# Appendix A – Polity game and Nash equilibria

The polity game has two stages. At stage 1 two candidates $i = A, B$ compete for election by simultaneously announcing binding policy offers to the public. At stage 2 the electorate $\mathrm{E}$ of finite size $E$ determines a winner by simple majority voting (with a coin toss in case of a tie). We distinguish between elections with compulsory and voluntary voting. In the former case, each voter $j \in \mathrm{E} = \{1,2,..., E\}$ must participate at costs and decides whether to *vote* for *A*, *B*, or 'blank' (i.e. for neither candidate). In the latter case, each voter decides whether to *participate* at costs and vote for *A* or *B*, or to *abstain* and bear no costs. After all voters have simultaneously made their decisions the election outcome is made public. The winning candidate *i* gets a payoff of $\rho_i = 1$ and the opponent $-i$ nothing, $\rho_{-i} = 0$. There are no costs related to making policy offers.[41] Voters receive their benefits according to the victor's policy offer. Candidates and voters are assumed to be rational (risk neutral) players who aim at maximizing their own expected payoffs.

### A.1 Policy offers, voter preferences, and group formation

A policy offer $p_i = (p_{i_1} \; ... \; p_{i_j} \; ... \; p_{i_E})$ by candidate $i = A, B$ is a vector that represents a distribution of a given budget $W = W_A = W_B$, assumed the same for both candidates, across voters. Voter *j* is either selected by candidate *i* ( $p_{i_j} = 1$) or not ( $p_{i_j} = 0$ ). Denote the number of selected voters by $i = A, B$ as

$$P_i \equiv \sum_{j=1}^{E} p_{i_j} \; . \qquad (\mathrm{A.1})$$

A policy offer can consist of any combination of selected voters for which $1 \leq P_i \leq E$ holds. Thus, each candidate can choose from $2^E - 1$ possible combinations. We assume that voters are *a priori* identical, with zero-income and -endowment for each. Then, each voter *j*'s benefit promised by candidate $i = A, B$ is given by

$$w_{i_j} = \begin{cases} W/P_i & if \qquad p_{i_j} = 1 \\ 0 & if \qquad p_{i_j} = 0. \end{cases} \qquad (\mathrm{A.2})$$

Hence, a selected voter's benefit $w_{i_j}$ is decreasing in $P_i$.

Next, we describe voters' preferences with respect to their own pecuniary payoffs, which are derived in the same way for compulsory and voluntary voting. Define the benefit-differential for voter *j* by

$$\Delta w_j \equiv \left| w_{A_j} - w_{B_j} \right| . \qquad (\mathrm{A.3})$$

[41] Hence, candidates face a winner-takes-all situation in a constant sum game.

Let $d_j \in \{A, B, 0\}$ be $j$'s preference for either candidate $i = A, B$, or neither. This is given by

$$d_j = \begin{cases} i & if \quad w_{i_j} > w_{-i_j} \quad \text{(hence } \Delta w_j > 0) \\ 0 & if \quad w_{i_j} = w_{-i_j} \quad \text{(hence } \Delta w_j = 0). \end{cases} \qquad (A.4)$$

We denote voters with a strict preference ($\Delta w_j > 0$) for either candidate *i* by $j_i$ and indifferent voters ($\Delta w_j = 0$) by $j_0$. Then, define supporter group $G_i$ as set of the $N_i$ voters { $j_i$, $i = A, B$ }, and the group of indifferent voters $G_0$ as the $N_0$ voters $j_0$. In summary, groups and their sizes are endogenously formed by voters' preferences, which are based on their benefit-differentials as generated by both policy offers. The electorate $\mathrm{E}$ is split in $N_A$ voters in $G_A$, $N_B$ voters in $G_B$ and $N_0$ voters in $G_0$, with $N_A + N_B + N_0 = E$.

Denote the vector sum of both policy offers as $\hat{p} \equiv p_A + p_B$. This sum gives a first grasp of possible group patterns:

$$\begin{aligned} \hat{p} &= p_A + p_B \\ &= [(p_{A_1} + p_{B_1}) \ \dots \ (p_{A_j} + p_{B_j}) \ \dots \ (p_{A_E} + p_{B_E})] \\ &= (\hat{p}_1 \ \dots \ \hat{p}_j \ \dots \ \hat{p}_E) \end{aligned} \qquad (A.5)$$

Voter $j = 1, 2, ..., E$ may encounter three different situations: he may be selected (*i*) by neither candidate, $\hat{p}_j = 0$, yielding $\Delta w_j = 0$; (*ii*) by only candidate *i*, $\hat{p}_j = 1$, yielding $\Delta w_j = w_{i_j} > 0$; (*iii*) by both candidates, $\hat{p}_j = 2$, yielding $\Delta w_j \geq 0$. Note that $\Delta w_j = 0$ can not only arise in (*i*) but also in (*iii*), in case both candidates select identical numbers $P_i$ of voters.

Lemmas 1 and 2 give an exhaustive description of the existence of groups and benefit-differentials. In brief, as a result of both policy offers up to four distinct benefit-differentials may arise across voters. Note that at most one group can consist of voters with distinct differentials. This group supports the candidate *i* who selects fewer voters, $P_i < P_{-i}$, $i = A, B$. Moreover, the number of distinct benefit-differentials within a group cannot exceed two.

The following can be said about groups and their patterns:

**LEMMA 1** *(existence of groups)*:

(*a*) $p_A = p_B$ (identity) $\Leftrightarrow$ $G_A = G_B = \varnothing$.

(*b*) $p_A \neq p_B$ (difference) $\Leftrightarrow$ $G_A \neq \varnothing \wedge G_B \neq \varnothing$

$\wedge$ (*b.1*) $G_A \cup G_B \subset \mathrm{E} \Rightarrow G_0 \neq \varnothing$.

$\wedge$ (*b.2*) $G_A \cup G_B = \mathrm{E} \wedge P_A = P_B \neq E/2 \Rightarrow G_0 \neq \varnothing$.

$\wedge$ (*b.3*) $G_A \cup G_B = \mathrm{E} \wedge P_A = P_B = E/2 \Rightarrow G_0 = \varnothing$.

$\wedge$ (*b.4*) $G_A \cup G_B = \mathrm{E} \wedge P_A \neq P_B \Rightarrow G_0 = \varnothing$.

**LEMMA 2** *(existence of benefit-differentials)*:

($a$) $p_A = p_B$ (identity) $\Leftrightarrow$ $\Delta w_h = 0, \forall h \in \mathrm{E}$.

($b$) $p_A \neq p_B$ (difference)

$\wedge$ ($b.1$) $G_A \cup G_B \subset \mathrm{E}$

$\wedge$ ($b.1.1$) $P_A \neq P_B \wedge G_A \cap G_B = \varnothing$ (separation)

$\Leftrightarrow \exists h \in \mathrm{E}; \exists h' \in \mathrm{E}; \exists h'' \in \mathrm{E}$, $h \neq h' \neq h''$ s.t. $\Delta w_h > 0, \Delta w_{h'} > 0, \Delta w_{h''} = 0$.

$\wedge$ ($b.1.2$) $P_A \neq P_B \wedge G_A \cap G_B \neq \varnothing$ (overlapping)

$\Leftrightarrow \exists h \in \mathrm{E}; \exists h' \in \mathrm{E}; \exists h'' \in \mathrm{E}; \exists h''' \in \mathrm{E}$, $h \neq h' \neq h'' \neq h'''$ s.t. $\Delta w_h > 0$, $\Delta w_{h'} > 0, \Delta w_{h''} > 0, \Delta w_{h'''} = 0$.

$\wedge$ ($b.2$) $(G_A \cup G_B \subset \mathrm{E} \wedge P_A = P_B) \vee (G_A \cup G_B = \mathrm{E} \wedge P_A = P_B \neq E/2)$

$\Leftrightarrow \exists h \in \mathrm{E}; \exists h' \in \mathrm{E}$, $h \neq h'$ s.t. $\Delta w_h > 0$, $\Delta w_{h'} = 0$.

$\wedge$ ($b.3$) $G_A \cup G_B = \mathrm{E}$

$\wedge$ ($b.3.1$) $P_A = P_B = E/2 \Leftrightarrow \Delta w_h > 0, \forall h \in \mathrm{E}$.

$\wedge$ ($b.3.2$) $P_A \neq P_B \wedge G_A \cap G_B = \varnothing$ (separation)

$\Leftrightarrow \exists h \in \mathrm{E}; \exists h' \in \mathrm{E}$, $h \neq h'$ s.t. $\Delta w_h > 0$, $\Delta w_{h'} > 0$.

$\wedge$ ($b.3.3$) $P_A \neq P_B \wedge G_A \cap G_B \neq \varnothing$ (overlapping)

$\Leftrightarrow \exists h \in \mathrm{E}; \exists h' \in \mathrm{E}; \exists h'' \in \mathrm{E}$, $h \neq h' \neq h''$ s.t. $\Delta w_h > 0, \Delta w_{h'} > 0, \Delta w_{h''} > 0$.

*Proofs* straightforward

Next, we show two examples following lemmas 2($b.1.1$) and 2($b.1.2$). Suppose $E = 5$ and $W = 5$ in both examples.

*Example 1* [cf. lemma 2($b.1.1$)]:

$$p_A = (0 \quad 0 \quad 0 \quad 1 \quad 0) \qquad w_A = (0 \quad 0 \quad 0 \quad 5 \quad 0)$$
$$p_B = (0 \quad 1 \quad 1 \quad 0 \quad 1) \qquad w_B = (0 \quad 5/3 \quad 5/3 \quad 0 \quad 5/3)$$
$$\hat{p} = (0 \quad 1 \quad 1 \quad 1 \quad 1) \qquad \Delta w = (0 \quad 5/3 \quad 5/3 \quad 5 \quad 5/3)$$

The numbers of selected voters are $P_A = 1$ and $P_B = 3$. Since no $\hat{p}_j = 2$ occurs, policy offers do not overlap. There are two supporter groups: $G_A$ (only voter 4, hence $N_A = 1$) and $G_B$ (voters 2, 3, and 5, hence $N_B = 3$), and a group $G_0$ with one indifferent voter (voter 1, hence $N_0 = 1$). Moreover, we find three distinct benefit-differentials: $\Delta w_4 = 5$, $\Delta w_2 = \Delta w_3 = \Delta w_5 = 5/3$, and $\Delta w_1 = 0$. Each group contains just one differential.

