cell represents a statistical summary of the values possible using PSATSIM(*X*, *Y*), where *X* is a pcset of the X-axis cardinality and *Y* is a pcset of the Y-axis cardinality (or vice versa). The number of distinct PSATSIM values (lower right corner of each cell) might seem extraordinarily large in some instances (e.g., there are 242 different PSATSIM(#4/#8, #6) values). To

PSATV[036]:	<0.00, 0.00, 1.00, 0.00, 0.00, 1.00>	#PSATV[036] = 2.00
PSATV[048]:	<0.00, 0.00, 0.00, 1.00, 0.00, 0.00>	#PSATV[048] = 1.00

Figure 19. PSATV[036] and PSATV[048]

X [012678]	<420243>	PSATV:	<	0.80	0.33	0.00	0.00	0.80	1.00	>
Y [0369]	<004002>	PSATV:	<	0.00	0.00	1.00	0.00	0.00	1.00	>

Step 1: Compare the vectors, creating a difference vector:

PSATV(*X*) : PSATV(*Y*) = 0.80 0.33 1.00 0.00 0.80 0.00

Step 2: Add together the values in the difference vectors: = **2.93**

Step 3: Divide the sum from step 2 by the number 6 (the largest potential PSATSIM diffV sum) to complete the PSATSIM function:

PSATSIM(*X*, *Y*) = 2.93/6.00 = **0.49**

Figure 20. PSATSIM(X, Y) comparison of [012678] and [0369]

$$\mathrm{PSATSIM}(X, Y) = \frac{\sum_{n=1}^{6} \left(\left| \mathrm{PSATV}(X)_n - \mathrm{PSATV}(Y)_n \right| \right)}{6}$$

Figure 21. Formal definition of PSATSIM(X, Y) measure

	#2/#10	#3/#9	#4/#8	#5/#7	#6
#2/#10	*0.000* 0.333 0.333 0.278 2				
#3/#9	0.000 0.500 0.056 0.335 16	*0.000* 0.583 0.139 0.321 17			
#4/#8	0.153 0.556 0.153 0.390 25	0.000 0.611 0.056 0.345 64	0.000 0.583 0.097 0.294 40		
#5/#7	0.208 0.667 0.208 0.450 35	0.125 0.625 0.125 0.364 43	0.056 0.597 0.056 0.309 76	0.000 0.625 0.083 0.259 50	
#6	0.244 0.667 0.244 0.471 74	0.156 0.661 0.156 0.385 143	0.092 0.633 0.092 0.322 242	0.000 0.689 0.031 0.267 248	0.000 0.689 0.061 0.219 153

Key to figures in value group matrix:

v	*w*
x	
y	*z*

v = smallest PSATSIM number *w* = largest PSATSIM number
x = smallest non-zero PSATSIM number
y = average PSATSIM value *z* = number of distinct PSATSIM values

The value 0.000 is italicized when it only occurs trivially (when comparing a sc to itself or to its complement)

Figure 22. PSATSIM value group matrix

reflect this degree of precision, all similarity-index values have been rounded to three significant digits in the value-group matrices.[44]

The same patterns identified when examining the SATSIM value-group matrix in Figure 18 remain invariant in the PSATSIM value-group comparisons. I commented, in the former case, that 'almost without exception, the average value increases with the cardinality difference.' When discussing PSATSIM, we can remove the word "almost" from that observation and make it more systematic. Without exception, the primary determinant of average PSATSIM values in any value group is the difference in cardinality between the two set classes; once again the secondary determinant is the size of the set classes being compared.

The overall range of possible values using PSATSIM is a bit smaller than that yielded by SATSIM. SATSIM produces values ranging from 0.00 to 0.74; PSATSIM yields values ranging from 0.00 to 0.69. Unlike SATSIM, however, the #6 : #6 group gives us not only the widest range of values (the #5 : #6 group also has the same range), but also the largest possible single comparison (the most dissimilar set-class pair).

The smallest non-zero number yielded by PSATSIM is 0.03, between 5/7-7 [01267] / [0123678] and 6-7 [012678] (see Figure 23). The largest PSATSIM relation is found between any of the first-order all-combinatorial hexachords (6-1 [012345], 6-8 [023457], or 6-32 [024579]) and either the whole-tone collection (6-35 [02468a]) or its five/seven-note sub/superset (5/7-33 [02468] / [012468a]).[45] While the latter pair maximally saturates each of the even ics, the first-order all-combinatorial hexachords have the maximal total amount of the odd ics and the minimal amounts of ic4 and ic6. These comparisons are shown in Figure 24.

All the observations derived regarding the smallest non-zero SATSIM values hold true with PSATSIM as well. In summary, the values in the middle row of each cell of the PSATSIM value-group matrix increase in size from the top to the bottom of each column, except for the first value, which is the highest. Each diagonal decreases in size from left to right. The top cell in each column indicates the PSATSIM(X, Y) values where $\#X = \#Y$ and $X \neq Y$. While many characteristics of the PSATV and PSATSIM values strongly resemble their respective SATV and SATSIM values, I would be reluctant to propose PSATV as a replacement for SATV, both because PSATV provides less specific information than does SATV and also because it does not lend itself to weighted comparisons as readily as does the two-part SATV.

PSATV(5/7-7)	<	0.75	0.25	0.00	0.00	0.75	1.00	>	
PSATV(6-7)	<	0.80	0.33	0.00	0.00	0.80	1.00	>	
Diff. PSATV(5/7-7, 6-7)		0.05	0.08	0.00	0.00	0.05	0.00	=	0.18

PSATSIM = 0.18 / 6.00 = **0.03**

Figure 23. Closest non-equivalent PSATSIM SCs

PSATV(6-35 or 5/7-33)	<	0.00	1.00	0.00	1.00	0.00	1.00	>	
PSATV(6-1)	<	1.00	0.67	0.60	0.00	0.20	0.00	>	
PSATV(6-8)	<	0.60	0.67	0.60	0.00	0.60	0.00	>	
PSATV(6-32)	<	0.20	0.67	0.60	0.00	1.00	0.00	>	
Diff. PSATV(6-35, 6-1)		1.00	0.33	0.60	1.00	0.20	1.00	=	4.13
Diff. PSATV(6-35, 6-8)		0.60	0.33	0.60	1.00	0.60	1.00	=	4.13
Diff. PSATV(6-35, 6-32)		0.20	0.33	0.60	1.00	1.00	1.00	=	4.13

PSATSIM = 4.13 / 6.00 = **0.69**

Figure 24. Farthest PSATSIM relations

Forte# (3-)		1	2	3	4	5	6	7	8	9	10	11	12	
$SATV_{3A}$(6-7)	<	-2	+0	+0	-2	-0	+0	+0	+4	-2	+0	+0	+0	>
$SATV_{3B}$(6-7)	<	+2	-6	-6	+4	+8	-6	-6	-8	+2	-4	-6	-2	>

Figure 25. SATV3(6-7) [012678]

The Generalized Saturation Vector (SATV*n*)

To this point, the saturation vectors have shown only relative quantities of interval classes within a pcset. This was a useful limitation to demonstrate the construction of both vector classes and their associated similarity indices. One can, however, easily create saturation vectors (of either variety already demonstrated) that show relative content of any subset size.[46] As noted earlier, an interval class is not the same as a #2 subset: the former represents a distance between two elements, while the latter represents a collection of two elements (which are necessarily separated by some interval). That said, one can produce an equivalent structure to the ICV(*/Y/*) by performing EMB(*/X/*, */Y/*) six times, where */X/* cycles through all #2 scs (2-1 through 2-6), and by displaying the results in a six-argument array.

By extension, we could have a trichord-class SATV for scs larger than #3, or a hexachord-class SATV for scs larger than #6. In this section, SATV will be generalized to allow any or all subset-class cardinalities. The generalization, SATV*n*(*X*), is constructed exactly like our earlier SATV(*X*), but carries an extra variable, *n*, which signifies the cardinality of subsets. The interval-class SATV(*X*) will now be called SATV2(*X*) and the relation SATSIM will now be called SATSIM2(*X*) in keeping with this new convention.

Let us now return to the function SATEMB(*/X/*, */Y/*), first suggested at the beginning of this article. SATEMB(*/X/*, */Y/*) returns the degree to which */X/* is saturated in */Y/* compared to maximal and minimal EMB(*/X/*, */Y/*) values for any sc of #*Y*. If, for example, */X/* = 3-1 [012] and */Y/* = 6-7 [012678], EMB(*/X/*, */Y/*) = 2. There is, however, a maximum of four embedded [012]s in any hexachord (there are four in 6-1 [012345]) and the minimum number is zero (there are none in 6-35 [02468a], among other scs). Therefore, SATEMB(*/X/*, */Y/*) = < (max)–2, (min)+2 >.

We can now redefine SATV*n*(*Y*) as a complete listing of all SATEMB(*/X/*, */Y/*) values where each */X/* is an sc of cardinality *c*. The SATV3(6-7) is shown in Figure 25. In the second section of this article, minimal and maximal ic saturation was derived from the cyclic set classes. Naturally, one could do the same sort of thing with larger subset classes, examining imbricated chains of all the trichord-classes, tetrachord-classes, etc., to generate set classes that maximally saturate each sc of cardinality c.[47] However, the number of scs involved and the length of the process make this exercise prohibitively long. Appendix C contains a complete list of the max$c$, i and min$c$, i values where c = #2 through #12 and i includes the complete range of scs smaller than or equal to the superset cardinality.

Complementation and SATV*n*

Generalizing SATV*n* for any value of *n* reveals an interesting corollary to the SATV2 equivalence of complementary scs. Readers will recall that all complementary scs generate identical SATV2s, PSATV2s, and min-adjusted ICVs (MAV2). The same does *not* hold true with complementary scs using SATV*n* where $n > 2$. Figure 26 shows the SATV3s of complementary scs 5-1 [01234] and 7-1 [0123456]. While there are distinct similarities between the two SATV3_As, they are not precisely the same and their SATV3_Bs are markedly different, reflecting the significant differences in the range of possible SATEMB(#3, #5) and SATEMB(#3, #7) values.

Clearly, then, complementary scs would not be considered equivalences under an index that uses SATV*n*s where $n > 2$. However, just as SATV2 yielded an interesting complementary equivalence, a similar property is found when examining all the SATV*n*s of #10 scs. Figure 27 shows all the SATV*n*s (for n = 2 through 10) of sc 10-3. Notice that all complementary set classes are saturated to precisely the same degree (SATV3(10-3) = SATV9(10-3), SATV4(10-3) = SATV8(10-3), etc.). Similarly, complementary hexachord classes are also saturated to the same degree (e.g., SATEMB(6-z3, 10-3) = SATEMB(6-z36, 10-3) = < (max)–2, (min)+2 >). More formally, we can declare that when $\#Y = 2$ (and $\#\overline{Y} = 10$), SATEMB(Y, X) = SATEMB(Y, $\overline{X}$) and SATEMB(X,$\overline{Y}$) = SATEMB($\overline{X}$,$\overline{Y}$). That is to say all complementary set-class pairs (larger than #2) saturate the same interval classes (#2 scs) to the same degree and the complements of the interval classes (#10 scs) saturate complementary

Forte# (3-)		1	2	3	4	5	6	7	8	9	10	11	12	
$\text{SATV}_{3\text{A}}$(5-1)	<	-0	-0	-1	+0	+0	+1	+0	+0	+0	+0	+0	+0	>
$\text{SATV}_{3\text{B}}$(5-1)	<	+3	+4	+2	-3	-5	-2	-4	-6	-3	-4	-3	-1	>
Forte# (3-)		1	2	3	4	5	6	7	8	9	10	11	12	
$\text{SATV}_{3\text{A}}$(7-1)	<	-0	-0	-1	-3	+0	-3	+3	+0	+0	+0	+0	+0	>
$\text{SATV}_{3\text{B}}$(7-1)	<	+5	+7	+6	+4	-7	+3	-4	-10	-5	-4	-7	-2	>

Figure 26. SATV3s of complementary SCs 5-1 [01234] and 7-1 [0123456]

```
Forte# (2-)         1  2  3  4  5  6
SATV2A(10-3)    < +0 +0 -0 +0 +0 +0  >
SATV2B(10-3)    < -1 -1 +1 -1 -1 -1  >

Forte# (3-)         1  2  3  4  5  6  7  8  9 10 11 12
SATV3A(10-3)    < +0 -0 -0 +0 +0 +0 -0 +0 +0 -0 -0 +0  >
SATV3B(10-3)    < -2 +2 +2 -2 -4 -2 +2 -4 -2 +2 +2 -1  >

Forte# (4-)         1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29
SATV4A(10-3)    < +1 -2 -0 -2 +0 +0 -1 +0 +0 -0 -2 -0 -0 -2 +0 +0 -0 -0 +2 -1 +0 -2 +1 +0 +0 -0 -0 -0 +0  >
SATV4B(10-3)    < -2 +2 +2 +2 -4 -2 +1 -2 -2 +2 +2 +4 +4 +2 -2 -4 +2 +4 -4 +1 -3 +2 -2 -3 -2 +2 +4 +1 -2  >

Forte# (5-)         1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
SATV5A(10-3)    < -2 -2 -2 -2 +0 +0 +0 -1 +0 -0 -0 -0 +0 +0 +0 -0 -1 -0 +2 +0 -4 -1 -2 +0 -0 -2 -2 +2 -2 +0 -0 -0 +0 -1 -2 -0 -1 -0  >
SATV5B(10-3)    < +2 +4 +4 +2 -4 -4 -8 +2 -4 +4 +4 +1 -4 -4 -4 +4 +2 +2 -4 -4 +4 +2 +4 -4 +4 +2 +4 -4 +2 -4 +6 +4 -4 +2 +2 +2 +2 +2  >

Forte# (6-)         1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 ...
SATV6A(10-3)    < -2 -2 -2 +0 +0 +0 +0 -1 +0 -0 -0 +0 -0 -2 -2 +0 +0 +0 -2 -1 +2 +0 -0 -0 -2 +0 -0 -0 -0 -2 -2 -2 -2 +2 +0 -2 +0 +0 -0 -0 ...
SATV6B(10-3)    < +3 +4 +2 -2 -4 -3 -3 +3 -4 +2 +2 -4 +2 +6 +2 -4 -4 -4 +4 +1 -4 -6 +2 +2 +2 -2 +6 +2 +2 +2 +2 +3 +4 -4 -1 +2 -2 -3 +2 +2 ...
Forte# (6-)        41 42 43 44 45 46 47 48 49 50
SATV6A(cont.)      +0 -0 +0 -2 -0 -0 -2 +0 -0 -0  >
SATV6B(cont.)      -4 +2 -4 +4 +2 +2 +2 -2 +2 +2  >

Forte# (7-)         1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
SATV7A(10-3)    < -2 -2 -2 -2 +0 +0 +0 -1 +0 -0 -0 -0 +0 +0 +0 -0 -1 -0 +2 +0 -4 -1 -2 +0 -0 -2 -2 +2 -2 +0 -0 -0 +0 -1 -2 -0 -1 -0  >
SATV7B(10-3)    < +2 +4 +4 +2 -4 -4 -8 +2 -4 +4 +4 +1 -4 -4 -4 +4 +2 +2 -4 -4 +4 +2 +4 -4 +4 +2 +4 -4 +2 -4 +6 +4 -4 +2 +2 +2 +2 +2  >

Forte# (8-)         1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29
SATV8A(10-3)    < +1 -2 -0 -2 +0 +0 -1 +0 +0 -0 -2 -0 -0 -2 +0 +0 -0 -0 +2 -1 +0 -2 +1 +0 +0 -0 -0 -0 +0  >
SATV8B(10-3)    < -2 +2 +2 +2 -4 -2 +1 -2 -2 +2 +2 +4 +4 +2 -2 -4 +2 +4 -4 +1 -3 +2 -2 -3 -2 +2 +4 +1 -2  >

Forte# (9-)         1  2  3  4  5  6  7  8  9 10 11 12
SATV9A(10-3)    < +0 -0 -0 +0 +0 +0 -0 +0 +0 -0 -0 +0>
SATV9B(10-3)    < -2 +2 +2 -2 -4 -2 +2 -4 -2 +2 +2 -1>

Forte# (10-)        1  2  3  4  5  6
SATV10A(10-3)   < +0 +0 -0 +0 +0 +0  >
SATV10B(10-3)   < -1 -1 +1 -1 -1 -1  >
```

Figure 27. Complete SATV of SC 10-3 [012345679a]

Forte# (2-)		1	2	3	4	5	6	
SATV2A(6-z37)	<	-1	-3	+2	+1	+2	+1	>
SATV2B(6-z37)	<	+4	+3	-3	-3	-3	-2	>

Forte# (3-)		1	2	3	4	5	6	7	8	9	10	11	12	
SATV3A(6-z37)	<	-1	-2	+2	+2	+2	+1	+0	+2	+1	+0	+2	-1	>
SATV3B(6-z37)	<	+3	+4	-4	-4	-6	-5	-6	-10	-3	-4	-4	+1	>

Forte# (4-)		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	
SATV4A(6-z37)	<	-1	-2	-1	+0	-2	-1	+0	+0	+0	+0	+0	+0	+0	-0	+0	+0	+0	+0	+2	+0	+0	+0	+0	+1	+0	+0	+0	+0	-0	>
SATV4B(6-z37)	<	+2	+2	+1	-2	+2	+1	-3	-2	-2	-2	-2	-2	-2	+2	-2	-4	-3	-2	-4	-3	-6	-4	-3	-5	-3	-2	-2	-1	+2	>

Forte# (5-)		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	
SATV5A(6-z37)	<	-1	+0	+0	+0	-0	+0	+0	+0	+0	+0	+0	+0	-0	+0	+0	+0	-0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	>
SATV5B(6-z37)	<	+1	-2	-2	-2	+2	-2	-4	-1	-2	-2	-2	-1	+2	-2	-2	-2	+1	-2	-2	-2	-6	-1	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-6	-1	-2	-2	-1	-2	>

Figure 28. SATVn(6-z37 [012348]) for n = 2 through 5

set class pairs (larger than #2) to the same degree. This is true of no other complementary sc cardinalities.

SATSIM*n*: A Similarity Index to Compare Scs Using SATV*n*

In order to compare two set classes using the expanded SATV*n*, we will use a generalized version of SATSIM(*X*, *Y*) that is capable of comparing SATV*n*s, where *n* represents either a single cardinality or a range of cardinalities that are smaller than both #*X* and #*Y*. Any SATSIM*n*(*X*, *Y*) that uses all applicable SATV*n* values (from $n = 2$ to the smaller of $n = \#X\text{-}1$ and $n = \#Y\text{-}1$) will be referred to as TSATSIM(*X*, *Y*) (the "total" saturation vector similarity index).

Figure 28 shows all significant SATV*n* vectors for 6-z37 [012348]. Figure 29 shows all possible SATV*n* vectors for 6-z4 [012456]. As one can see from a quick glance at the respective SATV2s, the two hexachords are SATV2 Z-related (and also ICV Z-related, since there are no SATV2 Z-relations among same-sized scs that are not also ICV Z-relations). Therefore, SATSIM2(6-z37, 6-z4) = 0.00. For all other SATV*n*s (where $n > 2$), however, the two scs appear rather different. In fact, there are no ICV or SATV2 Z-related scs that are also SATV*n*>2 Z-related. Even the two all-interval tetrachords (4-z15 [0146] and 4-z29 [0137]) have different SATV3 vectors. Figure 30 shows the SATV3 vectors of 6-z37 and 6-z4 and provides a SATSIM3 comparison of the two. SATSIM3 is calculated precisely the same as SATSIM2, described earlier. Like SATV2(*X*), the values in SATV3(*X*) (and any SATV*n*(*X*)) add up to constant cardinalities for constant *n* and #*X* values. The list of SATV*n* cardinalities is given in Figure 31.

Rather than rigorously examining SATSIM2, SATSIM3, SATSIM4, etc., as separate similarity indices and providing value group comparisons and demonstrations of each, I will instead discuss TSATSIM—the total subset saturation similarity index. As mentioned above, TSATSIM is an amalgam of all possible and non-trivial SATSIM*n* relations. A possible SATSIM*n*(*X*, *Y*) relation is one in which $n \leq \#X$ and $n \leq \#Y$ (i.e., we must be examining subsets of at least the cardinality of the smaller pcset being compared). A non-trivial relation is one in which $n < \#X$ and $n < \#Y$. A SATSIM*n*(*X*, *Y*) relation where $n = \#X$ or #*Y* is considered trivial because the smaller sc will be maximally saturated with itself and minimally saturated with all other scs of its cardinality.

We can now work through a TSATSIM comparison using the same ICV Z-related pair of hexachords (6-z37 and 6-z4). Because of its rather cumbersome length, this vector comparison will be demonstrated only once, and not in as much detail as the other ones. Since these two scs are ICV (and SATV2) Z-related, the SATSIM2(6-z37 and 6-z4) comparison

Forte# (2-)		1	2	3	4	5	6	
SATV2A(6-z4)	<	-1	-3	+2	+1	+2	+1	>
SATV2B(6-z4)	<	+4	+3	-3	-3	-3	-2	>

Forte# (3-)		1	2	3	4	5	6	7	8	9	10	11	12	
SATV3A(6-z4)	<	-2	+2	-2	-2	+2	+2	+2	+2	+0	+0	+0	+0	>
SATV3B(6-z4)	<	+2	-4	+4	+4	-6	-4	-4	-10	-4	-4	-6	-2	>

Forte# (4-)		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	
SATV4A(6-z4)	<	+0	-2	-1	-0	-2	+0	-1	-1	+0	+0	-0	+0	+0	+0	-0	+0	+0	+0	+0	+0	+1	+0	+0	+0	+0	+0	+0	+0	+0	>
SATV4B(6-z4)	<	-3	+2	+1	+2	+2	-2	+2	+1	-2	-2	+2	-2	-2	-2	+2	-4	-3	-2	-6	-3	-5	-4	-3	-6	-3	-2	-2	-1	-2	>

Forte# (5-)		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	
SATV5A(6-z4)	<	+0	+0	-0	+0	+0	-0	+0	+0	-0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	+0	>
SATV5B(6-z4)	<	-2	-2	+2	-2	-2	+2	-4	-1	+2	-2	-2	-1	-2	-2	-2	-2	-1	-2	-2	-2	-6	-1	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-6	-1	-2	-2	-1	-2	>

Figure 29. SATV*n*(6-z4 [012456]) for *n* = 2 through 5

$SATV_3$(6-z37)	<	-1	-2	+2	+2	+2	+1	+0	+2	+1	+0	+2	-1	>
	<	+3	+4	-4	-4	-6	-5	-6	-10	-3	-4	-4	+1	>
$SATV_3$(6-z4)	<	-2	+2	-2	-2	+2	+2	+2	+2	+0	+0	+0	+0	>
	<	+2	-4	+4	+4	-6	-4	-4	-10	-4	-4	-6	-2	>
Diff. $SATV_3$(6-z4, 6-z37)	1,1	2,2	2,2	2,2	0,0	1,1	2,2	0,0	1,1	0,0	2,2	1,1		

$\sum$ Diff. $SATV_3$(6-z4, 6-z37) = 28

#$SATV_3$(6-z4) = 70; #$SATV_3$(6-z37) = 70 *(see figure 31 below)*

$SATSIM_3$(6-z4, 6-z37) = 28 / 140 = **0.20**

Figure 30. SATSIM3(6-z4, 6-z37)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	6	12	18	21	28	21	18	12	6	0	0
3	0	0	0	12	29	42	70	74	77	61	27	0	0
4	0	0	0	0	29	45	83	114	164	142	87	0	0
5	0	0	0	0	0	38	80	107	185	208	160	0	0
6	0	0	0	0	0	0	50	73	145	213	178	0	0
7	0	0	0	0	0	0	0	38	82	134	160	0	0
8	0	0	0	0	0	0	0	0	29	50	87	0	0
9	0	0	0	0	0	0	0	0	0	12	27	0	0
10	0	0	0	0	0	0	0	0	0	0	6	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 31. Cardinalities of Saturation Vectors ($SATV_n$) for all cardinalities of supersets (columns) and subsets (rows).

$=\frac{0}{56}= 0.00$. As noted above, the SATSIM3 comparison $=\frac{28}{140}= 0.20$. The SATSIM4 comparison $= \frac{40}{166} = 0.24$; and the SATSIM5 comparison $= \frac{24}{160} = 0.15$. The TSATSIM value is calculated by dividing the sum of the numerators by the sum of the denominators. Therefore:

$$\text{TSATSIM(6-z37, 6-z4)} = \frac{0 + 28 + 40 + 24}{56 + 140 + 166 + 160} = \frac{92}{522} = 0.18$$

Depending on how one wanted to weight this comparison, one could also derive a total subset saturation index by averaging the individual SATSIM*n* values. This would give equal weight to each SATSIM*n* comparison rather than slanting the TSATSIM comparison toward the SATSIM*n* comparison(s) where *n* is closest to 6 (i.e., the comparisons with the great-

	#3	#4	#5	#6	#7	#8	#9	#10
#3	*0.000* 0.500 0.167 0.312 4							
#4	0.000 0.567 0.033 0.345 18	*0.000* 0.383 0.085 0.243 9						
#5	0.061 0.636 0.061 0.363 18	0.064 0.473 0.064 0.287 44	*0.000* 0.370 0.111 0.227 16					
#6	0.100 0.700 0.100 0.421 23	0.069 0.524 0.069 0.347 58	0.000 0.484 0.062 0.265 103	*0.000* 0.368 0.084 0.201 36				
#7	0.061 0.636 0.061 0.363 18	0.141 0.563 0.141 0.382 56	0.132 0.502 0.132 0.335 98	0.080 0.432 0.080 0.266 173	*0.000* 0.396 0.118 0.240 50			
#8	0.000 0.567 0.033 0.345 18	0.155 0.613 0.155 0.397 58	0.215 0.548 0.215 0.426 96	0.125 0.488 0.125 0.355 178	0.028 0.480 0.028 0.300 240	*0.000* 0.465 0.095 0.257 81		
#9	*0.000* 0.500 0.167 0.312 4	0.192 0.575 0.192 0.390 42	0.226 0.557 0.226 0.429 80	0.197 0.577 0.197 0.416 157	0.177 0.549 0.177 0.391 169	0.119 0.528 0.119 0.342 172	*0.000* 0.568 0.190 0.311 32	
#10	0.000 0.500 0.056 0.318 10	0.188 0.550 0.188 0.394 25	0.219 0.618 0.219 0.432 56	0.305 0.608 0.305 0.452 105	0.272 0.591 0.272 0.441 103	0.212 0.585 0.212 0.432 96	0.156 0.556 0.156 0.418 42	*0.000* 0.533 0.380 0.377 10

Figure 32. TSATSIM value group matrix

est number of elements). This second way of deriving the comparison follows:

Average of all non-trivial SATSIMn(6-z37, 6-z4) =

$$\frac{0.00 + 0.20 + 0.24 + 0.15}{4} = 0.15$$

This value is lower than that TSATSIM value that we first calculated because the SATV2 Z-relation is now weighted as one quarter of the total comparison, whereas it only amounted to a little more than ten percent in the earlier index. One can certainly make a case for calculating the index either way. I call the first method TSATSIM(X, Y) and the second method AvgSATSIMn(X, Y). The value-group matrix for TSATSIM(X, Y) is given in Figure 32 and the sc-specific summary of values appears in Appendix D; the value-group matrix for AvgSATSIMn(X, Y) is given in Figure 33, with the sc-specific summary of values in Appendix E.

The range of TSATSIM values is flatter than the range of SATV2 values. There are fewer possible low numbers, largely because of the decreased probability of similar embedding patterns of #3 and larger subset classes. The highest TSATSIM values are not as high as any of the other indices we have examined because of the very large SATVn cardinalities when $n > 3$. Dividing by the combined cardinality of the vectors creates a remarkably large denominator, which invariably leads to a relatively low dissimilarity level.

Both TSATSIM and AvgSATSIMn have more limitations than SATSIM2. Neither can compare dyad classes to any other set class because both use SATV2 through SATV((#X < #Y)–1) as data. When either #X or #Y = 2, TSATSIM or AvgSATSIMn would have an impossible range of values to compare. Also, when the smaller of the two scs, X or Y, is a trichord, TSATSIM and AvgSATSIMn are the same as SATSIM2, and therefore yield the complementary equivalences that are not present in other (larger) TSATSIM and AvgSATSIMn comparisons. This means that when comparing a pcset larger than a hexachord to a trichord, the larger the difference, the *smaller* the average TSATSIM or AvgSATSIMn comparison would be. This is the opposite of what happens when two scs that are both larger than a trichord are compared using those indices. In both cases, the larger the difference, the higher the average comparison value.[48] Despite these admitted quirks, TSATSIM and AvgSATSIMn are both useful indices in cases where an analyst cares to use a saturation-based index, but also wants to differentiate between Z-related and complementary set classes.

TSATSIM and AvgSATSIMn both differentiate Z-related and complementary scs (as before, all values of 0.000 in the value-group matrices are italicized when maximal similarity occurs trivially—i.e., when the two sets being compared are members of the same sc). Interestingly, there are two special cases of TSATSIM and AvgSATSIMn equivalence.

	#3	#4	#5	#6	#7	#8	#9	#10
#3	*0.000* 0.500 0.167 0.312 4							
#4	0.000 0.567 0.033 0.345 18	*0.000* 0.416 0.069 0.252 13						
#5	0.061 0.636 0.061 0.363 18	0.061 0.493 0.061 0.289 234	*0.000* 0.407 0.092 0.232 32					
#6	0.100 0.700 0.100 0.421 23	0.074 0.541 0.074 0.345 294	0.000 0.517 0.059 0.263 904	*0.000* 0.417 0.082 0.204 136				
#7	0.061 0.636 0.061 0.363 18	0.129 0.567 0.129 0.356 283	0.096 0.525 0.096 0.314 623	0.082 0.471 0.082 0.263 1049	*0.000* 0.418 0.108 0.237 229			
#8	0.000 0.567 0.033 0.345 18	0.104 0.594 0.104 0.361 145	0.176 0.552 0.176 0.382 605	0.166 0.517 0.166 0.348 837	0.054 0.497 0.054 0.295 628	*0.000* 0.462 0.083 0.255 189		
#9	*0.000* 0.500 0.167 0.312 4	0.139 0.556 0.139 0.375 82	0.206 0.575 0.206 0.400 251	0.213 0.595 0.213 0.403 354	0.163 0.560 0.163 0.376 264	0.125 0.529 0.125 0.337 213	*0.000* 0.533 0.191 0.306 41	
#10	0.000 0.500 0.056 0.318 10	0.170 0.560 0.170 0.400 34	0.235 0.636 0.235 0.448 130	0.301 0.637 0.301 0.476 185	0.274 0.612 0.274 0.455 138	0.215 0.583 0.215 0.435 113	0.132 0.539 0.132 0.408 48	*0.000* 0.525 0.386 0.374 12

Figure 33. AvgSATSIMn value-group matrix

These are between the set-class pairs 5-21[01458] / 6-20[014589] and 5-33[02468] / 6-35[02468a]. Not surprisingly, these two set-class pairs are also maximally similar using the SATSIM2 index.

PSATV*n* and its Associated Similarity Index

Naturally, it is possible to create one-part proportional vectors from the generalized SATV*n*. Constructing such vectors and associated similarity indices is accomplished in precisely the same manner shown in the sections above that cover the interval-class PSATV and PSATSIM. Because the mechanics are identical, I will withhold further discussion of them in this article.

Conclusions

Much atonal analysis has relied—directly or indirectly—on data from functions and measures such as Lewin's EMB and COV and Forte's K and Kh. These tools all answer the question "is element *x* embedded in or does it embed some other element *y*?" with either a yes or no answer (K and Kh) or with a number that tells one how many *x* are embedded in or embed *y* (EMB and COV). While such information is undoubtedly useful, I hope it is enriched by the frames of reference that saturation-based tools provide.

Saturation-based data, by the nature of their design, make *a priori* adjustments for cardinality. This forms the primary methodological difference between earlier similarity indices and the methods for comparing sets introduced in this article. While the fundamental concern in creating most new similarity indices has been how one might construct a measure that better adjusts for cardinality, my primary focus has been on creating data that are themselves more cardinality neutral.

Accordingly, SATSIM*n*(*X*, *Y*), PSATSIM(*X*, *Y*), TSATSIM(*X*, *Y*), and AvgSATSIM*n*(*X*, *Y*), are all based upon Robert Morris's ASIM(*X*, *Y*), a relatively simple and straightforward method for comparing two interval-class vectors. Certainly, these new indices are not the only algorithms by which saturation values can be measured. Systems based upon standard deviation, such as those designed by Isaacson and Hermann, present but two of many other possibilities.[49] In fact, saturation vectors can be used in *any* vector-based measure of resemblance, and I hope they will prove helpful in relating sets (of equal or unequal size), regardless of the technique one uses in drawing the comparisons.