*Example 2* [cf. lemma 2($b.1.2$)]:

$$p_A = (1 \quad 1 \quad 1 \quad 0 \quad 0) \qquad w_A = (5/3 \quad 5/3 \quad 5/3 \quad 0 \quad 0)$$
$$p_B = (0 \quad 0 \quad 1 \quad 0 \quad 1) \qquad w_B = (0 \quad 0 \quad 5/2 \quad 0 \quad 5/2)$$
$$\hat{p} = (1 \quad 1 \quad 2 \quad 0 \quad 1) \qquad \Delta w = (5/3 \quad 5/3 \quad 5/6 \quad 0 \quad 5/2)$$

There are $P_A = 3$ and $P_B = 2$ selected voters. This time policy offers overlap since $\hat{p}_3 = 2$. Two supporter groups are formed: $G_A$ (voters 1 and 2, hence $N_A = 2$) and $G_B$ (voters 3 and 5, hence $N_B = 2$), and a group $G_0$ with one indifferent voter (voter 4, hence $N_0 = 1$). This simple example suffices to show the maximal number of four distinct benefit-differentials: $\Delta w_5 = 5/2$, $\Delta w_1 = \Delta w_2 = 5/3$, $\Delta w_3 = 5/6$, and $\Delta w_4 = 0$. $G_B$ contains two distinct differentials.

## A.2 Compulsory voting

### A.2.1 Voter behavior

In elections with compulsory voting participation is required. We assume identical voting costs to all voters within the range $c \in (0,1)$[42]. Voters who abstain must pay a penalty larger than $c$, making this decision a strictly dominated strategy. At the ballot box voters decide whether to vote for *A*, *B*, or neither candidate. Applying iterated weak dominance, voters will vote sincerely for the preferred candidate and assuming for the moment blank votes by indifferent voters, the election outcome is merely determined by the group sizes $N_A$ and $N_B$. In case of $N_i > N_{-i}$ candidate $i$ comes into power, and in case of $N_i = N_{-i}$ a coin toss selects the victor. Each voter $j_i$'s pecuniary payoff $\pi_{j_i}$, $i = A, B$, is given by

$$\pi_{j_i} = \begin{cases} W/P_i - c & if \quad N_i > N_{-i} \\ W/2P_i + p_{-i_j} W/2P_{-i} - c & if \quad N_i = N_{-i} \\ p_{-i_j} W/P_{-i} - c & if \quad N_i < N_{-i} \end{cases} \tag{A.6}$$

and that of each indifferent voter $j_0$ by $\pi_{j0} = -c$ ($\pi_{j0} = W / P_i - c$) if selected by neither (both) candidate(s).

However, indifferent voters must participate in elections too and it is not obvious that they will all vote blank. Alternatively, suppose that voters have lexicographical preferences with own pecuniary concerns as the most important argument, followed by a second argument, e.g. concerns about equality or sympathy for candidates. For indifferent voters the value of the most important argument is zero, hence their second argument comes into play. In the following, we will discuss four alternative decision rules with respect to a (possible) second argument important for indifferent voters $j_0$ (for convenience, we define indifferent voters only based on the most important argument as those with $\Delta w = 0$):

[42] Since participation is compulsory, costs can be interpreted as 'sunk' costs. One may want to relate the upper bound of the costs to the budget *W*, but nothing would change in the presentation of our analysis.

1. *Random rule*:

$$d_{j_0} = \begin{cases} A & with \quad prob(A) = 1/3 \\ B & with \quad prob(B) = 1/3 \\ 0 & with \quad prob(0) \ = 1/3 \end{cases} \tag{A.7a}$$

This rule represents voters who derive no additional value from other than the most important argument, i.e. the benefit-differential. Hence, they choose each alternative with equal probability.[43]

For the following rules, we suppose that the second most important argument matters.

2. *Neutral rule*:

$$d_{j_0} = 0\,. \tag{A.7b}$$

In this rule the concern is not to harm (privilege) any candidate or, indirectly, any other voter. Hence, $j_0$ votes blank.

3. *Egalitarian rule*:

$$d_{j_0} = \begin{cases} A & if \quad P_A > P_B \\ B & if \quad P_A < P_B \\ 0 & if \quad P_A = P_B\,. \end{cases} \tag{A.7c}$$

This rule takes as second argument the budget distribution across *other* voters. The decision is to vote for the candidate who selects more voters (with lower benefit for each).[44]

4. *Elitist rule*:

$$d_{j_0} = \begin{cases} A & if \quad P_A < P_B \\ B & if \quad P_A > P_B \\ 0 & if \quad P_A = P_B\,. \end{cases} \tag{A.7d}$$

Our last rule is diametrically opposed to the egalitarian rule. Again, the second argument concerns the budget distribution across other voters, however, this time $j_0$ votes for the candidate who selects fewer voters (with higher benefit for each).

[43] Of course, all possible distributions of *prob*(*A*), *prob*(*B*), and *prob*(0) are optimal. Equal probabilities are only chosen to stress the distinction with the systematic decision making in the following rules.
[44] With further, more elaborate assumptions on the impact of one's vote on future policy offers, the egalitarian rule may also be interpreted as a 'risk aversion rule'. In such a rule, voters prefer future benefits that are smaller but more likely to be offered to them over those which are larger but less likely. Candidates may tend to select more voters by realizing that otherwise indifferent voters may vote against them.

Note that for $P_A = P_B$ the egalitarian and elitist rule leave voters $j_0$ indifferent concerning both the most important and the second most important argument. One could formulate decision rules for a third argument, and so on. For simplicity, however, we restrict our analysis to two arguments only. How these affect policy offers and election outcomes will be described next.

### A.2.2 Candidate behavior

Each candidate $i = A, B$ seeks to maximize her own payoff $\rho_i$, or equivalently, her probability of winning the election $prob_i(win)$. A *pure strategy* for *i* is a policy offer $p_i(k)$ from the set $\mathrm{P_E}$ = $\{p_i(1),..., p_i(k),..., p_i(2^E - 1)\}$ of all $2^E - 1$ possible offers in $\mathrm{E}$ of size *E*. A *mixed strategy* $\sigma_i$ = $\{\sigma_i(1),...,\sigma_i(k),...,\sigma_i(2^E - 1)\}$ is a probability distribution over all pure strategies, with $\sigma_i(k)$ being the probability *i* assigns to $p_i(k), \forall k$, $\sum_{k=1}^{2^E-1}\sigma_i(k) = 1$. Candidate $i$'s winning probability depends on the group sizes $N_A, N_B, N_0$, and the decision rule used by voters $j_0$. For simplicity, we assume the same lexicographical preferences for all voters, which is common knowledge. We now specify $prob_i(win)$ for each proposed decision rule in turn.

1. $prob_i(win)$ *for the random rule* ( A.7a ) :

$$prob_i(win) = \begin{cases} 1 & if \quad N_i > \lfloor E/2 \rfloor \\ prob_i(majority) + 1/2\ prob_i(tie) & if \quad N_i \le \lfloor E/2 \rfloor \ \wedge\ N_{-i} \le \lfloor E/2 \rfloor \\ 0 & if \quad N_{-i} > \lfloor E/2 \rfloor \end{cases} \qquad (\text{A.8a})$$

and $prob_{-i}(win) = prob_i(lose) = 1 - prob_i(win)$, $i = A, B$ where

$$prob_i(majority) = \sum_{k=\max[0,\Delta N]}^{\lfloor (N_0+\Delta N)/2 \rfloor} \sum_{l=1}^{N_0-k+\Delta N} \binom{N_0}{k - \Delta N + l} \binom{N_0 - k + \Delta N - l}{k} \left(\frac{1}{3}\right)^{N_0}$$

and $$prob_i(tie) = \sum_{k=\max[0,\Delta N]}^{\lfloor (N_0+\Delta N)/2 \rfloor} \binom{N_0}{k - \Delta N} \binom{N_0 - k + \Delta N}{k} \left(\frac{1}{3}\right)^{N_0},$$

with $\Delta N = N_i - N_{-i}$ and $$\left(\frac{1}{3}\right)^{N_0} = \left(\frac{1}{3}\right)^{k-\Delta N+l} \left(\frac{2}{3}\right)^{N_0-k+\Delta N-l} \left(\frac{1}{2}\right)^{N_0-k+\Delta N-l} = \left(\frac{1}{3}\right)^{k-\Delta N} \left(\frac{2}{3}\right)^{N_0-k+\Delta N} \left(\frac{1}{2}\right)^{N_0-k+\Delta N},$$

where *i* will (not) receive a vote by an indifferent voter with probability 1/3 (2/3), and the conditional probability that $-i$ will receive a vote by an indifferent voter (given that *i* does not receive it) by (1/3)/(1/3 + 1/3) = 1/2. From the middle line in ( A.8a ), it follows that $\Delta N \le N_0$. Note that $N_i = N_{-i}$ is a special event of the middle case in ( A.8a ), which gives $prob_i(win) = 1/2$. In the expression $prob_i(majority)$, $k - \Delta N + l$ votes for candidate *i* versus *k* votes for her opponent represents the event that there are $l \ge 1$ more votes for $i$ than for $-i$, which considers $\Delta N$, the difference in votes between both non-indifferent voter groups. If $k - \Delta N + l$ indifferent voters vote for *i*, then there are

$N_0 - k + \Delta N - l$ remaining to possibly vote for $-i$. To account for this conditionality, we derived 1/2 as the conditional probability of support for $-i$ by indifferent voters. The expression for $prob_i(tie)$ is developed similar to that for $prob_i(majority)$, only $l$ is not considered.

2. $prob_i(win)$ *for the neutral rule* (A.7b):

$$prob_i(win) = \begin{cases} 1 & if \quad N_i > N_{-i} \\ 1/2 & if \quad N_i = N_{-i} \\ 0 & if \quad N_i < N_{-i}, \quad i = A, B. \end{cases} \tag{A.8b}$$

3. $prob_i(win)$ *for the egalitarian rule* (A.7c):

$$prob_i(win) = \begin{cases} 1 & if \quad N_i + \lambda N_0 > N_{-i} \\ 1/2 & if \quad N_i + \lambda N_0 = N_{-i} \\ 0 & if \quad N_i + \lambda N_0 < N_{-i}, \quad i = A, B, \end{cases}$$

with

$$\lambda = \begin{cases} 1 & if \quad P_i > P_{-i} \\ 0 & if \quad P_i = P_{-i} \\ -1 & if \quad P_i < P_{-i}. \end{cases} \tag{A.8c}$$

4. $prob_i(win)$ *for the elitist rule* (A.7d):

$$prob_i(win) = \begin{cases} 1 & if \quad N_i - \lambda N_0 > N_{-i} \\ 1/2 & if \quad N_i - \lambda N_0 = N_{-i} \\ 0 & if \quad N_i - \lambda N_0 < N_{-i}, \quad i = A, B, \end{cases} \tag{A.8d}$$

where $\lambda$ is derived in the same way as in (A.8c).

### A.2.3 Nash equilibria

In this section, we derive the subgame perfect Nash equilibria for the polity game with simultaneous policy offers and compulsory voting. At stage 2, applying iterative weak dominance, we only need to consider sincere votes that are cast for the preferred candidate and, in case of indifference, according to the decision rule at hand (cf. eqs (A.7a) to (A.7d)).[45] At stage 1, candidates anticipate these decisions. Then, we can construct for each of the four rules a constant sum normal form game for the candidate competition, with the cells representing all possible combinations of *A*'s and *B*'s policy offers and the cells' entries the expected payoff $Exp[\rho_A] = prob_A(win)$ of *A*. Because this is a constant

[45] Sincere voting is a weakly dominant strategy. Applying iterative strict dominance yields a plethora of Nash equilibria with at least one voter up to everybody voting insincerely. Only in cases where a voter's decision is pivotal, he strictly prefers voting sincerely.

sum game, $B$'s expected payoff is simply $Exp[\rho_B] = 1 - prob_A(win)$. We know by Nash's Theorem that for such a game at least one Nash equilibrium exists.

Appendix B gives numerical examples of subgame perfect Nash equilibria for electorate size 4. For the further analysis it is helpful to first introduce subsets of policy offers with equal numbers of selected voters and define (pure and mixed) balanced strategies.

**DEFINITION 1** (*subsets of pure strategies with equal numbers of selected voters*):

For the set $\mathrm{P}_{\mathrm{E}}$ of all $2^E - 1$ possible pure strategies $p_i$, $i = A, B$, we define subset $\mathrm{P}_e \subset \mathrm{P}_{\mathrm{E}}$, $e = 1,..., E$, as $\mathrm{P}_e \equiv \{p_i \in \mathrm{P}_{\mathrm{E}} | P_i = e\}$ and denote any $p_i \in \mathrm{P}_e$ by $p_{i,e}$. In words, the subset $\mathrm{P}_e$ is the set of pure strategies $p_{i,e}$ which all select the same number $e$ of voters.

**DEFINITION 2** (*pure and mixed balanced strategies for candidates*):

We define a *pure balanced strategy* $\overline{p}_{i,e}$, $e = 1,..., E$, for *i* as a mixed strategy on $\mathrm{P}_e$ in which all $p_{i,e}$ are played with equal probability and all $p_i \notin \mathrm{P}_e$ are played with probability 0. Note that $\overline{p}_{i,E} = p_{i,E}$. And, a *mixed balanced strategy* for *i* is defined as a probability distribution $\overline{\sigma}_i = \{\overline{\sigma}_{i,1},..., \overline{\sigma}_{i,e},..., \overline{\sigma}_{i,E}\}$, with $\overline{\sigma}_{i,e}$ being the probability she assigns to $\overline{p}_{i,e}$, $\forall e$, $\sum_{e=1}^{E} \overline{\sigma}_{i,e} = 1$. More formally, the number of elements in $\mathrm{P}_e$ is equal to

$$\#(p_{i,e}|E) = \binom{E}{e}. \tag{A.9}$$

Each $p_{i,e}$ in $\overline{p}_{i,e}$ is therefore played with probability

$$prob\left(p_{i,e}|\overline{p}_{i,e}, E\right) = \binom{E}{e}^{-1} \tag{A.10}$$

and each $p_{i,e}$ in $\overline{\sigma}_{i,e}$ with probability

$$prob\left(p_{i,e}|\overline{\sigma}_{i,e}, E\right) = \overline{\sigma}_{i,e}\binom{E}{e}^{-1}, \quad \text{with } \sum_{p_{i,e}} \overline{\sigma}_{i,e}\binom{E}{e}^{-1} = \overline{\sigma}_{i,e}. \tag{A.11}$$

Moreover, we will refer to a *pure unbalanced strategy* $\tilde{p}_{i,e}$, $e = 1,..., E$, as any (pure or mixed) strategy that can only result in *e* selected voters, except $\overline{p}_{i,e}$. And, we will refer to a *mixed unbalanced strategy* $\tilde{\sigma}_i$ as any mixed strategy, except $\overline{\sigma}_i$.

We now formulate and prove our proposition 2 for the polity game with compulsory (sincere) voting and $E > 6$. Note that when we refer to dominance, it is sometimes stochastic dominance, as will be clear from the context.

**PROPOSITION 2** (*Nash equilibrium policy offers per decision rule for compulsory voting*):

For compulsory voting and $E > 6$,

(*i*) with the random and neutral rule, a) there exists at least one subgame perfect Nash equilibrium in which both candidates use mixed balanced strategies; b) no subgame perfect Nash equilibrium exists in strategies that can only result in a unique number of selected voters; c) no equilibrium strategy uses with strictly positive probability any policy offer which selects $\lfloor E/4 \rfloor$ voters or less;

(*ii*) with the egalitarian rule, a) for *E* even (odd) any combination of strategies by both candidates that can only result in exactly $E/2+1$ ($\lceil E/2 \rceil$) selected voters constitutes a subgame perfect Nash equilibrium; b) no other subgame perfect Nash equilibria survive iterated weak dominance;

(*iii*) with the elitist rule, a) there exists at least one subgame perfect Nash equilibrium in which both candidates use mixed balanced strategies; b) no subgame perfect Nash equilibrium exists in strategies that can only result in a unique number of selected voters.

*Proof*:

*Of* (*i*): Note first that given $-i$ chooses a strategy that can only result in $e = 1,..., E$ selected voters, using $\overline{p}_{-i,e}$ is at least as good for her as using any $\tilde{p}_{-i,e}$. This is because adding any further $p_{-i,e}$ with strictly positive probability may only make it more difficult but never easier for her opponent *i* to pursue overlapping (separation) of policy offers if choosing $P_i < P_{-i}$ ($P_i > P_{-i}$). Moreover, $-i$'s strategy should be balanced, i.e. all possible $p_{-i,e}$ should be played with equal probability. This is because otherwise her opponent *i* may increase but never decrease her winning probability through optimizing by putting more probability weight on overlapping with (separating from) those $p_{-i,e}$ that are played with higher probability. Then, if for every *e* there is a $p_i$ against $\overline{p}_{-i,e}$ which yields $prob_i(win | p_i, \overline{p}_{-i,e}, E) > 1/2$, we know that no subgame perfect Nash equilibrium exists in strategies that can only result in a specific number of selected voters, including all $\tilde{p}_{-i,e}$ (recall the 'at least as good' property of $\overline{p}_{-i,e}$). For both the neutral and random rule, suppose $-i$ chooses $\overline{p}_{-i,P_{-i}}$, where

*a*) $P_{-i} < E/2$ ($P_{-i} < \lceil E/2 \rceil$); then *i* surely wins by selecting all *E* voters in the electorate;

*b*) $P_{-i} > E/2+1$ ($P_{-i} > \lceil E/2 \rceil$); then *i* surely wins by selecting any $P_i = P_{-i} - 1$ voters.

It remains to investigate the cases where $P_{-i} = E/2$ and $P_{-i} = E/2+1$ ($P_{-i} = \lceil E/2 \rceil$), for each of which we claim that *i* achieves $prob_i(win | p_i, \overline{p}_{-i,P_{-i}}, E) > 1/2$ by choosing any $P_i = P_{-i} - 1$ voters. To see that this is true, suppose without loss of generality that $P_i < P_{-i}$ (note that $P_i = P_{-i}$ always yields a tie). With the *neutral rule*, $i$ ties if both policy offers overlap with $\Omega \equiv P_{-i} - P_i$ voters and she gets a majority of votes if there is more overlap. Then, $i$'s probability of winning against $\overline{p}_{-i,e=P_{-i}}$ is given by

$$prob_i(win \mid p_{i,P_i}, \overline{p}_{-i,P_{-i}}, P_i < P_{-i}, E, neutral\ rule) =$$
$$\begin{cases} 1 & if \quad P_i > \lfloor E/2 \rfloor \\ \frac{1}{2}\binom{P_i}{\Omega}\binom{E-P_i}{P_i}\binom{E}{P_{-i}}^{-1} + \sum_{k=1}^{2P_i-P_{-i}}\binom{P_i}{\Omega+k}\binom{E-P_i}{P_i-k}\binom{E}{P_{-i}}^{-1} & if \quad \lceil P_{-i}/2 \rceil \le P_i \le \lfloor E/2 \rfloor \\ 0 & if \quad P_i < \lceil P_{-i}/2 \rceil. \end{cases} \qquad (A.12)$$

The three binomials of the first term in the middle line of $(A.12)$ give the probability of a tie, in which case the probability of winning is 1/2. The first two of these binomials partition the electorate into the number of voters selected by $i$ ($P_i$), recall that these can be any voters, and the number of voters not selected by $i$ ($E - P_i$). The number of cases with $\Omega$ overlaps of both policy offers is then given by both binomials jointly. For the second binomial, note that $P_{-i} - \Omega = P_i$. The third binomial gives the probability with which each possible $p_{-i,P_{-i}}$ in $\overline{p}_{-i,P_{-i}}$ is played (cf. $(A.10)$). The second term in the middle line gives $i$'s probability of getting a majority of votes and is essentially the same as the first term. However, now all possible numbers of overlap larger than $\Omega$ are considered.

With the *random rule*, $i$'s probability of winning against $\overline{p}_{-i,e=P_{-i}}$ is given by

$$prob_i(win \mid p_{i,P_i}, \overline{p}_{-i,P_{-i}}, P_i < P_{-i}, E, random\ rule) =$$
$$\begin{cases} 1 & if \quad P_i > \lfloor E/2 \rfloor \\ \sum_{l=\max[0,P_i+P_{-i}-E]}^{P_i} \sum_{r_i=\max[0,P_{-i}-P_i-l]}^{\lfloor E/2 \rfloor - P_i} \frac{1}{2}\binom{P_i}{l}\binom{E-P_i}{P_{-i}-l}\binom{E}{P_{-i}}^{-1} \\ \qquad \times \binom{E-P_i-P_{-i}+l}{r_i}\binom{E-P_i-P_{-i}+l-r_i}{P_i-P_{-i}+l+r_i}\left(\frac{1}{3}\right)^{E-P_i-P_{-i}+l} \\ + \sum_{l=\max[0,P_i+P_{-i}-E]}^{P_i} \sum_{r_i=\max[0,P_{-i}-P_i-l+1]}^{E-P_i-P_{-i}+l} \sum_{r_{-i}=0}^{\min[E-P_i-P_{-i}+l-r_i,P_i-P_{-i}+l+r_i-1]} \binom{P_i}{l}\binom{E-P_i}{P_{-i}-l}\binom{E}{P_{-i}}^{-1} & if \quad P_i \le \lfloor E/2 \rfloor \\ \qquad \times \binom{E-P_i-P_{-i}+l}{r_i}\binom{E-P_i-P_{-i}+l-r_i}{r_{-i}}\left(\frac{1}{3}\right)^{E-P_i-P_{-i}+l}, \end{cases}$$
$$(A.13)$$

where $l$ denotes the number of overlaps of both policy offers, $\left(\frac{1}{3}\right)^{E-P_i-P_{-i}+l} = \left(\frac{1}{3}\right)^{r_i}\left(\frac{2}{3}\right)^{E-P_{-i}-P_i+l_i-r_i}\left(\frac{1}{2}\right)^{E-P_{-i}-P_i+l-r_i}$ gives the conditional probability that $i$ will receive a random vote by an indifferent voter but not $-i$, and $r_i$ denotes the number of random votes for $i$. Note that $i$'s probability of winning is derived similarly as in $(A.12)$, however, this time the number of random votes by indifferent voters for both candidates are accounted for. The number of indifferent voters depends on $l$ and is equal to $E - P_{-i} - P_i + l$. From this number $r_i$ are randomly cast for $i$ and from the remaining $E - P_{-i} - P_i + l - r_i$ indifferent voters $r_{-i}$ votes are randomly cast for $-i$. For $i$ to tie (win) it must hold that $P_{-i} - l + r_{-i} = P_i + r_i \Leftrightarrow r_{-i} = P_i - P_{-i} + l + r_i$ ($r_{-i} < P_i - P_{-i} + l + r_i$).