NOTES

1. Howard Hanson employed such vector equivalence in his groundbreaking book, *Harmonic Materials of Modern Music* (Hanson 1960).
2. Summaries of many such relations and their methodological foundations can be found in Isaacson 1990, Hermann 1993, and Castrén 1994.
3. Following Lewin's convention (Lewin 1979–80a), the set class of pcset X will be denoted as $/X/$.
4. Following Morris's convention (Morris 1987), $\#/X/$ denotes the cardinality of (number of elements in) sc X.
5. We will, for now, assume that canonical forms of pcset X include only transpositions and inversions of X ($T_n(X)$ and $T_nI(X)$).
6. Because the use of set class is inherent to the notion of "abstract subset," AS($/X/$, $/Y/$) is functionally the same as AS(X, Y). If $X \subset Y$, $/X/$ is an abstract subset of $/Y/$. AS($/X/$, $/Y/$) is functionally equivalent to Morris's KI relation (1990, 277).
7. Lewin 1977, 1979–80a, and 1987. In his 1979–80 article, Lewin uses the form EMB($/X/$, Y), but comments that EMB($/X/$, $/Y/$) provides the same information (433). Consistent with my definition of AS, I prefer the latter because it deals entirely with set classes. In his 1987 book *Generalized Musical Intervals and Transformations*, Lewin more completely describes the differences between EMB(X, $/Y/$), EMB($/X/$, Y), and EMB($/X/$, $/Y/$) (106).
8. This abbreviation is adopted from Castrén (1994, 3) who, in turn, adopted it from Lewin (1987, 106–7). In this context, n is used as a variable representing cardinality.
9. Even though there is only one interval present in a set of cardinality 2, a dyad (or dyad class) is not the same as an interval. An interval represents a distance between elements; a dyad is a set with two members. Despite this important distinction, one might think of the interval-class vector as a #2 subset-class vector—a listing of the number and type of #2 subsets embedded within a pcset. While such a vector would be identical in appearance to the ICV, conceptualizing the elements as subsets rather than intervals will allow us to generalize outwards and produce other subset-class vectors (e.g., a trichord-class vector, tetrachord-class vector, etc.).
10. One could also define the ICV using Lewin's function COV($/X/$, $/Y/$). This function, also presented in Lewin 1979–80a (434), returns the number of distinct forms of $/Y/$ that cover—or include—a member of $/X/$. Where $\#X = 2$ (that is, when X is a dyad), COV($/X/$, $/Y/$) = EMB($/X/$, $/Y/$). COV and EMB will also return the same value in all cases where pcset X is inversionally symmetrical. When pcset X is not inversionally symmetrical, these two functions return different values. Morris 1987 (90) provides an example where these two functions serve different purposes. Since we will be dealing with abstract inclusion of all pcset classes in the course of this study, EMB($/X/$, $/Y/$) will serve as a more useful model than COV($/X/$, $/Y/$).
11. The function SATEMB can also be considered as a special case of the membership function (μ) used in "fuzzy" set theory. Where inclusion in classical ("crisp") set theory deals exclusively with whether or not element x is included in set A, fuzzy set theory allows a statement such as element x (or sc $/X/$) is only partially

a member of set A. While SATEMB does not model partial membership, it does model partial "saturation," that is, the degree to which /X/ is maximally and/or minimally embedded in pcset A, given what is possible in any pcset of $\#A$.

12. It does not matter whether we discuss specific sets or set classes since the interval-class vectors (and therefore saturation vectors) of a set are those for the set class to which it belongs.
13. Morris's SG(n) vectors provide a number of alternate means of determining equivalence. His first three set groups include the most common means of grouping pcsets: under transposition alone, transposition and inversion, T, I, and multiplication by 5 (Morris 1982).
14. Cf. Morris 1987, 128–35; Perle 1996; and Headlam 1996, 14–22.
15. Buchler 2000, 55.
16. Morris 1990, 179.
17. Buchler 2000, 56.
18. Ericksson 1986, 96–100.
19. This, too, is explained in greater detail in Buchler 2000.
20. For those readers who are not comfortable reading logical symbols: statements on the left in parentheses are conditions; the right-pointing arrows indicate what happens *if* the condition is true; and ~ indicates negation. So, the second line of the derivation would read "*If* it is not true that c mod p equals 0, *then* s is assigned the quantity to the right of the equal sign.
21. For example, when calculating the minimum number of ic5 in a pcset with fewer than six elements the first part of the equation yields a negative number, which indicates that the minimal number of ic5 in a set of that cardinality is zero.
22. The collection of mutliple-cyclic set classes also appears in Ericksson 1986 as "maxgroup 2" (2/4/6-cycle sets) and "maxgroup 3" (3/6-cycle sets), though Ericksson does not discuss their cyclic properties (Ericksson 1986, 97–99).
23. More formally, the difference values $\max(c, i) - \min(c, i)$ and $\max(d, i) - \min(d, i)$ are the same for all i where c and d are inverses, mod 12.
24. Ericksson 1986, 96.
25. Of course, this is to say nothing of the innumerable possibilities for orchestration into p-space and the structures that might exist on a much larger plane.
26. Kh is described in Forte 1973, 96–100. ZC is described in Morris 1982, 103–9; 1987, 74; 1990, 180; and 1997, 276.
27. The words "positive" and "negative" are in quotes because the values are not actually positive or negative. The sign indicates whether the ICV value is being compared to a smaller or larger ($\min(c, i)$ or $\max(c, i)$) value. +0 and –0 are opposite ICV(c, i) extremes: the first indicates minimal saturation of ic i in cardinality c; the second indicates maximal saturation of ic i in cardinality c.
28. This could easily be transformed into a similarity index by subtracting SATSIM(X, Y) values from 1.
29. Morris 1979–80, 450–1.
30. Buchler 1997, 51–5; and 2000, 71–4.
31. It is worth noting again that this final cardinality adjustment is similar to the construction of Morris's ASIM index (Morris 1979–80, 450–1).
32. One could argue that this is only a semantic difference in the case of maximally similar SATSIM relations (i.e., SATSIM(X, Y) = 0). While similarity measures

normally have no transitivity features, there is transitivity among maximally similar SATSIM relations. If SATSIM(X, Y) = 0 and SATSIM(Y, Z) = 0, then SATSIM(X, Z) = 0. Maximal SATSIM similarity therefore meets the criteria for equivalence; that is, it is reflexive, symmetrical, and transitive.

33. This is, in essence, an equivalence group of SATV Z-relations.

34. The $\mathrm{SATV_B}$ vector is not different for sets X and Y in the first SATSIM group in Figure 13 because the empty set/aggregate and the one-member/eleven-member sc both maximize *and* minimize all possible interval classes for their cardinalities. This is because there is only one set class that belongs to each of those cardinalities (#0/#12 and #1/#11).

35. SATSIM groups 2 through 6, shown in Figure 13, are all segments of Ki subcomplexes about 10-3, 10-6, 10-2, and/or 10-4 (see Kaplan 1990). The set classes in SATSIM group 1 would (almost trivially) be the progenitors/end-points of all Ki subcomplexes had Kaplan elected to extend the boundaries of Ki beyond #2 and #10 sets.

36. There are only 111 distinct $\mathrm{SATV_A}$s among the 4096 possible pcsets; by contrast, there are 224 different set classes ($\mathrm{T}_n/\mathrm{T}_n\mathrm{I}$ equivalence classes) among the 4096 possible pcsets (including the empty set).

37. This relationship would be further simplified by allowing T^nM/MI equivalence into our canon. These two pcset pairs would fold into a single pair since 5/7-4 is M/MI related to 5/7-29 and 6-2 is M/MI related to 6-33.

38. Castrén defines a value group as follows: "The value group *#X/#Y* contains the values that a given similarity index returns to the sc pairs in the comparison group *#X/#Y*" (Castrén, 5). A comparison group *#X/#Y* "contains all sc pairs {X,Y} such that X belongs to #X and Y belongs to #Y" (Castrén, 5).

39. When #X = #Y, the value 0.000 in the upper left corner is italicized if it represents only the trivial case of one sc compared with itself. If the upper left number is an unitalicized 0.000, there is some SATSIM Z-relation in the value group.

40. This observation can now be refined as follows: the primary determinant of average SATSIM values in any value group is the difference in cardinality between the two set classes. Thus, if $|\#W - \#X| > |\#Y - \#Z|$, SATSIM($W$, X) will, on average, yield a larger value than SATSIM(Y, Z). If, however, the difference between the cardinalities of W and X and Y and Z is the same, the secondary determinant of average SATSIM values is the size of the set classes being compared. If $|\#W - \#X| = |\#Y - \#Z|$ and $(\#W + \#X) > (\#Y + \#Z)$, SATSIM($W$, X) will, on average, yield a smaller value than SATSIM(Y, Z). This generalization does not always hold true; for example, the average SATSIM(X, Y) value where #X = 2 and #Y = 3 is smaller than the average SATSIM(X, Y) value where #X = 3 and #Y = 4. The difference between these two averages, however, is small (0.027) and does not undermine the above claim.

41. However, the average SATSIM(X, Y) comparison where #X or #Y = 2 or 10 is larger than the average SATSIM(X, Y) comparison where #X = #Y ≠ 2 or 10. SATSIM considers #2/#10 sets to be so distinct because they are maximally saturated with one ic and minimally saturated with all the others.

42. E.g., those from the same Ki subcomplex (Kaplan 1990) or ic maxpoint structure (Ericksson 1986). With some cycles, there is more than one cyclic sc of the same cardinality (e.g., 6-cycle scs). However, such cases always fall within different maxpoint structures/Ki subcomplexes/categories of multiple-cyclic scs.

43. Unlike SATV, the cardinality of PSATV is derived like the ICV:

$$\#\text{PSATV} = \sum_{n=1}^{6} (\text{PSATV}(X)_n)$$

44. These figures have been rounded from the 8-bit real numbers generated by computer when calculating the measurement values.
45. Recall that 6-32 and 5/7-33 form one of the special PSATV Z-relations.
46. One can also construct saturation-based *n*-class % vectors of the sort used by Castrén 1994 in his relations. See Buchler 1997, 68–72.
47. Cf. Lewin's discussion of TCH and RICH (transposition and retrograde inversion chains) in Lewin 1987, 180–88.
48. These difficulties are not unique to TSATSIM and AvgSATSIM*n*. Rather, they occur in every so-called "total" similarity index (the term is Castrén's), including Rahn's ATMEMB, Lewin's REL (when the TEST group includes all scs smaller than the sets being compared), and Castrén's RECREL.
49. Isaacson 1990 and Hermann 1993.

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APPENDIX A

Statistical summary of SATSIM2 relations for each cardinality pair #2/#10 through #6 sc compared to every other #2/#10 through #6 sc (SATSIM2 value group #2/#10 . . . #6:#2/#10 . . . #6)

Prime form	Forte	Average	Lowest	Highest
[01]	2-1	0.474	0.056	0.647
[02]	2-2	0.481	0.111	0.706
[03]	2-3	0.485	0.111	0.704
[04]	2-4	0.511	0.125	0.735
[05]	2-5	0.474	0.056	0.647
[06]	2-6	0.480	0.111	0.676
#2 Averages:		0.484	0.095	0.686
[012]	3-1	0.390	0.056	0.575
[013]	3-2	0.334	0.121	0.576
[014]	3-3	0.338	0.133	0.675
[024]	3-6	0.401	0.100	0.576
[015]	3-4	0.325	0.033	0.675
[025]	3-7	0.334	0.121	0.576
[016]	3-5	0.358	0.061	0.625
[026]	3-8	0.379	0.067	0.600
[036]	3-10	0.442	0.111	0.567
[027]	3-9	0.390	0.056	0.575
[037]	3-11	0.338	0.133	0.675
[048]	3-12	0.462	0.133	0.700
#3 Averages:		0.374	0.094	0.616
[0123]	4-1	0.374	0.051	0.556
[0124]	4-2	0.321	0.111	0.556
[0134]	4-3	0.290	0.077	0.609
[0125]	4-4	0.275	0.077	0.587
[0135]	4-11	0.283	0.077	0.556
[0235]	4-10	0.274	0.077	0.542
[0145]	4-7	0.340	0.111	0.565
[0126]	4-5	0.288	0.077	0.583
[0136]	4-13	0.242	0.077	0.583
[0236]	4-12	0.266	0.103	0.583
[0146]	4-z15	0.251	0.103	0.500
[0246]	4-21	0.441	0.087	0.590
[0156]	4-8	0.321	0.033	0.609
[0127]	4-6	0.301	0.051	0.609
[0137]	4-z29	0.251	0.103	0.500

Appendix A *(continued)*

[0237]	4-14	0.275	0.077	0.587
[0147]	4-18	0.262	0.103	0.583
[0247]	4-22	0.321	0.111	0.556
[0347]	4-17	0.317	0.103	0.587
[0157]	4-16	0.288	0.077	0.583
[0257]	4-23	0.374	0.051	0.556
[0167]	4-9	0.365	0.051	0.609
[0148]	4-19	0.377	0.103	0.583
[0248]	4-24	0.439	0.087	0.590
[0158]	4-20	0.340	0.111	0.565
[0258]	4-27	0.266	0.103	0.583
[0358]	4-26	0.290	0.077	0.609
[0268]	4-25	0.432	0.067	0.587
[0369]	4-28	0.445	0.125	0.567
#4 Averages:		0.321	0.085	0.575
[01234]	5-1	0.370	0.051	0.593
[01235]	5-2	0.298	0.077	0.571
[01245]	5-3	0.267	0.077	0.593
[01236]	5-4	0.257	0.020	0.593
[01246]	5-9	0.269	0.077	0.519
[01346]	5-10	0.264	0.082	0.593
[02346]	5-8	0.283	0.095	0.630
[01256]	5-6	0.256	0.077	0.592
[01356]	5-z12	0.220	0.041	0.630
[01237]	5-5	0.256	0.082	0.593
[01247]	5-z36	0.220	0.041	0.630
[01347]	5-16	0.264	0.095	0.592
[02347]	5-11	0.231	0.041	0.630
[01257]	5-14	0.256	0.082	0.593
[01357]	5-24	0.269	0.077	0.519
[02357]	5-23	0.298	0.077	0.571
[01457]	5-z18	0.220	0.041	0.612
[01267]	5-7	0.346	0.041	0.593
[01367]	5-19	0.261	0.095	0.667
[01248]	5-13	0.279	0.095	0.519
[01348]	5-z17	0.272	0.082	0.612
[01258]	5-z38	0.220	0.041	0.612
[01358]	5-27	0.267	0.077	0.593
[02358]	5-25	0.264	0.082	0.593
[01458]	5-21	0.343	0.095	0.667
[02458]	5-26	0.280	0.095	0.519

[03458]	5-z37	0.272	0.082	0.612
[01268]	5-15	0.292	0.143	0.704
[01368]	5-29	0.257	0.020	0.593
[02368]	5-28	0.280	0.095	0.593
[01468]	5-30	0.279	0.095	0.519
[02468]	5-33	0.524	0.103	0.653
[01568]	5-20	0.256	0.077	0.592
[01478]	5-22	0.285	0.095	0.630
[01369]	5-31	0.364	0.154	0.667
[01469]	5-32	0.264	0.095	0.592
[02469]	5-34	0.283	0.095	0.630
[02479]	5-35	0.370	0.051	0.593
#5 Averages:		0.283	0.077	0.600
[012345]	6-1	0.336	0.071	0.653
[012346]	6-2	0.286	0.020	0.647
[012356]	6-z3	0.277	0.071	0.706
[012456]	6-z4	0.279	0.071	0.647
[012347]	6-z36	0.277	0.071	0.706
[012357]	6-9	0.260	0.041	0.706
[012457]	6-z11	0.253	0.061	0.706
[013457]	6-z10	0.254	0.071	0.647
[023457]	6-8	0.266	0.041	0.647
[012367]	6-5	0.273	0.071	0.676
[012467]	6-z12	0.257	0.061	0.706
[013467]	6-z13	0.280	0.071	0.676
[012567]	6-z6	0.300	0.071	0.643
[012348]	6-z37	0.279	0.071	0.647
[012358]	6-z40	0.253	0.061	0.706
[012458]	6-15	0.268	0.071	0.647
[013458]	6-14	0.275	0.061	0.676
[023458]	6-z39	0.254	0.071	0.647
[012368]	6-z41	0.257	0.061	0.706
[012468]	6-22	0.289	0.071	0.647
[013468]	6-z24	0.254	0.071	0.647
[023468]	6-21	0.288	0.071	0.647
[012568]	6-z43	0.250	0.061	0.618
[013568]	6-z25	0.277	0.071	0.706
[023568]	6-z23	0.288	0.071	0.706
[014568]	6-16	0.268	0.071	0.647
[012378]	6-z38	0.300	0.071	0.643
[012478]	6-z17	0.250	0.061	0.618
[013478]	6-z19	0.270	0.041	0.647
[012578]	6-18	0.273	0.071	0.676

Appendix A *(continued)*

[013578]	6-z26	0.279	0.071	0.647
[012678]	6-7	0.354	0.041	0.647
[012369]	6-z42	0.280	0.071	0.676
[012469]	6-z46	0.254	0.071	0.647
[013469]	6-27	0.323	0.071	0.735
[023469]	6-z45	0.288	0.071	0.706
[012569]	6-z44	0.270	0.041	0.647
[013569]	6-z28	0.281	0.071	0.618
[012479]	6-z47	0.277	0.071	0.706
[013479]	6-z49	0.281	0.071	0.618
[012579]	6-z48	0.279	0.071	0.647
[013579]	6-34	0.288	0.071	0.647
[023579]	6-33	0.286	0.020	0.647
[014579]	6-31	0.268	0.071	0.647
[024579]	6-32	0.336	0.071	0.653
[013679]	6-30	0.317	0.071	0.735
[023679]	6-z29	0.280	0.071	0.676
[014679]	6-z50	0.280	0.071	0.676
[014589]	6-20	0.371	0.082	0.706
[02468a]	6-35	0.527	0.087	0.675
#6 Averages:		0.286	0.065	0.667

APPENDIX B

Statistical summary of PSATSIM2 relations for each cardinality pair #2/#10 through #6 sc compared to every other #2/#10 through #6 sc (PSATSIM2 value group #2/#10 . . . #6:#2/#10 . . . #6)

Prime form	Forte	Average	Lowest	Highest
[01]	2-1	0.413	0.083	0.667
[02]	2-2	0.420	0.056	0.633
[03]	2-3	0.419	0.167	0.667
[04]	2-4	0.472	0.153	0.656
[05]	2-5	0.413	0.083	0.667
[06]	2-6	0.424	0.139	0.633
#2 Averages:		0.427	0.113	0.654
[012]	3-1	0.378	0.069	0.583
[013]	3-2	0.285	0.111	0.583
[014]	3-3	0.316	0.083	0.611
[024]	3-6	0.417	0.056	0.611
[015]	3-4	0.314	0.139	0.611
[025]	3-7	0.285	0.111	0.583
[016]	3-5	0.352	0.056	0.556
[026]	3-8	0.386	0.083	0.661
[036]	3-10	0.453	0.125	0.600
[027]	3-9	0.378	0.069	0.583
[037]	3-11	0.316	0.083	0.611
[048]	3-12	0.472	0.153	0.656
#3 Averages:		0.363	0.095	0.604
[0123]	4-1	0.367	0.056	0.597
[0124]	4-2	0.317	0.097	0.569
[0134]	4-3	0.293	0.083	0.583
[0125]	4-4	0.269	0.097	0.597
[0135]	4-11	0.276	0.097	0.569
[0235]	4-10	0.271	0.097	0.583
[0145]	4-7	0.340	0.097	0.597
[0126]	4-5	0.274	0.097	0.528
[0136]	4-13	0.235	0.083	0.556
[0236]	4-12	0.264	0.097	0.500
[0146]	4-z15	0.240	0.097	0.458
[0246]	4-21	0.432	0.111	0.606
[0156]	4-8	0.316	0.111	0.583
[0127]	4-6	0.285	0.106	0.583
[0137]	4-z29	0.240	0.097	0.458

[0237]	4-14	0.269	0.097	0.597
[0147]	4-18	0.265	0.097	0.556
[0247]	4-22	0.317	0.097	0.569
[0347]	4-17	0.323	0.097	0.583
[0157]	4-16	0.274	0.097	0.528
[0257]	4-23	0.367	0.056	0.597
[0167]	4-9	0.375	0.056	0.611
[0148]	4-19	0.380	0.097	0.586
[0248]	4-24	0.431	0.111	0.583
[0158]	4-20	0.340	0.097	0.597
[0258]	4-27	0.264	0.097	0.500
[0358]	4-26	0.293	0.083	0.583
[0268]	4-25	0.437	0.083	0.633
[0369]	4-28	0.453	0.125	0.600
#4 Averages:		0.317	0.094	0.569
[01234]	5-1	0.370	0.056	0.625
[01235]	5-2	0.306	0.081	0.625
[01245]	5-3	0.276	0.083	0.611
[01236]	5-4	0.258	0.078	0.583
[01246]	5-9	0.268	0.083	0.514
[01346]	5-10	0.262	0.078	0.583
[02346]	5-8	0.286	0.083	0.597
[01256]	5-6	0.261	0.083	0.569
[01356]	5-z12	0.227	0.078	0.583
[01237]	5-5	0.257	0.083	0.583
[01247]	5-z36	0.227	0.078	0.583
[01347]	5-16	0.266	0.083	0.569
[02347]	5-11	0.245	0.083	0.611
[01257]	5-14	0.257	0.083	0.583
[01357]	5-24	0.268	0.083	0.514
[02357]	5-23	0.306	0.081	0.625
[01457]	5-z18	0.231	0.083	0.569
[01267]	5-7	0.367	0.031	0.625
[01367]	5-19	0.290	0.097	0.625
[01248]	5-13	0.278	0.083	0.528
[01348]	5-z17	0.288	0.092	0.597
[01258]	5-z38	0.231	0.083	0.569
[01358]	5-27	0.276	0.083	0.611
[02358]	5-25	0.262	0.078	0.583
[01458]	5-21	0.365	0.050	0.611
[02458]	5-26	0.279	0.083	0.528

[03458]	5-z37	0.288	0.092	0.597
[01268]	5-15	0.313	0.139	0.639
[01368]	5-29	0.258	0.078	0.583
[02368]	5-28	0.298	0.139	0.556
[01468]	5-30	0.278	0.083	0.528
[02468]	5-33	0.536	0.111	0.689
[01568]	5-20	0.261	0.083	0.569
[01478]	5-22	0.295	0.097	0.611
[01369]	5-31	0.372	0.097	0.625
[01469]	5-32	0.266	0.083	0.569
[02469]	5-34	0.286	0.083	0.597
[02479]	5-35	0.370	0.056	0.625
#5 Averages:		0.290	0.084	0.589
[012345]	6-1	0.344	0.064	0.689
[012346]	6-2	0.293	0.061	0.600
[012356]	6-z3	0.264	0.061	0.661
[012456]	6-z4	0.253	0.067	0.586
[012347]	6-z36	0.264	0.061	0.661
[012357]	6-9	0.255	0.061	0.600
[012457]	6-z11	0.245	0.061	0.661
[013457]	6-z10	0.236	0.067	0.586
[023457]	6-8	0.281	0.083	0.689
[012367]	6-5	0.275	0.061	0.633
[012467]	6-z12	0.252	0.061	0.628
[013467]	6-z13	0.278	0.061	0.633
[012567]	6-z6	0.310	0.067	0.633
[012348]	6-z37	0.253	0.067	0.586
[012358]	6-z40	0.245	0.061	0.661
[012458]	6-15	0.254	0.061	0.572
[013458]	6-14	0.279	0.083	0.661
[023458]	6-z39	0.236	0.067	0.586
[012368]	6-z41	0.252	0.061	0.628
[012468]	6-22	0.282	0.067	0.572
[013468]	6-z24	0.236	0.067	0.586
[023468]	6-21	0.285	0.067	0.572
[012568]	6-z43	0.248	0.069	0.558
[013568]	6-z25	0.264	0.061	0.661
[023568]	6-z23	0.273	0.061	0.628
[014568]	6-16	0.253	0.061	0.572
[012378]	6-z38	0.310	0.067	0.633
[012478]	6-z17	0.248	0.069	0.558
[013478]	6-z19	0.274	0.061	0.633
[012578]	6-18	0.275	0.061	0.633

Appendix B *(continued)*

[013578]	6-z26	0.253	0.067	0.586
[012678]	6-7	0.374	0.031	0.656
[012369]	6-z42	0.278	0.061	0.633
[012469]	6-z46	0.236	0.067	0.586
[013469]	6-27	0.304	0.061	0.633
[023469]	6-z45	0.273	0.061	0.628
[012569]	6-z44	0.274	0.061	0.633
[013569]	6-z28	0.269	0.069	0.558
[012479]	6-z47	0.264	0.061	0.661
[013479]	6-z49	0.269	0.069	0.558
[012579]	6-z48	0.253	0.067	0.586
[013579]	6-34	0.285	0.067	0.572
[023579]	6-33	0.293	0.061	0.600
[014579]	6-31	0.254	0.061	0.572
[024579]	6-32	0.344	0.064	0.689
[013679]	6-30	0.316	0.083	0.656
[023679]	6-z29	0.278	0.061	0.633
[014679]	6-z50	0.278	0.061	0.633
[014589]	6-20	0.382	0.050	0.661
[02468a]	6-35	0.536	0.111	0.689
#6 Averages:		0.281	0.065	0.620

APPENDIX C

Min(#*Y*, *X*) and max(#*Y*, *X*) values for all cardinalities of superset (#*Y*) and all possible subset classes (*X*) where #*X* < #*Y* (when #*X* = #*Y*, min(#*Y*, *X*) = 0 and max(#*Y*, *X*) = 1, except for #0, #1, #11 and #12 scs).

#Y	sc X	Forte#(X)	min(#Y,X)	max(#Y,X)
0	[]	0-0	1	1
1	[0]	1-0	1	1
2	[0]	1-0	2	2
3	[0]	1-0	3	3
3	[01]	2-1	0	2
3	[02]	2-2	0	2
3	[03]	2-3	0	2
3	[04]	2-4	0	3
3	[05]	2-5	0	2
3	[06]	2-6	0	1
4	[0]	1-0	4	4
4	[01]	2-1	0	3
4	[02]	2-2	0	3
4	[03]	2-3	0	4
4	[04]	2-4	0	3
4	[05]	2-5	0	3
4	[06]	2-6	0	2
4	[012]	3-1	0	2
4	[013]	3-2	0	2
4	[014]	3-3	0	2
4	[015]	3-4	0	2
4	[016]	3-5	0	4
4	[024]	3-6	0	2
4	[025]	3-7	0	2
4	[026]	3-8	0	4
4	[027]	3-9	0	2
4	[036]	3-10	0	4
4	[037]	3-11	0	2
4	[048]	3-12	0	1
5	[0]	1-0	5	5
5	[01]	2-1	0	4
5	[02]	2-2	0	4
5	[03]	2-3	0	4
5	[04]	2-4	1	4
5	[05]	2-5	0	4
5	[06]	2-6	0	2
5	[012]	3-1	0	3
5	[013]	3-2	0	4
5	[014]	3-3	0	3
5	[015]	3-4	0	3
5	[016]	3-5	0	5
5	[024]	3-6	0	3
5	[025]	3-7	0	4
5	[026]	3-8	0	6
5	[027]	3-9	0	3
5	[036]	3-10	0	4
5	[037]	3-11	0	3
5	[048]	3-12	0	1
5	[0123]	4-1	0	2
5	[0124]	4-2	0	2
5	[0134]	4-3	0	1
5	[0125]	4-4	0	2
5	[0126]	4-5	0	2
5	[0127]	4-6	0	1
5	[0145]	4-7	0	1
5	[0156]	4-8	0	1
5	[0167]	4-9	0	1
5	[0235]	4-10	0	1
5	[0135]	4-11	0	2
5	[0236]	4-12	0	2
5	[0136]	4-13	0	2
5	[0237]	4-14	0	2
5	[0146]	4-15	0	1
5	[0157]	4-16	0	2
5	[0347]	4-17	0	1
5	[0147]	4-18	0	2
5	[0148]	4-19	0	2
5	[0158]	4-20	0	1
5	[0246]	4-21	0	2
5	[0247]	4-22	0	2
5	[0257]	4-23	0	2
5	[0248]	4-24	0	2
5	[0268]	4-25	0	1
5	[0358]	4-26	0	1
5	[0258]	4-27	0	2

Appendix C *(continued)*

5	[0369]	4-28	0	1
5	[0137]	4-29	0	1
6	[0]	1-0	6	6
6	[01]	2-1	0	5
6	[02]	2-2	0	6
6	[03]	2-3	0	5
6	[04]	2-4	2	6
6	[05]	2-5	0	5
6	[06]	2-6	0	3
6	[012]	3-1	0	4
6	[013]	3-2	0	6
6	[014]	3-3	0	6
6	[015]	3-4	0	6
6	[016]	3-5	0	8
6	[024]	3-6	0	6
6	[025]	3-7	0	6
6	[026]	3-8	0	12
6	[027]	3-9	0	4
6	[036]	3-10	0	4
6	[037]	3-11	0	6
6	[048]	3-12	0	2
6	[0123]	4-1	0	3
6	[0124]	4-2	0	4
6	[0134]	4-3	0	2
6	[0125]	4-4	0	2
6	[0126]	4-5	0	4
6	[0127]	4-6	0	2
6	[0145]	4-7	0	3
6	[0156]	4-8	0	2
6	[0167]	4-9	0	2
6	[0235]	4-10	0	2
6	[0135]	4-11	0	2
6	[0236]	4-12	0	2
6	[0136]	4-13	0	2
6	[0237]	4-14	0	2
6	[0146]	4-15	0	2
6	[0157]	4-16	0	4
6	[0347]	4-17	0	3
6	[0147]	4-18	0	2
6	[0148]	4-19	0	6
6	[0158]	4-20	0	3
6	[0246]	4-21	0	6
6	[0247]	4-22	0	4
6	[0257]	4-23	0	3
6	[0248]	4-24	0	6

6	[0268]	4-25	0	3
6	[0358]	4-26	0	2
6	[0258]	4-27	0	2
6	[0369]	4-28	0	1
6	[0137]	4-29	0	2
6	[01234]	5-1	0	2
6	[01235]	5-2	0	2
6	[01245]	5-3	0	2
6	[01236]	5-4	0	2
6	[01237]	5-5	0	2
6	[01256]	5-6	0	2
6	[01267]	5-7	0	4
6	[02346]	5-8	0	1
6	[01246]	5-9	0	2
6	[01346]	5-10	0	2
6	[02347]	5-11	0	2
6	[01356]	5-12	0	1
6	[01248]	5-13	0	2
6	[01257]	5-14	0	2
6	[01268]	5-15	0	2
6	[01347]	5-16	0	2
6	[01348]	5-17	0	1
6	[01457]	5-18	0	2
6	[01367]	5-19	0	2
6	[01568]	5-20	0	2
6	[01458]	5-21	0	6
6	[01478]	5-22	0	1
6	[02357]	5-23	0	2
6	[01357]	5-24	0	2
6	[02358]	5-25	0	2
6	[02458]	5-26	0	2
6	[01358]	5-27	0	2
6	[02368]	5-28	0	2
6	[01368]	5-29	0	2
6	[01468]	5-30	0	2
6	[01369]	5-31	0	2
6	[01469]	5-32	0	2
6	[02468]	5-33	0	6
6	[02469]	5-34	0	1
6	[02479]	5-35	0	2
6	[01247]	5-36	0	2
6	[03458]	5-37	0	1
6	[01258]	5-38	0	2
7	[0]	1-0	7	7
7	[01]	2-1	2	6