For $E > 6$, with both the neutral and random rule (cf. (A.12) and (A.13)) it is readily verified that, according to our claim, if $i$ selects any $P_i = P_{-i} - 1$ voters, $prob_i(win|\cdot) > 1/2$ against $\overline{p}_{-i,P_{-i}=E/2}$, $\overline{p}_{-i,P_{-i}=E/2+1}$, and $\overline{p}_{-i,P_{-i}=\lceil E/2 \rceil}$. Hence, together with *a*) and *b*), there is no subgame perfect Nash equilibrium in candidates' strategies that can only result in a specific number of selected voters for $E > 6$ with both the neutral and random rule.

Next, we show that at least one subgame perfect Nash equilibrium in *mixed balanced strategies* exists for both rules.[46] Note that the 'at least as good' property that we used for pure strategies cannot be applied easily to mixed strategies. But suppose both candidates use $\overline{\sigma}_i$. Then, a 'reduced' constant sum normal form can be derived with cells only representing all possible combinations $\overline{p}_{i,e}$ of $A$ and $B$, hence compressing all $p_{i,e}$, and the cell's entries $A$'s expected probability of winning. By Nash's theorem and because we showed that there is no equilibrium in $\overline{p}_{i,e}$, there exists at least one subgame perfect Nash equilibrium in $\overline{\sigma}_i$ for this reduced normal form game. Knowing this, we need to show that no candidate can increase her winning probability by switching to any $\tilde{\sigma}_i$, hence returning to the original 'non-reduced' normal form game, given the opponent plays $\overline{\sigma}_{-i}$. However, any $\tilde{p}_i$ yields the same probability of winning as $\overline{p}_{i,e}$ against any $\overline{p}_{-i,e}$ used in $\overline{\sigma}_{-i}$ ('randomness' can be produced with only one candidate). Hence, $i$ cannot improve by unbalancing any part of her strategy. We conclude that with the neutral and the random rule at least one subgame perfect Nash equilibrium exists in which both candidates play mixed balanced strategies.

Finally, we show (by applying iterated weak dominance) that no equilibrium strategy uses any policy offer which selects $P_i \le \lfloor E/4 \rfloor$ voters. If $i$ picks all $P_i = E$ voters, she surely wins [ties; loses] against any $p_{-i,P_{-i}}$ which selects $P_{-i} < \lceil E/2 \rceil$ [$P_{-i} = E/2$ and $P_{-i} = E$; $\lfloor E/2 \rfloor < P_{-i} < E$] voters. In comparison, if $i$ picks $P_i < \lfloor E/4 \rfloor$ voters instead, she surely loses with the neutral rule [expects to lose more often with the random rule] against any $p_{-i,P_{-i}}$ which selects $P_{-i} \ge \lceil E/2 \rceil - 1$ voters. And, if $i$ picks $P_i = E/4$, she can at most tie with the neutral rule [expect to tie with the random rule] against $p_{-i,E/2}$. Hence, with the neutral and random rule $p_{i,E}$ (at least) weakly dominates any $p_{-i,P_{-i}}$ which selects $P_i \le \lfloor E/4 \rfloor$ voters and the latter strategies are not used in any subgame perfect Nash equilibrium. □

[46] Note that we do not elaborate on subgame perfect Nash equilibria in $\tilde{\sigma}_i$. We conjecture that such equilibria exist, however, only using minor unbalancing.

*Of* (*ii*): To demonstrate that with the *egalitarian rule* any combination of strategies that can only result in $P_i = E/2+1$ ($P_i = \lceil E/2 \rceil$) selected voters constitutes a subgame perfect Nash equilibrium but no other equilibria exist, we must show that these are the only strategies of *i* that yield $prob_i(win|\cdot) \geq 1/2$ against each possible strategy of –*i*, after applying iterated weak dominance. Again, for all *e*, $\overline{p}_{i,e}$ is at least as good as any $\tilde{p}_{i,e}$. Then, suppose *i* selects any $E/2+1$ ($\lceil E/2 \rceil$) voters and –*i* chooses $\overline{p}_{-i,e=P_{-i}}$, where

*a*) $P_{-i} < E/2$ ($P_{-i} < \lceil E/2 \rceil$); then, from her $P_i = E/2+1$ ($P_i = \lceil E/2 \rceil$) selected voters *i* automatically gains for every overlapping voter, whom she loses because $P_i > P_{-i}$, one indifferent voter's egalitarian vote, since $P_i + P_{-i} \leq E$. Hence, *i* always gets a majority of votes;

*b*) $P_{-i} = E/2$; then, by a similar argument as in *a*), from her $E/2+1$ selected voters *i* loses one overlapping voter for whom she gains no indifferent voter's egalitarian vote, since $P_i + P_{-i} = E+1 > E$. For every other overlapping voter whom *i* loses, however, she gains one indifferent voter's vote. Hence, *i* always gets $E/2$ votes and ties;

*c*) $P_{-i} = E/2+1$ ($P_{-i} = \lceil E/2 \rceil$); this, of course, always results in a tie;

*d*) $P_{-i} > E/2+1$ ($P_{-i} > \lceil E/2 \rceil$); then *i* always gets a majority of votes, because she receives at least the votes of her selected $E/2+1$ ($\lceil E/2 \rceil$) voters due to $P_i < P_{-i}$.

*a*)-*d*) show that any $P_i = E/2+1$ ($P_i = \lceil E/2 \rceil$) selected voters achieve $prob_i(win|\cdot) > 1/2$ against any $\overline{p}_{-i,P_{-i} \neq E/2 \wedge P_{-i} \neq E/2+1 (\lceil E/2 \rceil)}$ and $prob_i(win|\cdot) = 1/2$ against $\overline{p}_{-i,E/2}$ and $\overline{p}_{-i,E/2+1}$ ($\overline{p}_{-i,\lceil E/2 \rceil}$). For *E* even, as $p_{i,E/2+1}$, $p_{i,E/2}$ always wins [ties] against any $\overline{p}_{-i,P_{-i}}$ which selects $P_{-i} < E/2$ [$P_{-i} = E/2$ and $P_{-i} = E/2+1$] voters, however, it only ties against any $\overline{p}_{-i,P_{-i}}$ which selects $P_{-i} > E/2+1$ voters, for which $p_{i,E/2+1}$ always wins. Hence, any $p_{i,E/2+1}$ weakly dominates any $p_{i,E/2}$. Applying iterated weak dominance, it follows that only strategies which select $P_i = E/2+1$ ($P_i = \lceil E/2 \rceil$) voters can be part of a subgame perfect Nash equilibrium. □

For constructing the 'reduced' normal form game for the candidates with cells representing all possible combinations $\overline{p}_{i,e}$ of *A* and *B*, we next derive *i*'s victory probability for case (*ii*) in proposition 2. With the egalitarian rule, for $P_i < P_{-i}$ a tie occurs if $P_{-i} - \hat{\Omega} + (E - P_i - P_{-i} + \hat{\Omega}) = P_i \Leftrightarrow P_i = E/2$, where $\hat{\Omega}$ denotes the number of overlapping voters. Note that the occurrence of a tie does not depend on $\hat{\Omega}$. Then, *i*'s winning probability against $\overline{p}_{-i,e=P_{-i}}$ is given by

$$prob_i(win|p_{i,P_i}, \overline{p}_{-i,P_{-i}}, P_i < P_{-i}, E, egalitarian\, rule) = \begin{cases} 1 & if \quad P_i > \lfloor E/2 \rfloor \\ 1/2 & if \quad P_i = E/2 \\ 0 & if \quad P_i < \lceil E/2 \rceil . \end{cases} \qquad (A.14)$$

*Of* (*iii*): To demonstrate that with the *elitist rule* no Nash equilibrium uses strategies that can only result in a specific number of selected voters, we must show that there is always a $p_{i,e}$, $e = 1,\ldots,E$, for which $prob_i(win|\cdot) > 1/2$ against any possible strategy of $-i$. Once again, for all *e*, $\bar{p}_{i,e}$ is at least as good as any $\tilde{p}_{i,e}$. Then, suppose $-i$ chooses $\bar{p}_{-i,e=P_{-i}}$, where

*a*) $P_{-i} < E/2$ ($P_{-i} < \lceil E/2 \rceil$); then *i* surely wins by selecting all *E* voters in the electorate;

*b*) $P_{-i} = E/2$, then, by selecting any $P_i = E/2 - 1$ voters *i* never loses and expects to win more often than $-i$, because she ties [wins] if policy offers do not overlap [overlap at least once];

*c*) $P_{-i} = E/2 + 1$ ($P_{-i} = \lceil E/2 \rceil$); then, by selecting any $P_i = E/2$ ($P_i = \lfloor E/2 \rfloor$) voters *i* never loses and expects to win more often than $-i$, because she ties [wins] if policy offers overlap once [at least twice] (loses only in the single case where policy offers do not overlap but ties [for the more frequent cases, wins] if policy offers overlap once [at least twice]). Hence, *i* expects to win more often than $-i$;

*d*) $P_{-i} > E/2 + 1$ ($P_{-i} > \lceil E/2 \rceil$); then, by selecting any $P_i = P_{-i} - 1$ voters *i* surely wins, because she receives at least the votes of her selected voters, who already constitute a majority.

Hence, with the elitist rule no Nash equilibrium exists in strategies that can only result in a specific number of voters. Moreover, following the same argument as for the neutral and random rule, we know that at least one Nash equilibrium in mixed balanced strategies exists.