7	[02]	2-2	2	6	7	[01245]	5-3	0	4
7	[03]	2-3	2	6	7	[01236]	5-4	0	2
7	[04]	2-4	3	6	7	[01237]	5-5	0	3
7	[05]	2-5	2	6	7	[01256]	5-6	0	3
7	[06]	2-6	1	3	7	[01267]	5-7	0	5
7	[012]	3-1	0	5	7	[02346]	5-8	0	2
7	[013]	3-2	1	8	7	[01246]	5-9	0	2
7	[014]	3-3	0	7	7	[01346]	5-10	0	3
7	[015]	3-4	0	7	7	[02347]	5-11	0	2
7	[016]	3-5	2	9	7	[01356]	5-12	0	1
7	[024]	3-6	0	6	7	[01248]	5-13	0	2
7	[025]	3-7	1	8	7	[01257]	5-14	0	3
7	[026]	3-8	2	12	7	[01268]	5-15	0	2
7	[027]	3-9	0	5	7	[01347]	5-16	0	3
7	[036]	3-10	1	5	7	[01348]	5-17	0	2
7	[037]	3-11	0	7	7	[01457]	5-18	0	2
7	[048]	3-12	0	2	7	[01367]	5-19	0	3
7	[0123]	4-1	0	4	7	[01568]	5-20	0	3
7	[0124]	4-2	0	6	7	[01458]	5-21	0	6
7	[0134]	4-3	0	3	7	[01478]	5-22	0	2
7	[0125]	4-4	0	4	7	[02357]	5-23	0	4
7	[0126]	4-5	0	5	7	[01357]	5-24	0	2
7	[0127]	4-6	0	3	7	[02358]	5-25	0	3
7	[0145]	4-7	0	3	7	[02458]	5-26	0	2
7	[0156]	4-8	0	3	7	[01358]	5-27	0	4
7	[0167]	4-9	0	2	7	[02368]	5-28	0	3
7	[0235]	4-10	0	2	7	[01368]	5-29	0	2
7	[0135]	4-11	0	4	7	[01468]	5-30	0	2
7	[0236]	4-12	0	4	7	[01369]	5-31	0	3
7	[0136]	4-13	0	4	7	[01469]	5-32	0	3
7	[0237]	4-14	0	4	7	[02468]	5-33	0	6
7	[0146]	4-15	0	4	7	[02469]	5-34	0	2
7	[0157]	4-16	0	5	7	[02479]	5-35	0	3
7	[0347]	4-17	0	3	7	[01247]	5-36	0	2
7	[0147]	4-18	0	4	7	[03458]	5-37	0	2
7	[0148]	4-19	0	7	7	[01258]	5-38	0	2
7	[0158]	4-20	0	3	7	[012345]	6-1	0	2
7	[0246]	4-21	0	6	7	[012346]	6-2	0	2
7	[0247]	4-22	0	6	7	[012356]	6-3	0	2
7	[0257]	4-23	0	4	7	[012456]	6-4	0	1
7	[0248]	4-24	0	6	7	[012367]	6-5	0	1
7	[0268]	4-25	0	3	7	[012567]	6-6	0	1
7	[0358]	4-26	0	3	7	[012678]	6-7	0	1
7	[0258]	4-27	0	4	7	[023457]	6-8	0	1
7	[0369]	4-28	0	1	7	[012357]	6-9	0	1
7	[0137]	4-29	0	4	7	[013457]	6-10	0	2
7	[01234]	5-1	0	3	7	[012457]	6-11	0	1
7	[01235]	5-2	0	4	7	[012467]	6-12	0	2

Appendix C *(continued)*

7	[013467]	6-13	0	1	8	[012]	3-1	0	6
7	[013458]	6-14	0	2	8	[013]	3-2	4	10
7	[012458]	6-15	0	1	8	[014]	3-3	2	9
7	[014568]	6-16	0	1	8	[015]	3-4	0	9
7	[012478]	6-17	0	2	8	[016]	3-5	4	12
7	[012578]	6-18	0	1	8	[024]	3-6	0	6
7	[013478]	6-19	0	2	8	[025]	3-7	4	10
7	[014589]	6-20	0	1	8	[026]	3-8	4	12
7	[023468]	6-21	0	2	8	[027]	3-9	0	6
7	[012468]	6-22	0	2	8	[036]	3-10	2	8
7	[023568]	6-23	0	1	8	[037]	3-11	2	9
7	[013468]	6-24	0	2	8	[048]	3-12	0	2
7	[013568]	6-25	0	2	8	[0123]	4-1	0	5
7	[013578]	6-26	0	1	8	[0124]	4-2	0	8
7	[013469]	6-27	0	2	8	[0134]	4-3	0	4
7	[013569]	6-28	0	1	8	[0125]	4-4	0	6
7	[023679]	6-29	0	1	8	[0126]	4-5	0	8
7	[013679]	6-30	0	1	8	[0127]	4-6	0	4
7	[014579]	6-31	0	1	8	[0145]	4-7	0	4
7	[024579]	6-32	0	2	8	[0156]	4-8	0	4
7	[023579]	6-33	0	2	8	[0167]	4-9	0	3
7	[013579]	6-34	0	2	8	[0235]	4-10	0	4
7	[02468a]	6-35	0	1	8	[0135]	4-11	0	6
7	[012347]	6-36	0	2	8	[0236]	4-12	0	8
7	[012348]	6-37	0	1	8	[0136]	4-13	0	8
7	[012378]	6-38	0	1	8	[0237]	4-14	0	6
7	[023458]	6-39	0	2	8	[0146]	4-15	2	8
7	[012358]	6-40	0	1	8	[0157]	4-16	0	8
7	[012368]	6-41	0	2	8	[0347]	4-17	0	4
7	[012369]	6-42	0	1	8	[0147]	4-18	0	8
7	[012568]	6-43	0	2	8	[0148]	4-19	0	8
7	[012569]	6-44	0	2	8	[0158]	4-20	0	4
7	[023469]	6-45	0	1	8	[0246]	4-21	0	6
7	[012469]	6-46	0	2	8	[0247]	4-22	0	8
7	[012479]	6-47	0	2	8	[0257]	4-23	0	5
7	[012579]	6-48	0	1	8	[0248]	4-24	0	6
7	[013479]	6-49	0	1	8	[0268]	4-25	0	3
7	[014679]	6-50	0	1	8	[0358]	4-26	0	4
					8	[0258]	4-27	0	8
8	[0]	1-0	8	8	8	[0369]	4-28	0	2
8	[01]	2-1	4	7	8	[0137]	4-29	2	8
8	[02]	2-2	4	7	8	[01234]	5-1	0	4
8	[03]	2-3	4	8	8	[01235]	5-2	0	6
8	[04]	2-4	4	7	8	[01245]	5-3	0	6
8	[05]	2-5	4	7	8	[01236]	5-4	0	4
8	[06]	2-6	2	4	8	[01237]	5-5	0	4

8	[01256]	5-6	0	4
8	[01267]	5-7	0	8
8	[02346]	5-8	0	2
8	[01246]	5-9	0	4
8	[01346]	5-10	0	8
8	[02347]	5-11	0	4
8	[01356]	5-12	0	2
8	[01248]	5-13	0	4
8	[01257]	5-14	0	4
8	[01268]	5-15	0	4
8	[01347]	5-16	0	8
8	[01348]	5-17	0	2
8	[01457]	5-18	0	4
8	[01367]	5-19	0	8
8	[01568]	5-20	0	4
8	[01458]	5-21	0	7
8	[01478]	5-22	0	2
8	[02357]	5-23	0	6
8	[01357]	5-24	0	4
8	[02358]	5-25	0	8
8	[02458]	5-26	0	4
8	[01358]	5-27	0	6
8	[02368]	5-28	0	8
8	[01368]	5-29	0	4
8	[01468]	5-30	0	4
8	[01369]	5-31	0	8
8	[01469]	5-32	0	8
8	[02468]	5-33	0	6
8	[02469]	5-34	0	2
8	[02479]	5-35	0	4
8	[01247]	5-36	0	4
8	[03458]	5-37	0	2
8	[01258]	5-38	0	4
8	[012345]	6-1	0	3
8	[012346]	6-2	0	4
8	[012356]	6-3	0	4
8	[012456]	6-4	0	2
8	[012367]	6-5	0	4
8	[012567]	6-6	0	2
8	[012678]	6-7	0	2
8	[023457]	6-8	0	2
8	[012357]	6-9	0	2
8	[013457]	6-10	0	2
8	[012457]	6-11	0	2
8	[012467]	6-12	0	4
8	[013467]	6-13	0	4
8	[013458]	6-14	0	3
8	[012458]	6-15	0	2
8	[014568]	6-16	0	2
8	[012478]	6-17	0	4
8	[012578]	6-18	0	4
8	[013478]	6-19	0	3
8	[014589]	6-20	0	1
8	[023468]	6-21	0	4
8	[012468]	6-22	0	4
8	[023568]	6-23	0	4
8	[013468]	6-24	0	2
8	[013568]	6-25	0	4
8	[013578]	6-26	0	2
8	[013469]	6-27	0	8
8	[013569]	6-28	0	2
8	[023679]	6-29	0	2
8	[013679]	6-30	0	4
8	[014579]	6-31	0	2
8	[024579]	6-32	0	3
8	[023579]	6-33	0	4
8	[013579]	6-34	0	4
8	[02468a]	6-35	0	1
8	[012347]	6-36	0	2
8	[012348]	6-37	0	2
8	[012378]	6-38	0	2
8	[023458]	6-39	0	2
8	[012358]	6-40	0	2
8	[012368]	6-41	0	4
8	[012369]	6-42	0	2
8	[012568]	6-43	0	4
8	[012569]	6-44	0	3
8	[023469]	6-45	0	2
8	[012469]	6-46	0	2
8	[012479]	6-47	0	2
8	[012579]	6-48	0	2
8	[013479]	6-49	0	4
8	[014679]	6-50	0	4
8	[0123456]	7-1	0	2
8	[0123457]	7-2	0	2
8	[0123458]	7-3	0	2
8	[0123467]	7-4	0	2
8	[0123567]	7-5	0	2
8	[0123478]	7-6	0	2
8	[0123678]	7-7	0	4
8	[0234568]	7-8	0	1
8	[0123468]	7-9	0	2
8	[0123469]	7-10	0	2
8	[0134568]	7-11	0	2
8	[0123479]	7-12	0	1
8	[0124568]	7-13	0	2

Appendix C *(continued)*

8	[0123578]	7-14	0	2
8	[0124678]	7-15	0	2
8	[0123569]	7-16	0	2
8	[0124569]	7-17	0	1
8	[0145679]	7-18	0	2
8	[0123679]	7-19	0	4
8	[0125679]	7-20	0	2
8	[0124589]	7-21	0	2
8	[0125689]	7-22	0	1
8	[0234579]	7-23	0	2
8	[0123579]	7-24	0	2
8	[0234679]	7-25	0	2
8	[0134579]	7-26	0	2
8	[0124579]	7-27	0	2
8	[0135679]	7-28	0	4
8	[0124679]	7-29	0	2
8	[0124689]	7-30	0	2
8	[0134679]	7-31	0	8
8	[0134689]	7-32	0	2
8	[012468a]	7-33	0	2
8	[013468a]	7-34	0	1
8	[013568a]	7-35	0	2
8	[0123568]	7-36	0	2
8	[0134578]	7-37	0	1
8	[0124578]	7-38	0	2
9	[0]	1-0	9	9
9	[01]	2-1	6	8
9	[02]	2-2	6	8
9	[03]	2-3	6	8
9	[04]	2-4	6	9
9	[05]	2-5	6	8
9	[06]	2-6	3	4
9	[012]	3-1	3	7
9	[013]	3-2	6	12
9	[014]	3-3	6	12
9	[015]	3-4	6	12
9	[016]	3-5	6	13
9	[024]	3-6	3	7
9	[025]	3-7	6	12
9	[026]	3-8	8	13
9	[027]	3-9	3	7
9	[036]	3-10	3	8
9	[037]	3-11	6	12
9	[048]	3-12	1	3
9	[0123]	4-1	0	6
9	[0124]	4-2	4	10
9	[0134]	4-3	1	5
9	[0125]	4-4	4	8
9	[0126]	4-5	4	9
9	[0127]	4-6	0	5
9	[0145]	4-7	2	6
9	[0156]	4-8	1	5
9	[0167]	4-9	0	3
9	[0235]	4-10	0	4
9	[0135]	4-11	4	8
9	[0236]	4-12	2	10
9	[0136]	4-13	0	10
9	[0237]	4-14	4	8
9	[0146]	4-15	5	8
9	[0157]	4-16	4	9
9	[0347]	4-17	2	6
9	[0147]	4-18	2	10
9	[0148]	4-19	4	12
9	[0158]	4-20	2	6
9	[0246]	4-21	2	6
9	[0247]	4-22	4	10
9	[0257]	4-23	0	6
9	[0248]	4-24	2	6
9	[0268]	4-25	1	3
9	[0358]	4-26	1	5
9	[0258]	4-27	2	10
9	[0369]	4-28	0	2
9	[0137]	4-29	5	8
9	[01234]	5-1	0	5
9	[01235]	5-2	0	8
9	[01245]	5-3	2	8
9	[01236]	5-4	0	6
9	[01237]	5-5	0	6
9	[01256]	5-6	1	6
9	[01267]	5-7	0	9
9	[02346]	5-8	0	3
9	[01246]	5-9	2	6
9	[01346]	5-10	0	8
9	[02347]	5-11	2	6
9	[01356]	5-12	0	3
9	[01248]	5-13	2	6
9	[01257]	5-14	0	6
9	[01268]	5-15	1	4
9	[01347]	5-16	0	8
9	[01348]	5-17	1	3
9	[01457]	5-18	2	6

9	[01367]	5-19	0	8	9	[023679]	6-29	0	2
9	[01568]	5-20	1	6	9	[013679]	6-30	0	4
9	[01458]	5-21	2	12	9	[014579]	6-31	0	6
9	[01478]	5-22	0	3	9	[024579]	6-32	0	4
9	[02357]	5-23	0	8	9	[023579]	6-33	0	6
9	[01357]	5-24	2	6	9	[013579]	6-34	0	6
9	[02358]	5-25	0	8	9	[02468a]	6-35	0	1
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9	[01358]	5-27	2	8	9	[012348]	6-37	0	2
9	[02368]	5-28	2	8	9	[012378]	6-38	0	3
9	[01368]	5-29	0	6	9	[023458]	6-39	0	4
9	[01468]	5-30	2	6	9	[012358]	6-40	0	4
9	[01369]	5-31	0	10	9	[012368]	6-41	0	5
9	[01469]	5-32	0	8	9	[012369]	6-42	0	2
9	[02468]	5-33	1	6	9	[012568]	6-43	1	6
9	[02469]	5-34	0	3	9	[012569]	6-44	0	6
9	[02479]	5-35	0	5	9	[023469]	6-45	0	2
9	[01247]	5-36	0	5	9	[012469]	6-46	0	6
9	[03458]	5-37	1	3	9	[012479]	6-47	0	4
9	[01258]	5-38	2	6	9	[012579]	6-48	0	2
9	[012345]	6-1	0	4	9	[013479]	6-49	0	4
9	[012346]	6-2	0	6	9	[014679]	6-50	0	4
9	[012356]	6-3	0	6	9	[0123456]	7-1	0	3
9	[012456]	6-4	0	3	9	[0123457]	7-2	0	4
9	[012367]	6-5	0	5	9	[0123458]	7-3	0	3
9	[012567]	6-6	0	3	9	[0123467]	7-4	0	4
9	[012678]	6-7	0	2	9	[0123567]	7-5	0	4
9	[023457]	6-8	0	2	9	[0123478]	7-6	0	3
9	[012357]	6-9	0	4	9	[0123678]	7-7	0	5
9	[013457]	6-10	0	6	9	[0234568]	7-8	0	2
9	[012457]	6-11	0	4	9	[0123468]	7-9	0	4
9	[012467]	6-12	0	4	9	[0123469]	7-10	0	3
9	[013467]	6-13	0	4	9	[0134568]	7-11	0	2
9	[013458]	6-14	0	6	9	[0123479]	7-12	0	1
9	[012458]	6-15	0	6	9	[0124568]	7-13	0	6
9	[014568]	6-16	0	6	9	[0123578]	7-14	0	4
9	[012478]	6-17	0	4	9	[0124678]	7-15	0	2
9	[012578]	6-18	0	5	9	[0123569]	7-16	0	3
9	[013478]	6-19	0	6	9	[0124569]	7-17	0	3
9	[014589]	6-20	0	2	9	[0145679]	7-18	0	2
9	[023468]	6-21	0	6	9	[0123679]	7-19	0	4
9	[012468]	6-22	0	6	9	[0125679]	7-20	0	3
9	[023568]	6-23	0	4	9	[0124589]	7-21	0	6
9	[013468]	6-24	0	4	9	[0125689]	7-22	0	3
9	[013568]	6-25	0	6	9	[0234579]	7-23	0	4
9	[013578]	6-26	0	3	9	[0123579]	7-24	0	4
9	[013469]	6-27	0	8	9	[0234679]	7-25	0	3
9	[013569]	6-28	0	2	9	[0134579]	7-26	0	6