It is tedious but straightforward to show by applying iterated weak dominance that no pure strategy is weakly dominated. □

To construct the 'reduced' normal form game for the candidates with cells representing all possible combinations $\bar{p}_{i,e}$ of *A* and *B*, we next derive *i*'s victory probability for case (*iii*) in proposition 2. With the elitist rule, *i* ties if both policy offers overlap with $\breve{\Omega} \equiv P_{-i} - E/2 \geq 0$ voters (which follows from $P_{-i} - \breve{\Omega} = P_i + E - P_i - P_{-i} + \breve{\Omega}$) and she gets a majority of votes if there is more overlap. Then, for $P_i < P_{-i}$ *i*'s probability of winning against $\bar{p}_{-i,e=P_{-i}}$ is given for *E* even by

$$prob_i(win|p_{i,P_i}, \bar{p}_{-i,P_{-i}}, P_i < P_{-i}, E, eltitist\ rule) =$$
$$\begin{cases} 1 & if \quad P_i > E/2 \;\vee\; P_i < P_{-i} < E/2 \\ \frac{1}{2}\binom{P_i}{\breve{\Omega}}\binom{E-P_i}{E/2}\binom{E}{P_{-i}}^{-1} + \sum_{k=1}^{P_i-\breve{\Omega}}\binom{P_i}{\breve{\Omega}+k}\binom{E-P_i}{E/2-k}\binom{E}{P_{-i}}^{-1} & if \quad 0 \leq P_{-i} - E/2 \leq P_i \leq E/2 \\ 0 & if \quad 0 < P_i < P_{-i} - E/2 \leq E/2 \end{cases}$$

and for *E* odd by

$$
\begin{aligned}
&prob_i(win \,|\, p_{i,P_i}, \overline{p}_{-i,P_{-i}}, P_i < P_{-i}, E, eltitist\ rule) = \\
&\quad \begin{cases}
1 & if \quad P_i > \lfloor E/2 \rfloor \ \vee \ P_i < P_{-i} \le \lfloor E/2 \rfloor \\
\sum_{k=1}^{P_i - \breve{\Omega}} \binom{P_i}{P_j - \lfloor E/2 \rfloor + k} \binom{E - P_i}{\lfloor E/2 \rfloor - k} \binom{E}{P_{-i}}^{-1} & if \quad 0 < P_{-i} - \lfloor E/2 \rfloor \le P_i < \lceil E/2 \rceil \\
0 & if \quad 0 < P_i < P_{-i} - \lfloor E/2 \rfloor \le \lceil E/2 \rceil
\end{cases}
\end{aligned}
\tag{A.15}
$$

Note that $(\text{A.15})$ is derived in a similar way as $(\text{A.12})$ for the neutral rule.

## A.3 Voluntary voting

### A.3.1 Voter behavior and Nash equilibria of the subgames at the election stage

In this section we analyze elections with voluntary costly voting. Each voter must decide whether to participate at costs and vote for one of the candidates, or to abstain and bear no costs. Again, we assume identical voting costs to all voters within the range $c \in (0,1)$. For the participation decision, each voter $j$ has two pure strategies $v_j \in \{0,1\}$, where $v_j = 1$ denotes participation and $v_j = 0$ abstention. A mixed strategy profile of $j$ is given by the probability of participation $q_j$. Note that contrary to compulsory voting, the second argument of lexicographical preferences will never be used by indifferent voters. This is because voting costs are no longer sunk and indifferent voters rather avoid them than decrease the value of their most important argument. Hence, it is a strictly dominant strategy for voters $j_0$ to abstain ($v_{j_0} = 0$). Moreover, for non-indifferent voters $j_i$ the strategy to vote for candidate $-i$ is strictly dominated by abstention ($v_{j_i} = 0$). Thus, we can focus on participation decisions, with votes being cast sincerely for the preferred candidate.

For the cases where in one group $G_i$ two distinct benefit-differentials occur, we introduce further notations for subgroups: the $N_{i,H}$ denotes the number of voters $j_{i,H}$ with the higher (‘*H*’) differential in $G_{i,H}$ and the $N_{i,L}$ denotes the number of voters $j_{i,L}$ with the lower (‘*L*’) differential in $G_{i,L}$, where $G_{i,H} \cup G_{i,L} = G_i$ and $N_{i,H} + N_{i,L} = N_i$. Then, a pure strategy for voter $j_{i,H}$ ($j_{i,L}$) is denoted by $v_{j_{i,H}} \in \{0,1\}$ ($v_{j_{i,L}} \in \{0,1\}$). Mixed strategies are labeled $q_{i,H}$ and $q_{i,L}$, respectively. If $G_i$, $i = A, B$, contains a single benefit-differential, its aggregate participation is denoted by

$$V_i \equiv \sum_{j_i} v_{j_i} \tag{A.16}$$

and if it contains two benefit-differentials by

$$V_i \equiv \sum_{j_{i,H}} v_{j_{i,H}} + \sum_{j_{i,L}} v_{j_{i,L}} \,. \tag{A.17}$$

For later use, we denote aggregate participation by other voters in $G_i$ than $j_i$ by[47]

[47] To avoid extensive notations, what will be said for $j_i$ in the following will also hold for $j_{i,H}$ respectively $j_{i,L}$ (by simply replacing the notations), unless stated otherwise.

$$V_i^{-j_i} \equiv V_i - v_{j_i} \,. \tag{A.18}$$

Obviously, aggregate participation in $G_0$ is always $V_0 = 0$.

Candidate $i = A, B$ wins the election if $V_i > V_{-i}$, and a coin toss determines the winner in the event of $V_i = V_{-i}$. Hence, the payoff for a non-indifferent voter $j_i$ is given by

$$\pi_{j_i} = \begin{cases} W/P_i - v_{j_i} c & if \quad V_i > V_{-i} \\ W/2P_i + p_{-i_j} W/2P_{-i} - v_{j_i} c & if \quad V_i = V_{-i} \\ p_{-i_j} W/P_{-i} - v_{j_i} c & if \quad V_i < V_{-i} \end{cases} \tag{A.19}$$

and for an indifferent voter $j_0$ by $\pi_{j_0} = p_{i_j} W / P_i$.

Next, we will analyze participation decisions. Voter $j_i$ will vote with probability 1 (0) if his expected payoff of participation is higher (lower) than that of abstention, or

$$Exp\left[\pi_{j_i} \middle| v_{j_i} = 1\right] \underset{(<)}{>} Exp\left[\pi_{j_i} \middle| v_{j_i} = 0\right].$$

He will mix when the two are equal. Elaboration (cf. Palfrey and Rosenthal 1983) yields

$$prob(V_i^{-j_i} = V_{-i}) + prob(V_i^{-j_i} + 1 = V_{-i}) \underset{(<)}{>} \frac{2c}{\Delta w_{j_i}}, \tag{A.20}$$

where the left-hand side gives voter $j_i$'s probability of being pivotal (note that $\Delta w_{j_i} > 0$). It contains two components: the first gives the probability that his vote can turn a tie into a victory, and the second the probability that it can turn a defeat into a tie. Note that the expected benefit from voting is always negative for $c > \Delta w_{j_i}/2$, implying that a risk neutral voter will abstain.

Voter $j_i$'s participation decision $v_{j_i}$ depends on the actual group pattern that follows from the policy offers announced at the first stage. The results of lemmas 1 and 2 about the existence of groups and benefit-differentials are helpful in guiding the analysis of the participation decision for each possible group pattern. We distinguish between the following 3 (exhaustive) cases:

*Case 1*: Only $G_0$ exists, since all voters are indifferent [$\Delta w_j = 0, \forall j \in \mathrm{E}$; lemma 2(a)].

*Case 2*: There are two supporter groups $G_i$, $i = A, B$, and possibly $G_0$. Both supporter groups contain a single benefit-differential [lemmas 2(b.1.1), 2(b.2), 2(b.3.1), and 2(b.3.2)]. The following situations may occur:

(a) For $\forall j_i \wedge \forall j_{-i}$ we have $\Delta w_j/2 < c$.

(b) For $\forall j_i$ we have $\Delta w_j/2 \geq c$ and for $\forall j_{-i}$ we have $\Delta w_j/2 < c$.

(c) For $\forall j_i \wedge \forall j_{-i}$ we have $\Delta w_j/2 \geq c$.

*Case 3*: There are two supporter groups $G_i$, $i = A, B$, and possibly $G_0$. $G_i$ contains two different benefit-differentials, hence two subgroups $G_{i,H}$ and $G_{i,L}$ exist [lemmas 2(b.1.2) and 2(b.3.3)]. The following situations may occur:

(a) For $\forall j_{i,H} \wedge \forall j_{i,L} \wedge \forall j_{-i}$ we have $\Delta w_j / 2 < c$.

(b) For $\forall j_{i,H}$ we have $\Delta w_j / 2 \geq c$ and for $\forall j_{i,L} \wedge \forall j_{-i}$ we have $\Delta w_j / 2 < c$.

(c) For $\forall j_{i,H} \wedge \forall j_{i,L}$ we have $\Delta w_j / 2 \geq c$ and for $\forall j_{-i}$ we have $\Delta w_j / 2 < c$.

(d) For $\forall j_{i,H} \wedge \forall j_{-i}$ we have $\Delta w_j / 2 \geq c$ and for $\forall j_{i,L}$ we have $\Delta w_j / 2 < c$.

(e) For $\forall j_{i,H} \wedge \forall j_{i,L} \wedge \forall j_{-i}$ we have $\Delta w_j / 2 \geq c$.

Note that for case 3 there are only five different situations due the implicit restrictions that either $\Delta w_{j_{i,L}} \leq \Delta w_{j_{-i}} < \Delta w_{j_{i,H}}$ or $\Delta w_{j_{-i}} \leq \Delta w_{j_{i,L}} < \Delta w_{j_{i,H}}$. Importantly, as indifferent voters $j_0$, non-indifferent voters $j_i$ with $\Delta w_{j_i} / 2 < c$ will abstain with certainty too.

In the following, we derive (conditions for) the Nash equilibria of all possible subgames at the election stage with voluntary costly voting by specifying ( A.20 ). We focus on totally quasi-symmetric mixed strategy equilibria (cf. Palfrey and Rosenthal 1983), i.e. state ( A.20 ) as equality, where $q_i \in (0,1)$ and $q_{-i} \in (0,1)$.

*Group pattern 1*: Cases 1, 2(a), and 3(a) are trivial: everybody abstains.

*Group pattern 2*: Cases 2(b) and 3(b) are strategically equivalent to the *volunteer's dilemma game* (Diekmann 1985). Because $V_{-i} = 0$, the necessary and sufficient condition for $q \equiv q_i$ to be a best response is given for 2(b) by

$$(1-q)^{N_i - 1} = \frac{2c}{\Delta w_{j_i}}, \qquad ( A.21)$$

where the left-hand side gives $j_i$'s probability of being pivotal, i.e. only the probability of other group members' decisions creating a tie. Note that the condition for 3(b) is derived analogous, only $q$ ( $j_i$, $N_i$ ) must be replaced by $q_H \equiv q_{i,H}$ ( $j_{i,H}$, $N_{i,H}$ ).