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9	[0135679]	7-28	0	4
9	[0124679]	7-29	0	4
9	[0124689]	7-30	0	6
9	[0134679]	7-31	0	8
9	[0134689]	7-32	0	3
9	[012468a]	7-33	0	3
9	[013468a]	7-34	0	2
9	[013568a]	7-35	0	3
9	[0123568]	7-36	0	2
9	[0134578]	7-37	0	3
9	[0124578]	7-38	0	2
9	[01234567]	8-1	0	2
9	[01234568]	8-2	0	2
9	[01234569]	8-3	0	1
9	[01234578]	8-4	0	2
9	[01234678]	8-5	0	2
9	[01235678]	8-6	0	1
9	[01234589]	8-7	0	1
9	[01234789]	8-8	0	1
9	[01236789]	8-9	0	1
9	[02345679]	8-10	0	1
9	[01234579]	8-11	0	2
9	[01345679]	8-12	0	2
9	[01234679]	8-13	0	2
9	[01245679]	8-14	0	2
9	[01234689]	8-15	0	1
9	[01235789]	8-16	0	2
9	[01345689]	8-17	0	1
9	[01235689]	8-18	0	2
9	[01245689]	8-19	0	6
9	[01245789]	8-20	0	1
9	[0123468a]	8-21	0	2
9	[0123568a]	8-22	0	2
9	[0123578a]	8-23	0	2
9	[0124568a]	8-24	0	3
9	[0124678a]	8-25	0	1
9	[0134578a]	8-26	0	1
9	[0124578a]	8-27	0	2
9	[0134679a]	8-28	0	1
9	[01235679]	8-29	0	1
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10	[02]	2-2	8	9
10	[03]	2-3	8	9
10	[04]	2-4	8	9
10	[05]	2-5	8	9
10	[06]	2-6	4	5
10	[012]	3-1	6	8
10	[013]	3-2	12	14
10	[014]	3-3	12	14
10	[015]	3-4	12	14
10	[016]	3-5	12	16
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10	[025]	3-7	12	14
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10	[027]	3-9	6	8
10	[036]	3-10	6	8
10	[037]	3-11	12	14
10	[048]	3-12	2	3
10	[0123]	4-1	4	7
10	[0124]	4-2	8	12
10	[0134]	4-3	4	6
10	[0125]	4-4	8	12
10	[0126]	4-5	8	12
10	[0127]	4-6	4	6
10	[0145]	4-7	4	6
10	[0156]	4-8	4	6
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10	[0236]	4-12	8	12
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10	[0146]	4-15	10	12
10	[0157]	4-16	8	12
10	[0347]	4-17	4	6
10	[0147]	4-18	8	12
10	[0148]	4-19	8	14
10	[0158]	4-20	4	6
10	[0246]	4-21	4	7
10	[0247]	4-22	8	12
10	[0257]	4-23	4	7
10	[0248]	4-24	4	7
10	[0268]	4-25	2	4
10	[0358]	4-26	4	6
10	[0258]	4-27	8	12
10	[0369]	4-28	1	2
10	[0137]	4-29	10	12
10	[01234]	5-1	2	6
10	[01235]	5-2	4	10

10	[01245]	5-3	4	10
10	[01236]	5-4	6	10
10	[01237]	5-5	6	10
10	[01256]	5-6	6	10
10	[01267]	5-7	4	12
10	[02346]	5-8	2	5
10	[01246]	5-9	6	10
10	[01346]	5-10	6	10
10	[02347]	5-11	4	8
10	[01356]	5-12	3	4
10	[01248]	5-13	6	10
10	[01257]	5-14	6	10
10	[01268]	5-15	2	6
10	[01347]	5-16	6	10
10	[01348]	5-17	2	5
10	[01457]	5-18	6	8
10	[01367]	5-19	6	12
10	[01568]	5-20	6	10
10	[01458]	5-21	4	12
10	[01478]	5-22	2	5
10	[02357]	5-23	4	10
10	[01357]	5-24	6	10
10	[02358]	5-25	6	10
10	[02458]	5-26	6	10
10	[01358]	5-27	4	10
10	[02368]	5-28	6	12
10	[01368]	5-29	6	10
10	[01468]	5-30	6	10
10	[01369]	5-31	6	12
10	[01469]	5-32	6	10
10	[02468]	5-33	2	6
10	[02469]	5-34	2	5
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10	[01247]	5-36	6	8
10	[03458]	5-37	2	5
10	[01258]	5-38	6	8
10	[012345]	6-1	0	5
10	[012346]	6-2	2	8
10	[012356]	6-3	4	8
10	[012456]	6-4	2	4
10	[012367]	6-5	4	8
10	[012567]	6-6	1	4
10	[012678]	6-7	0	3
10	[023457]	6-8	0	4
10	[012357]	6-9	4	8
10	[013457]	6-10	4	6
10	[012457]	6-11	4	6
10	[012467]	6-12	4	8

10	[013467]	6-13	2	4
10	[013458]	6-14	0	8
10	[012458]	6-15	4	8
10	[014568]	6-16	4	8
10	[012478]	6-17	4	8
10	[012578]	6-18	4	8
10	[013478]	6-19	2	8
10	[014589]	6-20	0	2
10	[023468]	6-21	2	8
10	[012468]	6-22	2	8
10	[023568]	6-23	2	4
10	[013468]	6-24	4	6
10	[013568]	6-25	4	8
10	[013578]	6-26	2	4
10	[013469]	6-27	4	10
10	[013569]	6-28	2	4
10	[023679]	6-29	2	4
10	[013679]	6-30	2	6
10	[014579]	6-31	4	8
10	[024579]	6-32	0	5
10	[023579]	6-33	2	8
10	[013579]	6-34	2	8
10	[02468a]	6-35	0	1
10	[012347]	6-36	4	8
10	[012348]	6-37	2	4
10	[012378]	6-38	1	4
10	[023458]	6-39	4	6
10	[012358]	6-40	4	6
10	[012368]	6-41	4	8
10	[012369]	6-42	2	4
10	[012568]	6-43	4	8
10	[012569]	6-44	2	8
10	[023469]	6-45	2	4
10	[012469]	6-46	4	6
10	[012479]	6-47	4	8
10	[012579]	6-48	2	4
10	[013479]	6-49	2	4
10	[014679]	6-50	2	4
10	[0123456]	7-1	0	4
10	[0123457]	7-2	0	6
10	[0123458]	7-3	0	6
10	[0123467]	7-4	2	6
10	[0123567]	7-5	2	6
10	[0123478]	7-6	2	6
10	[0123678]	7-7	0	8
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10	[0123469]	7-10	2	6

Appendix C *(continued)*

10	[0134568]	7-11	0	4
10	[0123479]	7-12	1	2
10	[0124568]	7-13	2	6
10	[0123578]	7-14	2	6
10	[0124678]	7-15	0	4
10	[0123569]	7-16	2	6
10	[0124569]	7-17	0	3
10	[0145679]	7-18	2	4
10	[0123679]	7-19	2	8
10	[0125679]	7-20	2	6
10	[0124589]	7-21	0	8
10	[0125689]	7-22	0	3
10	[0234579]	7-23	0	6
10	[0123579]	7-24	2	6
10	[0234679]	7-25	2	6
10	[0134579]	7-26	2	6
10	[0124579]	7-27	0	6
10	[0135679]	7-28	2	8
10	[0124679]	7-29	2	6
10	[0124689]	7-30	2	6
10	[0134679]	7-31	2	8
10	[0134689]	7-32	2	6
10	[012468a]	7-33	0	4
10	[013468a]	7-34	0	3
10	[013568a]	7-35	0	4
10	[0123568]	7-36	2	4
10	[0134578]	7-37	0	3
10	[0124578]	7-38	2	4
10	[01234567]	8-1	0	3
10	[01234568]	8-2	0	4
10	[01234569]	8-3	0	2
10	[01234578]	8-4	0	4
10	[01234678]	8-5	0	4
10	[01235678]	8-6	0	2
10	[01234589]	8-7	0	2
10	[01234789]	8-8	0	2
10	[01236789]	8-9	0	2
10	[02345679]	8-10	0	2
10	[01234579]	8-11	0	4
10	[01345679]	8-12	0	4
10	[01234679]	8-13	0	4
10	[01245679]	8-14	0	4
10	[01234689]	8-15	2	4
10	[01235789]	8-16	0	4
10	[01345689]	8-17	0	2
10	[01235689]	8-18	0	4
10	[01245689]	8-19	0	6
10	[01245789]	8-20	0	2
10	[0123468a]	8-21	0	3
10	[0123568a]	8-22	0	4
10	[0123578a]	8-23	0	3
10	[0124568a]	8-24	0	3
10	[0124678a]	8-25	0	2
10	[0134578a]	8-26	0	2
10	[0124578a]	8-27	0	4
10	[0134679a]	8-28	0	1
10	[01235679]	8-29	2	4
10	[012345678]	9-1	0	2
10	[012345679]	9-2	0	2
10	[012345689]	9-3	0	2
10	[012345789]	9-4	0	2
10	[012346789]	9-5	0	4
10	[01234568a]	9-6	0	2
10	[01234578a]	9-7	0	2
10	[01234678a]	9-8	0	4
10	[01235678a]	9-9	0	2
10	[01234679a]	9-10	0	2
10	[01235679a]	9-11	0	2
10	[01245689a]	9-12	0	1
11	[0]	1-0	11	11
11	[01]	2-1	10	10
11	[02]	2-2	10	10
11	[03]	2-3	10	10
11	[04]	2-4	10	10
11	[05]	2-5	10	10
11	[06]	2-6	5	5
11	[012]	3-1	9	9
11	[013]	3-2	18	18
11	[014]	3-3	18	18
11	[015]	3-4	18	18
11	[016]	3-5	18	18
11	[024]	3-6	9	9
11	[025]	3-7	18	18
11	[026]	3-8	18	18
11	[027]	3-9	9	9
11	[036]	3-10	9	9
11	[037]	3-11	18	18
11	[048]	3-12	3	3
11	[0123]	4-1	8	8
11	[0124]	4-2	16	16
11	[0134]	4-3	8	8

11	[0125]	4-4	16	16
11	[0126]	4-5	16	16
11	[0127]	4-6	8	8
11	[0145]	4-7	8	8
11	[0156]	4-8	8	8
11	[0167]	4-9	4	4
11	[0235]	4-10	8	8
11	[0135]	4-11	16	16
11	[0236]	4-12	16	16
11	[0136]	4-13	16	16
11	[0237]	4-14	16	16
11	[0146]	4-15	16	16
11	[0157]	4-16	16	16
11	[0347]	4-17	8	8
11	[0147]	4-18	16	16
11	[0148]	4-19	16	16
11	[0158]	4-20	8	8
11	[0246]	4-21	8	8
11	[0247]	4-22	16	16
11	[0257]	4-23	8	8
11	[0248]	4-24	8	8
11	[0268]	4-25	4	4
11	[0358]	4-26	8	8
11	[0258]	4-27	16	16
11	[0369]	4-28	2	2
11	[0137]	4-29	16	16
11	[01234]	5-1	7	7
11	[01235]	5-2	14	14
11	[01245]	5-3	14	14
11	[01236]	5-4	14	14
11	[01237]	5-5	14	14
11	[01256]	5-6	14	14
11	[01267]	5-7	14	14
11	[02346]	5-8	7	7
11	[01246]	5-9	14	14
11	[01346]	5-10	14	14
11	[02347]	5-11	14	14
11	[01356]	5-12	7	7
11	[01248]	5-13	14	14
11	[01257]	5-14	14	14
11	[01268]	5-15	7	7
11	[01347]	5-16	14	14
11	[01348]	5-17	7	7
11	[01457]	5-18	14	14
11	[01367]	5-19	14	14
11	[01568]	5-20	14	14
11	[01458]	5-21	14	14
11	[01478]	5-22	7	7
11	[02357]	5-23	14	14
11	[01357]	5-24	14	14
11	[02358]	5-25	14	14
11	[02458]	5-26	14	14
11	[01358]	5-27	14	14
11	[02368]	5-28	14	14
11	[01368]	5-29	14	14
11	[01468]	5-30	14	14
11	[01369]	5-31	14	14
11	[01469]	5-32	14	14
11	[02468]	5-33	7	7
11	[02469]	5-34	7	7
11	[02479]	5-35	7	7
11	[01247]	5-36	14	14
11	[03458]	5-37	7	7
11	[01258]	5-38	14	14
11	[012345]	6-1	6	6
11	[012346]	6-2	12	12
11	[012356]	6-3	12	12
11	[012456]	6-4	6	6
11	[012367]	6-5	12	12
11	[012567]	6-6	6	6
11	[012678]	6-7	3	3
11	[023457]	6-8	6	6
11	[012357]	6-9	12	12
11	[013457]	6-10	12	12
11	[012457]	6-11	12	12
11	[012467]	6-12	12	12
11	[013467]	6-13	6	6
11	[013458]	6-14	12	12
11	[012458]	6-15	12	12
11	[014568]	6-16	12	12
11	[012478]	6-17	12	12
11	[012578]	6-18	12	12
11	[013478]	6-19	12	12
11	[014589]	6-20	2	2
11	[023468]	6-21	12	12
11	[012468]	6-22	12	12
11	[023568]	6-23	6	6
11	[013468]	6-24	12	12
11	[013568]	6-25	12	12
11	[013578]	6-26	6	6
11	[013469]	6-27	12	12
11	[013569]	6-28	6	6
11	[023679]	6-29	6	6
11	[013679]	6-30	6	6
11	[014579]	6-31	12	12
11	[024579]	6-32	6	6

Appendix C *(continued)*

11	[023579]	6-33	12	12
11	[013579]	6-34	12	12
11	[02468a]	6-35	1	1
11	[012347]	6-36	12	12
11	[012348]	6-37	6	6
11	[012378]	6-38	6	6
11	[023458]	6-39	12	12
11	[012358]	6-40	12	12
11	[012368]	6-41	12	12
11	[012369]	6-42	6	6
11	[012568]	6-43	12	12
11	[012569]	6-44	12	12
11	[023469]	6-45	6	6
11	[012469]	6-46	12	12
11	[012479]	6-47	12	12
11	[012579]	6-48	6	6
11	[013479]	6-49	6	6
11	[014679]	6-50	6	6
11	[0123456]	7-1	5	5
11	[0123457]	7-2	10	10
11	[0123458]	7-3	10	10
11	[0123467]	7-4	10	10
11	[0123567]	7-5	10	10
11	[0123478]	7-6	10	10
11	[0123678]	7-7	10	10
11	[0234568]	7-8	5	5
11	[0123468]	7-9	10	10
11	[0123469]	7-10	10	10
11	[0134568]	7-11	10	10
11	[0123479]	7-12	5	5
11	[0124568]	7-13	10	10
11	[0123578]	7-14	10	10
11	[0124678]	7-15	5	5
11	[0123569]	7-16	10	10
11	[0124569]	7-17	5	5
11	[0145679]	7-18	10	10
11	[0123679]	7-19	10	10
11	[0125679]	7-20	10	10
11	[0124589]	7-21	10	10
11	[0125689]	7-22	5	5
11	[0234579]	7-23	10	10
11	[0123579]	7-24	10	10
11	[0234679]	7-25	10	10
11	[0134579]	7-26	10	10
11	[0124579]	7-27	10	10
11	[0135679]	7-28	10	10
11	[0124679]	7-29	10	10
11	[0124689]	7-30	10	10
11	[0134679]	7-31	10	10
11	[0134689]	7-32	10	10
11	[012468a]	7-33	5	5
11	[013468a]	7-34	5	5
11	[013568a]	7-35	5	5
11	[0123568]	7-36	10	10
11	[0134578]	7-37	5	5
11	[0124578]	7-38	10	10
11	[01234567]	8-1	4	4
11	[01234568]	8-2	8	8
11	[01234569]	8-3	4	4
11	[01234578]	8-4	8	8
11	[01234678]	8-5	8	8
11	[01235678]	8-6	4	4
11	[01234589]	8-7	4	4
11	[01234789]	8-8	4	4
11	[01236789]	8-9	2	2
11	[02345679]	8-10	4	4
11	[01234579]	8-11	8	8
11	[01345679]	8-12	8	8
11	[01234679]	8-13	8	8
11	[01245679]	8-14	8	8
11	[01234689]	8-15	8	8
11	[01235789]	8-16	8	8
11	[01345689]	8-17	4	4
11	[01235689]	8-18	8	8
11	[01245689]	8-19	8	8
11	[01245789]	8-20	4	4
11	[0123468a]	8-21	4	4
11	[0123568a]	8-22	8	8
11	[0123578a]	8-23	4	4
11	[0124568a]	8-24	4	4
11	[0124678a]	8-25	2	2
11	[0134578a]	8-26	4	4
11	[0124578a]	8-27	8	8
11	[0134679a]	8-28	1	1
11	[01235679]	8-29	8	8
11	[012345678]	9-1	3	3
11	[012345679]	9-2	6	6
11	[012345689]	9-3	6	6
11	[012345789]	9-4	6	6
11	[012346789]	9-5	6	6
11	[01234568a]	9-6	3	3
11	[01234578a]	9-7	6	6

11	[01234678a]	9-8	6	6
11	[01235678a]	9-9	3	3
11	[01234679a]	9-10	3	3
11	[01235679a]	9-11	6	6
11	[01245689a]	9-12	1	1
11	[0123456789]	10-1	2	2
11	[012345678a]	10-2	2	2
11	[012345679a]	10-3	2	2
11	[012345689a]	10-4	2	2
11	[012345789a]	10-5	2	2
11	[012346789a]	10-6	1	1
11	[0123456789a]	11-1	1	1
12	[0]	1-0	12	12
12	[01]	2-1	12	12
12	[02]	2-2	12	12
12	[03]	2-3	12	12
12	[04]	2-4	12	12
12	[05]	2-5	12	12
12	[06]	2-6	6	6
12	[012]	3-1	12	12
12	[013]	3-2	24	24
12	[014]	3-3	24	24
12	[015]	3-4	24	24
12	[016]	3-5	24	24
12	[024]	3-6	12	12
12	[025]	3-7	24	24
12	[026]	3-8	24	24
12	[027]	3-9	12	12
12	[036]	3-10	12	12
12	[037]	3-11	24	24
12	[048]	3-12	4	4
12	[0123]	4-1	12	12
12	[0124]	4-2	24	24
12	[0134]	4-3	12	12
12	[0125]	4-4	24	24
12	[0126]	4-5	24	24
12	[0127]	4-6	12	12
12	[0145]	4-7	12	12
12	[0156]	4-8	12	12
12	[0167]	4-9	6	6
12	[0235]	4-10	12	12
12	[0135]	4-11	24	24
12	[0236]	4-12	24	24
12	[0136]	4-13	24	24
12	[0237]	4-14	24	24
12	[0146]	4-15	24	24
12	[0157]	4-16	24	24
12	[0347]	4-17	12	12
12	[0147]	4-18	24	24
12	[0148]	4-19	24	24
12	[0158]	4-20	12	12
12	[0246]	4-21	12	12
12	[0247]	4-22	24	24
12	[0257]	4-23	12	12
12	[0248]	4-24	12	12
12	[0268]	4-25	6	6
12	[0358]	4-26	12	12
12	[0258]	4-27	24	24
12	[0369]	4-28	3	3
12	[0137]	4-29	24	24
12	[01234]	5-1	12	12
12	[01235]	5-2	24	24
12	[01245]	5-3	24	24
12	[01236]	5-4	24	24
12	[01237]	5-5	24	24
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12	[01267]	5-7	24	24
12	[02346]	5-8	12	12
12	[01246]	5-9	24	24
12	[01346]	5-10	24	24
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12	[01356]	5-12	12	12
12	[01248]	5-13	24	24
12	[01257]	5-14	24	24
12	[01268]	5-15	12	12
12	[01347]	5-16	24	24
12	[01348]	5-17	12	12
12	[01457]	5-18	24	24
12	[01367]	5-19	24	24
12	[01568]	5-20	24	24
12	[01458]	5-21	24	24
12	[01478]	5-22	12	12
12	[02357]	5-23	24	24
12	[01357]	5-24	24	24
12	[02358]	5-25	24	24
12	[02458]	5-26	24	24
12	[01358]	5-27	24	24
12	[02368]	5-28	24	24
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12	[01468]	5-30	24	24
12	[01369]	5-31	24	24
12	[01469]	5-32	24	24
12	[02468]	5-33	12	12
12	[02469]	5-34	12	12
12	[02479]	5-35	12	12

12	[01247]	5-36	24	24
12	[03458]	5-37	12	12
12	[01258]	5-38	24	24
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12	[012346]	6-2	24	24
12	[012356]	6-3	24	24
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12	[012367]	6-5	24	24
12	[012567]	6-6	12	12
12	[012678]	6-7	6	6
12	[023457]	6-8	12	12
12	[012357]	6-9	24	24
12	[013457]	6-10	24	24
12	[012457]	6-11	24	24
12	[012467]	6-12	24	24
12	[013467]	6-13	12	12
12	[013458]	6-14	24	24
12	[012458]	6-15	24	24
12	[014568]	6-16	24	24
12	[012478]	6-17	24	24
12	[012578]	6-18	24	24
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12	[014589]	6-20	4	4
12	[023468]	6-21	24	24
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12	[023568]	6-23	12	12
12	[013468]	6-24	24	24
12	[013568]	6-25	24	24
12	[013578]	6-26	12	12
12	[013469]	6-27	24	24
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12	[023679]	6-29	12	12
12	[013679]	6-30	12	12
12	[014579]	6-31	24	24
12	[024579]	6-32	12	12
12	[023579]	6-33	24	24
12	[013579]	6-34	24	24
12	[02468a]	6-35	2	2
12	[012347]	6-36	24	24
12	[012348]	6-37	12	12
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12	[012358]	6-40	24	24
12	[012368]	6-41	24	24
12	[012369]	6-42	12	12
12	[012568]	6-43	24	24
12	[012569]	6-44	24	24
12	[023469]	6-45	12	12
12	[012469]	6-46	24	24
12	[012479]	6-47	24	24
12	[012579]	6-48	12	12
12	[013479]	6-49	12	12
12	[014679]	6-50	12	12
12	[0123456]	7-1	12	12
12	[0123457]	7-2	24	24
12	[0123458]	7-3	24	24
12	[0123467]	7-4	24	24
12	[0123567]	7-5	24	24
12	[0123478]	7-6	24	24
12	[0123678]	7-7	24	24
12	[0234568]	7-8	12	12
12	[0123468]	7-9	24	24
12	[0123469]	7-10	24	24
12	[0134568]	7-11	24	24
12	[0123479]	7-12	12	12
12	[0124568]	7-13	24	24
12	[0123578]	7-14	24	24
12	[0124678]	7-15	12	12
12	[0123569]	7-16	24	24
12	[0124569]	7-17	12	12
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12	[0123679]	7-19	24	24
12	[0125679]	7-20	24	24
12	[0124589]	7-21	24	24
12	[0125689]	7-22	12	12
12	[0234579]	7-23	24	24
12	[0123579]	7-24	24	24
12	[0234679]	7-25	24	24
12	[0134579]	7-26	24	24
12	[0124579]	7-27	24	24
12	[0135679]	7-28	24	24
12	[0124679]	7-29	24	24
12	[0124689]	7-30	24	24
12	[0134679]	7-31	24	24
12	[0134689]	7-32	24	24
12	[012468a]	7-33	12	12
12	[013468a]	7-34	12	12
12	[013568a]	7-35	12	12
12	[0123568]	7-36	24	24
12	[0134578]	7-37	12	12
12	[0124578]	7-38	24	24
12	[01234567]	8-1	12	12

12	[01234568]	8-2	24	24	12	[0134578a]	8-26	12	12
12	[01234569]	8-3	12	12	12	[0124578a]	8-27	24	24
12	[01234578]	8-4	24	24	12	[0134679a]	8-28	3	3
12	[01234678]	8-5	24	24	12	[01235679]	8-29	24	24
12	[01235678]	8-6	12	12	12	[012345678]	9-1	12	12
12	[01234589]	8-7	12	12	12	[012345679]	9-2	24	24
12	[01234789]	8-8	12	12	12	[012345689]	9-3	24	24
12	[01236789]	8-9	6	6	12	[012345789]	9-4	24	24
12	[02345679]	8-10	12	12	12	[012346789]	9-5	24	24
12	[01234579]	8-11	24	24	12	[01234568a]	9-6	12	12
12	[01345679]	8-12	24	24	12	[01234578a]	9-7	24	24
12	[01234679]	8-13	24	24	12	[01234678a]	9-8	24	24
12	[01245679]	8-14	24	24	12	[01235678a]	9-9	12	12
12	[01234689]	8-15	24	24	12	[01234679a]	9-10	12	12
12	[01235789]	8-16	24	24	12	[01235679a]	9-11	24	24
12	[01345689]	8-17	12	12	12	[01245689a]	9-12	4	4
12	[01235689]	8-18	24	24	12	[0123456789]	10-1	12	12
12	[01245689]	8-19	24	24	12	[012345678a]	10-2	12	12
12	[01245789]	8-20	12	12	12	[012345679a]	10-3	12	12
12	[0123468a]	8-21	12	12	12	[012345689a]	10-4	12	12
12	[0123568a]	8-22	24	24	12	[012345789a]	10-5	12	12
12	[0123578a]	8-23	12	12	12	[012346789a]	10-6	6	6
12	[0124568a]	8-24	12	12	12	[0123456789a]	11-1	12	12
12	[0124678a]	8-25	6	6	12	[0123456789ab]	12-1	1	1

APPENDIX D

Statistical summary of TSATSIM relations for each #3 through #10 sc compared to every other #3 through #10 sc (TSATSIM value group #3 . . . #10:#3 . . . #10)

Prime form	Forte	Average	Lowest	Highest
[012]	3-1	0.381	0.056	0.575
[013]	3-2	0.327	0.121	0.576
[014]	3-3	0.330	0.133	0.675
[024]	3-6	0.385	0.100	0.576
[015]	3-4	0.313	0.033	0.675
[025]	3-7	0.327	0.121	0.576
[016]	3-5	0.355	0.061	0.625
[026]	3-8	0.363	0.067	0.600
[036]	3-10	0.443	0.111	0.567
[027]	3-9	0.381	0.056	0.575
[037]	3-11	0.330	0.133	0.675
[048]	3-12	0.444	0.133	0.700
#3 Averages:		0.365	0.094	0.616
[0123]	4-1	0.374	0.064	0.567
[0124]	4-2	0.337	0.100	0.512
[0134]	4-3	0.335	0.128	0.563
[0125]	4-4	0.310	0.109	0.542
[0135]	4-11	0.321	0.128	0.537
[0235]	4-10	0.336	0.128	0.613
[0145]	4-7	0.343	0.127	0.533
[0126]	4-5	0.306	0.085	0.567
[0136]	4-13	0.306	0.100	0.575
[0236]	4-12	0.305	0.109	0.525
[0146]	4-z15	0.294	0.085	0.507
[0246]	4-21	0.404	0.076	0.567
[0156]	4-8	0.342	0.033	0.549
[0127]	4-6	0.340	0.118	0.567
[0137]	4-z29	0.294	0.085	0.507
[0237]	4-14	0.310	0.109	0.542
[0147]	4-18	0.298	0.100	0.542
[0247]	4-22	0.337	0.100	0.512
[0347]	4-17	0.338	0.100	0.525
[0157]	4-16	0.306	0.085	0.567
[0257]	4-23	0.374	0.064	0.567
[0167]	4-9	0.380	0.100	0.613
[0148]	4-19	0.344	0.090	0.567

[0248]	4-24	0.402	0.069	0.567
[0158]	4-20	0.343	0.127	0.533
[0258]	4-27	0.305	0.109	0.525
[0358]	4-26	0.335	0.128	0.563
[0268]	4-25	0.404	0.067	0.549
[0369]	4-28	0.407	0.164	0.599
#4 Averages:		0.339	0.100	0.552
[01234]	5-1	0.355	0.061	0.575
[01235]	5-2	0.311	0.109	0.588
[01245]	5-3	0.305	0.109	0.601
[01236]	5-4	0.293	0.100	0.557
[01246]	5-9	0.304	0.111	0.513
[01346]	5-10	0.305	0.100	0.554
[02346]	5-8	0.319	0.118	0.566
[01256]	5-6	0.305	0.118	0.522
[01356]	5-z12	0.295	0.127	0.545
[01237]	5-5	0.303	0.118	0.545
[01247]	5-z36	0.290	0.130	0.545
[01347]	5-16	0.308	0.100	0.533
[02347]	5-11	0.289	0.128	0.618
[01257]	5-14	0.303	0.118	0.545
[01357]	5-24	0.304	0.111	0.513
[02357]	5-23	0.311	0.109	0.588
[01457]	5-z18	0.284	0.114	0.504
[01267]	5-7	0.347	0.061	0.601
[01367]	5-19	0.313	0.130	0.551
[01248]	5-13	0.315	0.111	0.518
[01348]	5-z17	0.314	0.109	0.592
[01258]	5-z38	0.284	0.114	0.504
[01358]	5-27	0.305	0.109	0.601
[02358]	5-25	0.305	0.100	0.554
[01458]	5-21	0.357	0.091	0.601
[02458]	5-26	0.307	0.111	0.505
[03458]	5-z37	0.314	0.109	0.592
[01268]	5-15	0.331	0.131	0.601
[01368]	5-29	0.293	0.100	0.557
[02368]	5-28	0.315	0.130	0.520
[01468]	5-30	0.315	0.111	0.518
[02468]	5-33	0.422	0.091	0.636
[01568]	5-20	0.305	0.118	0.522
[01478]	5-22	0.322	0.100	0.583
[01369]	5-31	0.330	0.128	0.636
[01469]	5-32	0.308	0.100	0.533

Appendix D *(continued)*

[02469]	5-34	0.319	0.118	0.566
[02479]	5-35	0.355	0.061	0.575
#5 Averages:		0.315	0.108	0.560
[012345]	6-1	0.323	0.083	0.599
[012346]	6-2	0.295	0.100	0.575
[012356]	6-z3	0.288	0.100	0.625
[012456]	6-z4	0.290	0.107	0.600
[012347]	6-z36	0.284	0.107	0.625
[012357]	6-9	0.279	0.100	0.575
[012457]	6-z11	0.274	0.100	0.550
[013457]	6-z10	0.277	0.107	0.525
[023457]	6-8	0.288	0.115	0.599
[012367]	6-5	0.282	0.092	0.625
[012467]	6-z12	0.285	0.084	0.575
[013467]	6-z13	0.305	0.100	0.625
[012567]	6-z6	0.301	0.107	0.625
[012348]	6-z37	0.295	0.107	0.600
[012358]	6-z40	0.274	0.100	0.550
[012458]	6-15	0.275	0.100	0.500
[013458]	6-14	0.280	0.115	0.608
[023458]	6-z39	0.279	0.100	0.525
[012368]	6-z41	0.287	0.100	0.575
[012468]	6-22	0.291	0.084	0.550
[013468]	6-z24	0.279	0.100	0.525
[023468]	6-21	0.296	0.084	0.558
[012568]	6-z43	0.282	0.100	0.525
[013568]	6-z25	0.288	0.100	0.625
[023568]	6-z23	0.313	0.100	0.650
[014568]	6-16	0.272	0.100	0.500
[012378]	6-z38	0.301	0.107	0.625
[012478]	6-z17	0.280	0.084	0.525
[013478]	6-z19	0.280	0.100	0.547
[012578]	6-18	0.282	0.092	0.625
[013578]	6-z26	0.290	0.107	0.600
[012678]	6-7	0.339	0.062	0.625
[012369]	6-z42	0.308	0.100	0.625
[012469]	6-z46	0.277	0.107	0.525
[013469]	6-27	0.318	0.080	0.700
[023469]	6-z45	0.312	0.107	0.650
[012569]	6-z44	0.280	0.100	0.547
[013569]	6-z28	0.310	0.100	0.600