*Group pattern 3*: Case 3(c) is also strategically equivalent to the *volunteer's dilemma game*, but two different benefit-differentials in $G_i$ must be considered. Define $q_H \equiv q_{i,H}$ and $q_L \equiv q_{i,L}$. Then, the necessary and sufficient condition for $q_H$ to be a best response is given by

$$(1-q_H)^{N_{i,H}-1}(1-q_L)^{N_{i,L}} = \frac{2c}{\Delta w_{j_{i,H}}}$$

and for $q_L$ by

$$(1-q_H)^{N_{i,H}}(1-q_L)^{N_{i,L}-1} = \frac{2c}{\Delta w_{j_{i,L}}}, \qquad (A.22)$$

where the left-hand sides give the probability of being pivotal, i.e. only the probability of other group members' ($G_i$) decisions creating a tie. Together, both conditions characterize all ($q_H, q_L$)-equilibria.

*Group pattern 4*: Cases 2(c) and 3(d) are strategically equivalent to the *standard participation games* (Palfrey and Rosenthal 1983). Define $q \equiv q_i$ and $\tilde{q} \equiv q_{-i}$. For 2(c), the necessary and sufficient condition for $q$ to be a best response is then given by

$$\sum_{k=0}^{\min[N_i-1,N_{-i}]} \binom{N_i-1}{k}\binom{N_{-i}}{k} q^k(1-q)^{N_i-1-k}\tilde{q}^k(1-\tilde{q})^{N_{-i}-k} + \sum_{k=0}^{\min[N_i-1,N_{-i}-1]} \binom{N_i-1}{k}\binom{N_{-i}}{k+1} q^k(1-q)^{N_i-1-k}\tilde{q}^{k+1}(1-\tilde{q})^{N_{-i}-1-k} = \frac{2c}{\Delta w_{j_i}}$$

and for $\tilde{q}$ by

$$\sum_{k=0}^{\min[N_i,N_{-i}-1]} \binom{N_i}{k}\binom{N_{-i}-1}{k} q^k(1-q)^{N_i-k}\tilde{q}^k(1-\tilde{q})^{N_{-i}-1-k} + \sum_{k=0}^{\min[N_i-1,N_{-i}-1]} \binom{N_i}{k+1}\binom{N_{-i}-1}{k} q^k(1-q)^{N_i-1-k}\tilde{q}^k(1-\tilde{q})^{N_{-i}-1-k} = \frac{2c}{\Delta w_{j_{-i}}}, \qquad (A.23)$$

where the respective upper (lower) term on the left-hand side of each condition gives the probability of a tie (victory for one's own group) created by all other voters' decisions than $i$ respectively $-i$. Both best responses together characterize all $(q,\tilde{q})$-equilibria. The first term on the left hand side of each condition gives the (binomial) probability that there is a tie of $k$ votes between the $N_{-i}$ members in the other group and the $N_i - 1$ other members of $j_i$'s own group ($j_i$ can turn a tie into a victory). The second term gives the (binomial) probability that the other group outvotes $j_i$'s co-members by one vote ($j_i$ can turn a defeat into a tie). The conditions for 3(d) are derived analogous, only $q$ ($j_i$, $N_i$) must be replaced by $q_H \equiv q_{i,H}$ ($j_{i,H}$, $N_{i,H}$).

*Group pattern 5*: Case 3(e) is a straightforward modification of the standard participation game, which demands one extra condition as compared to (A.23), because there are three distinct benefit-differentials. Define $q_H \equiv q_{i,H}$, $q_L \equiv q_{i,L}$, and $\tilde{q} \equiv q_{-i}$. Then, a necessary and sufficient conditions for $q_H$ to be a best response is given by

$$\sum_{k=0}^{\min[N_i-1,N_{-i}]}\sum_{m=\max[0,k-N_{i,H}+1]}^{\min[k,N_{i,L}]}\binom{N_{i,H}-1}{k-m}\binom{N_{i,L}}{m}\binom{N_{-i}}{k}q_H^{k-m}(1-q_H)^{N_{i,H}-1-k+m}q_L^m(1-q_L)^{N_{i,L}-m}\tilde{q}^k(1-\tilde{q})^{N_{-i}-k}$$

$$+\sum_{k=0}^{\min[N_i-1,N_{-i}-1]}\sum_{m=\max[0,k-N_{i,H}+1]}^{\min[k,N_{i,L}]}\binom{N_{i,H}-1}{k-m}\binom{N_{i,L}}{m}\binom{N_{-i}}{k+1}q_H^{k-m}(1-q_H)^{N_{i,H}-1-k+m}q_L^m(1-q_L)^{N_{i,L}-m}\tilde{q}^{k+1}(1-\tilde{q})^{N_{-i}-k-1}$$

$$=\frac{2c}{\Delta w_{j_{i,H}}},$$

for $q_L$ by

$$\sum_{k=0}^{\min[N_i-1,N_{-i}]}\sum_{m=\max[0,k-N_{i,H}]}^{\min[k,N_{i,L}-1]}\binom{N_{i,H}}{k-m}\binom{N_{i,L}-1}{m}\binom{N_{-i}}{k}q_H^{k-m}(1-q_H)^{N_{i,H}-k-m}q_L^m(1-q_L)^{N_{i,L}-1-m}\tilde{q}^k(1-\tilde{q})^{N_{-i}-k}$$

$$+\sum_{k=0}^{\min[N_i-1,N_{-i}-1]}\sum_{m=\max[0,k-N_{i,H}]}^{\min[k,N_{i,L}-1]}\binom{N_{i,H}}{k-m}\binom{N_{i,L}-1}{m}\binom{N_{-i}}{k+1}q_H^{k-m}(1-q_H)^{N_{i,H}-k+m}q_L^m(1-q_L)^{N_{i,L}-1-m}\tilde{q}^{k+1}(1-\tilde{q})^{N_{-i}-1-k}$$

$$=\frac{2c}{\Delta w_{j_{i,L}}}$$

and for $\tilde{q}$ by

$$\sum_{k=0}^{\min[N_i,N_{-1}-1]}\sum_{m=\max[0,k-N_{i,H}]}^{\min[k,N_{i,L}]}\binom{N_{i,H}}{k-m}\binom{N_{i,L}}{m}\binom{N_{-i}-1}{k}q_H^{k-m}(1-q_H)^{N_{i,H}-k+m}q_L^m(1-q_L)^{N_{i,L}-m}\tilde{q}^k(1-\tilde{q})^{N_{-i}-1-k}$$

$$+\sum_{k=0}^{\min[N_i-1,N_{-i}-1]}\sum_{m=\max[0,k+1-N_{i,H}]}^{\min[k+1,N_{i,L}]}\binom{N_{i,H}}{k+1-m}\binom{N_{i,L}}{m}\binom{N_{-i}-1}{k}q_H^{k+1-m}(1-q_H)^{N_{i,H}-k-1+m}q_L^m(1-q_L)^{N_{i,L}-m}\tilde{q}^k(1-\tilde{q})^{N_{-i}-1-k}$$

$$=\frac{2c}{\Delta w_{j_{-i}}}. \qquad (\text{A.24})$$

Together, the three conditions characterize all $(q_H, q_L, \tilde{q})$-equilibria.

### A.3.2 Candidate behavior and subgame perfect Nash equilibria

Each candidate $i = A, B$ maximizes her own payoff $\rho_i$, respectively her winning probability $prob_i(win)$. She anticipates that voters with $\Delta w_j/2 < c$, including the indifferent voters, will abstain. Then, given all remaining voters with $\Delta w_j/2 \geq c$, $i$'s winning probability is given by

$$prob_i(win) = prob_i(majority) + \frac{1}{2}prob(tie), \qquad (\text{A.25})$$

where the elaboration of the right-hand side depends on the actual group pattern as described above. Of course, $-i$'s probability of winning is given by $prob_{-i}(win) = 1 - prob_i(win)$.

In the following we derive (conditions for) the winning probabilities of all possible subgames at the election stage with voluntary costly voting, for which we derived totally quasi-symmetric mixed strategy equilibria [cf. (A.21) to (A.24)], by specifying the right-hand side of (A.25).

*Group pattern 1*: Cases 1, 2(a), and 3(a) are trivial. A coin is tossed since nobody participates. Hence, $prob_i(win) = 1/2$, $i = A, B$.

*Group pattern 2*: For case 2(b), the volunteers' dilemma game, $prob_i(win)$ with $q \equiv q_i$ has the components

$$prob_i(majority) = \sum_{l=1}^{N_i} \binom{N_i}{l} q^l (1-q)^{N_i - l} \quad \text{and} \quad prob(tie) = (1-q)^{N_i} . \tag{A.26}$$

For case 3(b), these probabilities are derived analogous, only $q$ ($N_i$) must be replaced by $q_H \equiv q_{i,H}$ ($N_{i,H}$).

*Group pattern 3*: For case 3(c), the modified volunteers' dilemma game with two different benefit-differentials in $G_i$, $prob_i(win)$ with $q_H \equiv q_{i,H}$ and $q_L \equiv q_{i,L}$ has the components

$$prob_i(majority) = \sum_{l=1}^{N_i} \sum_{m=\max[0, l-N_{i,H}]}^{\min[l, N_i - N_{i,H}]} \binom{N_{i,H}}{l-m} \binom{N_i - N_{i,H}}{m} q_H{}^{l-m} (1-q_H)^{N_{i,H}-l+m} q_L{}^m (1-q_L)^{N_i - N_{i,H} - m}$$

and

$$prob(tie) = (1-q_H)^{N_{i,H}} (1-q_L)^{N_i - N_{i,H}} . \tag{A.27}$$

*Group pattern 4*: For cases 2(c) and 3(d), the standard participation games, $prob_i(win)$ with $q \equiv q_i$ and $\tilde{q} \equiv q_{-i}$ has the components

$$prob_i(majority) = \sum_{k=0}^{\min[N_i, N_{-i}]} \sum_{l=1}^{N_i - k} \binom{N_i}{k+l} \binom{N_{-i}}{k} q^{k+l} (1-q)^{N_i - k - l} \tilde{q}^k (1-\tilde{q})^{N_{-i} - k}$$

and

$$prob(tie) = \sum_{k=0}^{\min[N_i, N_{-i}]} \binom{N_i}{k} \binom{N_{-i}}{k} q^k (1-q)^{N_i - k} \tilde{q}^k (1-\tilde{q})^{N_{-i} - k} . \tag{A.28}$$

For case 3(d), these probabilities are derived analogous, only $q$ ($N_i$) must be replaced by $q_H \equiv q_{i,H}$ ($N_{i,H}$).

*Group pattern 5*: For case 3(e), the modified participation game with two different benefit-differentials in $G_i$, $prob_i(win)$ with $q_H \equiv q_{i,H}$, $q_L \equiv q_{i,L}$, and $\tilde{q} \equiv q_{-i}$ has the components

$$prob_i(majority) = \sum_{k=0}^{\min[N_i, N_{-i}]} \sum_{l=1}^{N_i - k} \sum_{m=\max[0, k+l-N_{i,H}]}^{\min[k+l, N_i - N_{i,H}]} \binom{N_{i,H}}{k+l-m} \binom{N_{i,L}}{m} \binom{N_{-i}}{k}$$
$$\times\; q_H^{k+l-m} (1-q_H)^{N_{i,H}-k-l+m} q_L^m (1-q_L)^{N_{i,L}-m} \tilde{q}^k (1-\tilde{q})^{N_{-i}-k}$$

and

$$prob(tie) = \sum_{k=0}^{\min[N_i, N_{-i}]} \sum_{m=\max[0, k-N_{i,H}]}^{\min[k, N_i - N_{i,H}]} \binom{N_{i,H}}{k-m} \binom{N_{i,L}}{m} \binom{N_{-i}}{k}$$
$$\times\; q_H^{k-m} (1-q_H)^{N_{i,H}-k+m} q_L^m (1-q_L)^{N_{i,L}-m} \tilde{q}^k (1-\tilde{q})^{N_{-i}-k} . \tag{A.29}$$

Finally, the subgame perfect Nash equilibria can be derived by using backwards induction. For all possible combinations of policy offers, the conditions for the totally quasi-symmetric mixed strategy equilibria at the election stage are described by $(A.21)$ to $(A.24)$. These, or other Nash equilibria, are anticipated by the candidates at the first stage. Then, as for compulsory voting, constant sum normal forms can be derived for the candidate competition stage, with the cells representing all possible combinations of *A*'s and *B*'s policy offers and the cells' entries the expected payoff $Exp[\rho_A] = prob_A(win)$ of *A*, as described by $(A.25)$ to $(A.29)$. Because the candidates play a constant sum game, $Exp[\rho_B] = 1 - prob_A(win)$. Appendix B gives examples for $E = W = 2,3,4$ and different voting costs of $c = 0.2$, 0.4, and 0.6. Because of computational complexity and multiple equilibria at the election stage we do not provide numerical solutions for larger electorates.

# Appendix B – Numerical equilibrium examples

For compulsory voting, figures B.1a-d present for each of the decision rules we considered the normal form of the policy game with $E = 4$ and $W = 4$. Numbers in the cells are payoffs to the row player (candidate *A*). The payoffs of the column player (candidate *B*), which are not shown, are equal to 1 minus *A*'s respective payoff. The 4 voters are labeled 1,2,3, and 4, respectively, and each candidate's pure strategies are represented by all possible combinations {1},{2},…,{1,2,3,4} of specific voters. Subgame perfect Nash equilibria in pure strategies are shown by gray shaded cells. For the random and neutral rules (egalitarian rule) any possible combination of strategies that can only results in two (three, using iterated weak dominance) selected votes constitutes an equilibrium. There is no subgame perfect Nash equilibrium in pure strategies for the elitist rule. It has, among many others, one mixed strategy equilibrium where both candidates play combinations {1},{2},{3,4},{1,2,3}, and {1,2,4} with equal probability of 1/5 each and another one where both play combinations {1,3},{2,4}, and {1,2,3,4} with equal probability of 1/3 each.[48]

Similarly, for voluntary voting figures B.2a-c show examples for $E = W = 3$ and varying participation costs. For $c = 0.2$ and $c = 0.6$, any combination of strategies that can only result in two selected voters constitutes a subgame perfect Nash equilibrium at the candidate competition stage. For $c = 0.4$, the only subgame perfect Nash equilibrium is where both candidates choose the egalitarian policy offer.

[48] We used Gambit (McKelvey et al. 2005) to compute the mixed strategy equilibria.

**FIGURE B.1a**: CANDIDATES' (STAGE 1) NORMAL FORM FOR COMPULSORY VOTING: NEUTRAL RULE ( $E = W = 4$ )

*Candidate B*

*Candidate A*

| | {1} | {2} | {3} | {4} | {1,2} | {1,3} | {1,4} | {2,3} | {2,4} | {3,4} | {1,2,3} | {1,2,4} | {1,3,4} | {2,3,4} | {1,2,3,4} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| {1} | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| {2} | ½ | ½ | ½ | ½ | ½ | 0 | 0 | ½ | ½ | 0 | 0 | 0 | 0 | 0 | 0 |
| {3} | ½ | ½ | ½ | ½ | 0 | ½ | 0 | ½ | 0 | ½ | 0 | 0 | 0 | 0 | 0 |
| {4} | ½ | ½ | ½ | ½ | 0 | 0 | ½ | 0 | ½ | ½ | 0 | 0 | 0 | 0 | 0 |
| {1,2} | ½ | ½ | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 1 | 1 | ½ | ½ | ½ |
| {1,3} | ½ | 1 | ½ | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 1 | ½ | 1 | ½ | ½ |
| {1,4} | ½ | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | 1 | ½ | ½ |
| {2,3} | 1 | ½ | ½ | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 1 | ½ | ½ | 1 | ½ |
| {2,4} | 1 | ½ | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | ½ | 1 | ½ |
| {3,4} | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | 1 | ½ |
| {1,2,3} | 1 | 1 | 1 | 1 | 0 | 0 | ½ | 0 | ½ | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,2,4} | 1 | 1 | 1 | 1 | 0 | ½ | 0 | ½ | 0 | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,3,4} | 1 | 1 | 1 | 1 | ½ | 0 | 0 | ½ | ½ | 0 | ½ | ½ | ½ | ½ | 1 |
| {2,3,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | 0 | 0 | 0 | ½ | ½ | ½ | ½ | 1 |
| {1,2,3,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | ½ |

**FIGURE B.1b**: CANDIDATES' (STAGE 1) NORMAL FORM FOR COMPULSORY VOTING: RANDOM RULE ( $E = W = 4$ )

*Candidate B*

*Candidate A*

| | {1} | {2} | {3} | {4} | {1,2} | {1,3} | {1,4} | {2,3} | {2,4} | {3,4} | {1,2,3} | {1,2,4} | {1,3,4} | {2,3,4} | {1,2,3,4} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| {1} | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 0 | 0 |
| {2} | ½ | ½ | ½ | ½ | ½ | 1/6 | 1/6 | ½ | ½ | 1/6 | 1/6 | 1/6 | 0 | 1/6 | 0 |
| {3} | ½ | ½ | ½ | ½ | 1/6 | ½ | 1/6 | ½ | 1/6 | ½ | 1/6 | 0 | 1/6 | 1/6 | 0 |
| {4} | ½ | ½ | ½ | ½ | 1/6 | 1/6 | ½ | 1/6 | ½ | ½ | 0 | 1/6 | 1/6 | 1/6 | 0 |
| {1,2} | ½ | ½ | 5/6 | 5/6 | ½ | ½ | ½ | ½ | ½ | ½ | 5/6 | 5/6 | ½ | ½ | ½ |
| {1,3} | ½ | 5/6 | ½ | 5/6 | ½ | ½ | ½ | ½ | ½ | ½ | 5/6 | ½ | 5/6 | ½ | ½ |
| {1,4} | ½ | 5/6 | 5/6 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 5/6 | 5/6 | ½ | ½ |
| {2,3} | 5/6 | ½ | ½ | 5/6 | ½ | ½ | ½ | ½ | ½ | ½ | 5/6 | ½ | ½ | 5/6 | ½ |
| {2,4} | 5/6 | ½ | 5/6 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 5/6 | ½ | 5/6 | ½ |
| {3,4} | 5/6 | 5/6 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 5/6 | 5/6 | ½ |
| {1,2,3} | 5/6 | 5/6 | 5/6 | 1 | 1/6 | 1/6 | ½ | 1/6 | ½ | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,2,4} | 5/6 | 5/6 | 1 | 5/6 | 1/6 | ½ | 1/6 | ½ | 1/6 | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,3,4} | 5/6 | 1 | 5/6 | 5/6 | ½ | 1/6 | 1/6 | ½ | ½ | 1/6 | ½ | ½ | ½ | ½ | 1 |
| {2,3,4} | 1 | 5/6 | 5/6 | 5/6 | ½ | ½ | ½ | 1/6 | 1/6 | 1/6 | ½ | ½ | ½ | ½ | 1 |
| {1,2,3,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | ½ |

**FIGURE B.1c**: CANDIDATES' (STAGE 1) NORMAL FORM FOR COMPULSORY VOTING: EGALITARIAN RULE ( $E = W = 4$ )

*Candidate B*

*Candidate A*

| | {1} | {2} | {3} | {4} | {1,2} | {1,3} | {1,4} | {2,3} | {2,4} | {3,4} | {1,2,3} | {1,2,4} | {1,3,4} | {2,3,4} | {1,2,3,4} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| {1} | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| {2} | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| {3} | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| {4} | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| {1,2} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ |
| {1,3} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ |
| {1,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ |
| {2,3} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ |
| {2,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ |
| {3,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ |
| {1,2,3} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,2,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,3,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 |
| {2,3,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,2,3,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | ½ |

**FIGURE B.1d**: CANDIDATES' (STAGE 1) NORMAL FORM FOR COMPULSORY VOTING: ELITIST RULE ( $E = W = 4$ )

*Candidate B*

*Candidate A*

| | {1} | {2} | {3} | {4} | {1,2} | {1,3} | {1,4} | {2,3} | {2,4} | {3,4} | {1,2,3} | {1,2,4} | {1,3,4} | {2,3,4} | {1,2,3,4} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| {1} | ½ | ½ | ½ | ½ | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 0 | 0 |
| {2} | ½ | ½ | ½ | ½ | 1 | ½ | ½ | 1 | 1 | ½ | ½ | ½ | 0 | ½ | 0 |
| {3} | ½ | ½ | ½ | ½ | ½ | 1 | ½ | 1 | ½ | 1 | ½ | 0 | ½ | ½ | 0 |
| {4} | ½ | ½ | ½ | ½ | ½ | ½ | 1 | ½ | 1 | 1 | 0 | ½ | ½ | ½ | 0 |
| {1,2} | 0 | 0 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | 1 | ½ | ½ | ½ |
| {1,3} | 0 | ½ | 0 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | ½ | 1 | ½ | ½ |
| {1,4} | 0 | ½ | ½ | 0 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | 1 | ½ | ½ |
| {2,3} | ½ | 0 | 0 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | ½ | ½ | 1 | ½ |
| {2,4} | ½ | 0 | ½ | 0 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | ½ | 1 | ½ |
| {3,4} | ½ | ½ | 0 | 0 | ½ | ½ | ½ | ½ | ½ | ½ | ½ | ½ | 1 | 1 | ½ |
| {1,2,3} | ½ | ½ | ½ | 1 | 0 | 0 | ½ | 0 | ½ | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,2,4} | ½ | ½ | 1 | ½ | 0 | ½ | 0 | ½ | 0 | ½ | ½ | ½ | ½ | ½ | 1 |
| {1,3,4} | ½ | 1 | ½ | ½ | ½ | 0 | 0 | ½ | ½ | 0 | ½ | ½ | ½ | ½ | 1 |
| {2,3,4} | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 0 | 0 | 0 | ½ | ½ | ½ | ½ | 1 |
| {1,2,3,4} | 1 | 1 | 1 | 1 | ½ | ½ | ½ | ½ | ½ | ½ | 0 | 0 | 0 | 0 | ½ |