[012479]	6-z47	0.284	0.107	0.625
[013479]	6-z49	0.307	0.100	0.600
[012579]	6-z48	0.295	0.107	0.600
[013579]	6-34	0.296	0.084	0.558
[023579]	6-33	0.295	0.100	0.575
[014579]	6-31	0.275	0.100	0.500
[024579]	6-32	0.323	0.083	0.599
[013679]	6-30	0.330	0.100	0.700
[023679]	6-z29	0.308	0.100	0.625
[014679]	6-z50	0.305	0.100	0.625
[014589]	6-20	0.336	0.090	0.600
[02468a]	6-35	0.392	0.069	0.675
#6 Averages:		0.296	0.098	0.590
[0123456]	7-1	0.346	0.061	0.573
[0123457]	7-2	0.309	0.121	0.575
[0123467]	7-4	0.296	0.133	0.515
[0123567]	7-5	0.300	0.139	0.515
[0123458]	7-3	0.296	0.135	0.590
[0123468]	7-9	0.306	0.118	0.528
[0123568]	7-z36	0.293	0.144	0.507
[0124568]	7-13	0.304	0.118	0.500
[0134568]	7-11	0.292	0.144	0.591
[0234568]	7-8	0.314	0.135	0.557
[0123478]	7-6	0.299	0.140	0.504
[0123578]	7-14	0.300	0.139	0.515
[0124578]	7-z38	0.290	0.129	0.500
[0134578]	7-z37	0.307	0.130	0.574
[0123678]	7-7	0.337	0.061	0.576
[0124678]	7-15	0.332	0.149	0.566
[0123469]	7-10	0.306	0.129	0.515
[0123569]	7-16	0.307	0.129	0.519
[0124569]	7-z17	0.307	0.130	0.574
[0123479]	7-z12	0.296	0.144	0.502
[0123579]	7-24	0.306	0.118	0.528
[0124579]	7-27	0.296	0.135	0.590
[0134579]	7-26	0.306	0.118	0.514
[0234579]	7-23	0.309	0.121	0.575
[0123679]	7-19	0.320	0.133	0.563
[0124679]	7-29	0.296	0.133	0.515
[0134679]	7-31	0.352	0.028	0.636
[0234679]	7-25	0.306	0.129	0.515
[0125679]	7-20	0.299	0.140	0.504
[0135679]	7-28	0.325	0.149	0.521

Appendix D *(continued)*

[0145679]	7-z18	0.290	0.129	0.500
[0124589]	7-21	0.332	0.120	0.575
[0124689]	7-30	0.304	0.118	0.500
[0134689]	7-32	0.307	0.129	0.519
[0125689]	7-22	0.307	0.125	0.561
[012468a]	7-33	0.383	0.133	0.636
[013468a]	7-34	0.314	0.135	0.557
[013568a]	7-35	0.346	0.061	0.573
#7 Averages:		0.311	0.123	0.544
[01234567]	8-1	0.370	0.067	0.585
[01234568]	8-2	0.339	0.133	0.566
[01234578]	8-4	0.326	0.152	0.553
[01234678]	8-5	0.337	0.133	0.567
[01235678]	8-6	0.342	0.133	0.556
[01234569]	8-3	0.338	0.133	0.566
[01234579]	8-11	0.327	0.146	0.550
[01234679]	8-13	0.337	0.128	0.531
[01235679]	8-z29	0.337	0.095	0.510
[01245679]	8-14	0.326	0.152	0.553
[01345679]	8-12	0.337	0.128	0.504
[02345679]	8-10	0.332	0.146	0.558
[01234589]	8-7	0.352	0.140	0.571
[01234689]	8-z15	0.337	0.095	0.510
[01235689]	8-18	0.331	0.134	0.534
[01245689]	8-19	0.378	0.119	0.567
[01345689]	8-17	0.351	0.146	0.580
[01234789]	8-8	0.347	0.033	0.538
[01235789]	8-16	0.337	0.133	0.567
[01245789]	8-20	0.352	0.140	0.571
[01236789]	8-9	0.384	0.089	0.567
[0123468a]	8-21	0.397	0.100	0.613
[0123568a]	8-22	0.339	0.133	0.566
[0124568a]	8-24	0.395	0.133	0.567
[0123578a]	8-23	0.370	0.067	0.585
[0124578a]	8-27	0.337	0.128	0.504
[0134578a]	8-26	0.338	0.133	0.566
[0124678a]	8-25	0.406	0.067	0.613
[0134679a]	8-28	0.387	0.028	0.569
#8 Averages:		0.351	0.116	0.558
[012345678]	9-1	0.396	0.142	0.557

[012345679]	9-2	0.372	0.167	0.557
[012345689]	9-3	0.360	0.167	0.539
[012345789]	9-4	0.364	0.167	0.557
[012346789]	9-5	0.396	0.167	0.548
[01234568a]	9-6	0.402	0.164	0.554
[01234578a]	9-7	0.372	0.167	0.557
[01234678a]	9-8	0.409	0.167	0.577
[01235678a]	9-9	0.396	0.142	0.557
[01234679a]	9-10	0.404	0.171	0.568
[01235679a]	9-11	0.360	0.167	0.539
[01245689a]	9-12	0.437	0.119	0.575
#9 Averages:		0.389	0.159	0.557
[0123456789]	10-1	0.414	0.056	0.566
[012345678a]	10-2	0.420	0.111	0.590
[012345679a]	10-3	0.420	0.111	0.601
[012345689a]	10-4	0.420	0.156	0.587
[012345789a]	10-5	0.414	0.056	0.566
[012346789a]	10-6	0.455	0.111	0.618
#10 Averages:		0.424	0.100	0.588

APPENDIX E

Statistical summary of AvgSATSIMn relations for each #3 through #10 sc compared to every other #3 through #10 sc (AvgSATSIMn value group #3 . . . #10:#3 . . . #10)

Prime form	Forte	Average	Lowest	Highest
[012]	3-1	0.381	0.056	0.575
[013]	3-2	0.327	0.121	0.576
[014]	3-3	0.330	0.133	0.675
[024]	3-6	0.385	0.100	0.576
[015]	3-4	0.313	0.033	0.675
[025]	3-7	0.327	0.121	0.576
[016]	3-5	0.355	0.061	0.625
[026]	3-8	0.363	0.067	0.600
[036]	3-10	0.443	0.111	0.567
[027]	3-9	0.381	0.056	0.575
[037]	3-11	0.330	0.133	0.675
[048]	3-12	0.444	0.133	0.700
#3 Averages:		0.365	0.094	0.616
[0123]	4-1	0.371	0.061	0.556
[0124]	4-2	0.328	0.112	0.500
[0134]	4-3	0.319	0.125	0.530
[0125]	4-4	0.296	0.102	0.541
[0135]	4-11	0.305	0.125	0.539
[0235]	4-10	0.316	0.125	0.557
[0145]	4-7	0.338	0.125	0.533
[0126]	4-5	0.297	0.090	0.567
[0136]	4-13	0.285	0.101	0.550
[0236]	4-12	0.290	0.115	0.542
[0146]	4-z15	0.277	0.069	0.467
[0246]	4-21	0.408	0.079	0.567
[0156]	4-8	0.334	0.033	0.552
[0127]	4-6	0.328	0.103	0.557
[0137]	4-z29	0.277	0.069	0.467
[0237]	4-14	0.296	0.102	0.541
[0147]	4-18	0.285	0.101	0.542
[0247]	4-22	0.328	0.112	0.500
[0347]	4-17	0.328	0.101	0.521
[0157]	4-16	0.297	0.090	0.567
[0257]	4-23	0.371	0.061	0.556
[0167]	4-9	0.374	0.089	0.594
[0148]	4-19	0.348	0.094	0.567

[0248]	4-24	0.406	0.074	0.567
[0158]	4-20	0.338	0.125	0.533
[0258]	4-27	0.290	0.115	0.542
[0358]	4-26	0.319	0.125	0.530
[0268]	4-25	0.405	0.067	0.531
[0369]	4-28	0.415	0.139	0.584
#4 Averages:		0.330	0.098	0.541
[01234]	5-1	0.356	0.061	0.575
[01235]	5-2	0.305	0.105	0.568
[01245]	5-3	0.292	0.102	0.598
[01236]	5-4	0.283	0.085	0.542
[01246]	5-9	0.290	0.108	0.514
[01346]	5-10	0.295	0.101	0.538
[02346]	5-8	0.307	0.115	0.590
[01256]	5-6	0.291	0.109	0.521
[01356]	5-z12	0.278	0.116	0.516
[01237]	5-5	0.290	0.112	0.535
[01247]	5-z36	0.272	0.122	0.516
[01347]	5-16	0.296	0.106	0.513
[02347]	5-11	0.272	0.113	0.622
[01257]	5-14	0.290	0.112	0.535
[01357]	5-24	0.290	0.108	0.514
[02357]	5-23	0.305	0.105	0.568
[01457]	5-z18	0.267	0.099	0.506
[01267]	5-7	0.345	0.059	0.609
[01367]	5-19	0.301	0.124	0.543
[01248]	5-13	0.301	0.108	0.518
[01348]	5-z17	0.301	0.092	0.590
[01258]	5-z38	0.267	0.099	0.506
[01358]	5-27	0.292	0.102	0.598
[02358]	5-25	0.295	0.101	0.538
[01458]	5-21	0.352	0.094	0.621
[02458]	5-26	0.296	0.108	0.494
[03458]	5-z37	0.301	0.092	0.590
[01268]	5-15	0.320	0.140	0.636
[01368]	5-29	0.283	0.085	0.542
[02368]	5-28	0.304	0.124	0.524
[01468]	5-30	0.301	0.108	0.518
[02468]	5-33	0.441	0.094	0.636
[01568]	5-20	0.291	0.109	0.521
[01478]	5-22	0.310	0.101	0.609
[01369]	5-31	0.336	0.126	0.636
[01469]	5-32	0.296	0.106	0.513

Appendix E *(continued)*

[02469]	5-34	0.307	0.115	0.590
[02479]	5-35	0.356	0.061	0.575
#5 Averages:		0.305	0.103	0.557
[012345]	6-1	0.326	0.082	0.618
[012346]	6-2	0.293	0.085	0.575
[012356]	6-z3	0.287	0.095	0.625
[012456]	6-z4	0.288	0.102	0.600
[012347]	6-z36	0.284	0.100	0.625
[012357]	6-9	0.275	0.094	0.575
[012457]	6-z11	0.271	0.089	0.554
[013457]	6-z10	0.272	0.100	0.534
[023457]	6-8	0.283	0.108	0.615
[012367]	6-5	0.281	0.088	0.625
[012467]	6-z12	0.281	0.082	0.575
[013467]	6-z13	0.302	0.096	0.625
[012567]	6-z6	0.302	0.090	0.625
[012348]	6-z37	0.292	0.100	0.600
[012358]	6-z40	0.271	0.089	0.554
[012458]	6-15	0.273	0.094	0.519
[013458]	6-14	0.277	0.108	0.632
[023458]	6-z39	0.274	0.094	0.528
[012368]	6-z41	0.283	0.096	0.575
[012468]	6-22	0.289	0.082	0.561
[013468]	6-z24	0.274	0.094	0.528
[023468]	6-21	0.293	0.082	0.578
[012568]	6-z43	0.276	0.096	0.530
[013568]	6-z25	0.287	0.095	0.625
[023568]	6-z23	0.311	0.096	0.650
[014568]	6-16	0.270	0.094	0.527
[012378]	6-z38	0.302	0.090	0.625
[012478]	6-z17	0.274	0.082	0.525
[013478]	6-z19	0.277	0.083	0.573
[012578]	6-18	0.281	0.088	0.625
[013578]	6-z26	0.288	0.102	0.600
[012678]	6-7	0.344	0.059	0.625
[012369]	6-z42	0.303	0.093	0.625
[012469]	6-z46	0.272	0.100	0.534
[013469]	6-27	0.321	0.094	0.700
[023469]	6-z45	0.309	0.100	0.650
[012569]	6-z44	0.277	0.083	0.573
[013569]	6-z28	0.304	0.093	0.600

[012479]	6-z47	0.284	0.100	0.625
[013479]	6-z49	0.303	0.096	0.600
[012579]	6-z48	0.292	0.100	0.600
[013579]	6-34	0.293	0.082	0.578
[023579]	6-33	0.293	0.085	0.575
[014579]	6-31	0.273	0.094	0.519
[024579]	6-32	0.326	0.082	0.618
[013679]	6-30	0.330	0.096	0.700
[023679]	6-z29	0.303	0.093	0.625
[014679]	6-z50	0.302	0.096	0.625
[014589]	6-20	0.344	0.094	0.637
[02468a]	6-35	0.419	0.074	0.675
#6 Averages:		0.295	0.092	0.596
[0123456]	7-1	0.349	0.061	0.579
[0123457]	7-2	0.304	0.121	0.584
[0123467]	7-4	0.285	0.125	0.515
[0123567]	7-5	0.288	0.129	0.531
[0123458]	7-3	0.286	0.117	0.602
[0123468]	7-9	0.292	0.113	0.523
[0123568]	7-z36	0.275	0.125	0.522
[0124568]	7-13	0.293	0.113	0.504
[0134568]	7-11	0.276	0.133	0.612
[0234568]	7-8	0.303	0.130	0.568
[0123478]	7-6	0.286	0.137	0.509
[0123578]	7-14	0.288	0.129	0.531
[0124578]	7-z38	0.272	0.108	0.517
[0134578]	7-z37	0.296	0.113	0.595
[0123678]	7-7	0.337	0.061	0.576
[0124678]	7-15	0.320	0.148	0.598
[0123469]	7-10	0.295	0.121	0.516
[0123569]	7-16	0.294	0.121	0.520
[0124569]	7-z17	0.296	0.113	0.595
[0123479]	7-z12	0.277	0.129	0.523
[0123579]	7-24	0.292	0.113	0.523
[0124579]	7-27	0.286	0.117	0.602
[0134579]	7-26	0.295	0.113	0.506
[0234579]	7-23	0.304	0.121	0.584
[0123679]	7-19	0.305	0.130	0.558
[0124679]	7-29	0.285	0.125	0.515
[0134679]	7-31	0.351	0.054	0.636
[0234679]	7-25	0.295	0.121	0.516
[0125679]	7-20	0.286	0.137	0.509
[0135679]	7-28	0.309	0.139	0.523

[0145679]	7-z18	0.272	0.108	0.517
[0124589]	7-21	0.332	0.094	0.596
[0124689]	7-30	0.293	0.113	0.504
[0134689]	7-32	0.294	0.121	0.520
[0125689]	7-22	0.299	0.119	0.577
[012468a]	7-33	0.406	0.109	0.636
[013468a]	7-34	0.303	0.130	0.568
[013568a]	7-35	0.349	0.061	0.579
#7 Averages:		0.302	0.115	0.552
[01234567]	8-1	0.364	0.067	0.575
[01234568]	8-2	0.325	0.132	0.561
[01234578]	8-4	0.304	0.144	0.558
[01234678]	8-5	0.316	0.133	0.567
[01235678]	8-6	0.325	0.133	0.556
[01234569]	8-3	0.319	0.117	0.567
[01234579]	8-11	0.306	0.137	0.558
[01234679]	8-13	0.305	0.123	0.527
[01235679]	8-z29	0.305	0.083	0.482
[01245679]	8-14	0.304	0.144	0.558
[01345679]	8-12	0.310	0.123	0.518
[02345679]	8-10	0.313	0.137	0.559
[01234589]	8-7	0.340	0.134	0.564
[01234689]	8-z15	0.305	0.083	0.482
[01235689]	8-18	0.305	0.129	0.535
[01245689]	8-19	0.369	0.104	0.577
[01345689]	8-17	0.334	0.138	0.583
[01234789]	8-8	0.335	0.033	0.542
[01235789]	8-16	0.316	0.133	0.567
[01245789]	8-20	0.340	0.134	0.564
[01236789]	8-9	0.376	0.080	0.558
[0123468a]	8-21	0.397	0.100	0.594
[0123568a]	8-22	0.325	0.132	0.561
[0124568a]	8-24	0.396	0.133	0.567
[0123578a]	8-23	0.364	0.067	0.575
[0124578a]	8-27	0.310	0.123	0.518
[0134578a]	8-26	0.319	0.117	0.567
[0124678a]	8-25	0.404	0.067	0.563
[0134679a]	8-28	0.396	0.054	0.567
#8 Averages:		0.335	0.112	0.554
[012345678]	9-1	0.386	0.125	0.546

[012345679]	9-2	0.352	0.167	0.566
[012345689]	9-3	0.343	0.167	0.566
[012345789]	9-4	0.340	0.166	0.582
[012346789]	9-5	0.376	0.141	0.569
[01234568a]	9-6	0.390	0.153	0.558
[01234578a]	9-7	0.352	0.167	0.566
[01234678a]	9-8	0.389	0.167	0.583
[01235678a]	9-9	0.386	0.125	0.546
[01234679a]	9-10	0.406	0.134	0.539
[01235679a]	9-11	0.343	0.167	0.566
[01245689a]	9-12	0.434	0.127	0.595
#9 Averages:		0.375	0.151	0.565
[0123456789]	10-1	0.425	0.056	0.593
[012345678a]	10-2	0.434	0.111	0.637
[012345679a]	10-3	0.433	0.111	0.636
[012345689a]	10-4	0.435	0.132	0.601
[012345789a]	10-5	0.425	0.056	0.593
[012346789a]	10-6	0.464	0.111	0.632
#10 Averages:		0.436	0.096	0.615