**FIGURE B.2a**: CANDIDATES' (STAGE 1) NORMAL FORM FOR VOLUNTARY VOTING
$(E = W = 3,\ c = .2)$

*Candidate B*

*Candidate A*

| | {1} | {2} | {3} | {1,2} | {1,3} | {2,3} | {1,2,3} |
|---|---|---|---|---|---|---|---|
| {1} | ½ | ½ | ½ | ½ | ½ | .017 | .038 |
| {2} | ½ | ½ | ½ | ½ | .017 | ½ | .038 |
| {3} | ½ | ½ | ½ | .017 | ½ | ½ | .038 |
| {1,2} | ½ | ½ | .983 | ½ | ½ | ½ | .826 |
| {1,3} | ½ | .983 | ½ | ½ | ½ | ½ | .826 |
| {2,3} | .983 | ½ | ½ | ½ | ½ | ½ | .826 |
| {1,2,3} | .962 | .962 | .962 | .174 | .174 | .174 | ½ |

**FIGURE B.2b**: CANDIDATES' (STAGE 1) NORMAL FORM FOR VOLUNTARY VOTING
$(E = W = 3,\ c = .4)$

*Candidate B*

*Candidate A*

| | {1} | {2} | {3} | {1,2} | {1,3} | {2,3} | {1,2,3} |
|---|---|---|---|---|---|---|---|
| {1} | ½ | ½ | ½ | ½ | ½ | .071 | .174 |
| {2} | ½ | ½ | ½ | ½ | .071 | ½ | .174 |
| {3} | ½ | ½ | ½ | .071 | ½ | ½ | .174 |
| {1,2} | ½ | ½ | .929 | ½ | ½ | ½ | 0 |
| {1,3} | ½ | .929 | ½ | ½ | ½ | ½ | 0 |
| {2,3} | .929 | ½ | ½ | ½ | ½ | ½ | 0 |
| {1,2,3} | .826 | .826 | .826 | 1 | 1 | 1 | ½ |

**FIGURE B.1C**: CANDIDATES' (STAGE 1) NORMAL FORM FOR VOLUNTARY VOTING
$(E = W = 3,\ c = .6)$

*Candidate B*

*Candidate A*

| | {1} | {2} | {3} | {1,2} | {1,3} | {2,3} | {1,2,3} |
|---|---|---|---|---|---|---|---|
| {1} | ½ | ½ | ½ | ½ | ½ | .174 | 1 |
| {2} | ½ | ½ | ½ | ½ | .174 | ½ | 1 |
| {3} | ½ | ½ | ½ | .174 | ½ | ½ | 1 |
| {1,2} | ½ | ½ | .826 | ½ | ½ | ½ | ½ |
| {1,3} | ½ | .826 | ½ | ½ | ½ | ½ | ½ |
| {2,3} | .826 | ½ | ½ | ½ | ½ | ½ | ½ |
| {1,2,3} | 0 | 0 | 0 | ½ | ½ | ½ | ½ |

# Appendix C – Instructions for treatment VP [*VS*, *CP*]

[*Translation from Dutch*]

Welcome to our experiment on decision-making. Everybody receives 10 Guilders for participation in the experiment. Depending on your *own choices* and the *choices of other participants*, you may earn more money today. Your earnings in the experiment are expressed in *tokens*. 4 tokens are worth one Guilder. At the end of the experiment your total earnings in tokens will be exchanged into Guilders and paid to you in cash. The payment will remain *anonymous*. No other participant will be informed about your payment.

**Please remain quiet and do not communicate with other participants during the entire experiment. Raise you hand if you have any question. One of us will come to you to answer them.**

*Rounds, 'participants A' and 'participants B'*

The experiment consists of *51 rounds*. Each round consists of two parts, *part A* and *part B*. At the *beginning of the experiment* the computer program will randomly split all participants (14) [*(18), (14)*] into 2 [*(6), (2)*] *participants A* and 12 *participants B*. You will then receive information whether you are of type participant *A* or participant *B*. **Note that your type will not change during the entire experiment.** Each participant *A* will be asked to make decisions only in part *A* of each round and each participant *B* will be asked to make decisions only in part *B* of each round. You will not know who of the other participants is of type participant *A* and who participant *B*.

*Choices participants A*

At the beginning of part *A* *in each round* *both* [*for VS*: *all*] participants *A* will be asked to make choices. When a participant *A* makes choices, no other participant (neither *A* nor *B*) will know these choices.

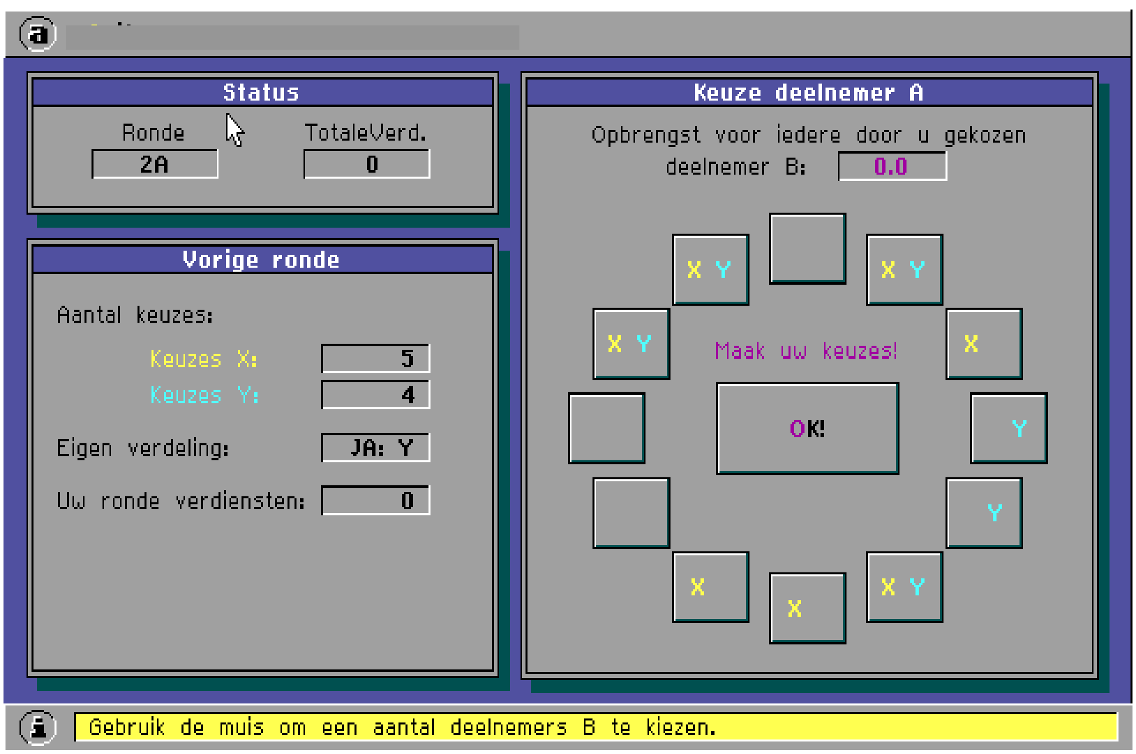
Each participant *A* will be asked to distribute a fixed *round budget* of 18 tokens across the participants *B*. This is done by selecting a number of 1, 2, …, 12 participants *B* (each participant *A* must select at least one participant *B*). The *revenues for participants B* are calculated as follows:

1. Each participant *B* receives **1 token**.
2. Each **selected** participant *B* receives in addition **18 tokens divided by the number of selected participants *B* (18/number participants *B* = *b*)**.

There are the following possible revenues for participants *B*:

| | *Number of selected participants B by one participant A* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Revenue participants B* | *1* | *2* | *3* | *4* | *5* | *6* | *7* | *8* | *9* | *10* | *11* | *12* |
| selected | **19.0** | **10.0** | **7.0** | **5.5** | **4.6** | **4.0** | **3.6** | **3.3** | **3.0** | **2.8** | **2.6** | **2.5** |
| not selected | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** | **1** |

The selection of participants *B* by participants *A* is made as follows. Each participant *B* is represented by a *button* (see the figure below). **The position of a button for a participant *B* will not change during the entire experiment.** If you want to select a specific participant *B*, then click the respective button with the mouse. The color of this button will then switch into *purple*. If you want to change your choice, click the button again with the mouse and the color switches back into *gray*. You can select *all* combinations of participants *B* in which at least one participant *B* is selected. The *selected* participants *B* are represented by all purple buttons. The participants *B* who are *not selected* are represented by all gray buttons. If you are ready with your choices, click the "OK!" button with the mouse or press the key "O".




Translation from Dutch: Round = 'Ronde'; Total earnings = 'TotaleVerd.'; Previous round = 'Vorige ronde'; Choices = 'Keuzes'; Own distribution = 'Eigen verdeling'; Your round earnings = 'Uw ronde verdiensten'; Yes = 'Ja'; Choice participant A = 'Keuze deelnemer A'; Revenue for each participant B selected by you = 'Opbrengst voor iedere door u gekozen deelnemer B'; Make your choices! = 'Maak uw Keuzes!'; Use the mouse to select a number of participants B = 'Gebruik de muis om een aantal deelnemers B te kiezen'.

*Division of participants A into participant X and participant Y*

From now on one participant *A* will be named *participant X* and the other *participant Y*. Their choices will be named *distribution X* and *distribution Y*. **In part *B* of the current round, both participants *A* and all participants *B* will then receive information about the choices of participant *X* and participant *Y*.**

[*In VS: After all 6 participants A have made their choices,* **2** *participants A will be randomly drawn by the computer program. Each participant A has the same chance of being drawn. The chosen participants will be named participant X and participant Y. Their choices will be named distribution X and distribution Y.* ***In part B of the current round, all participants A and all participants B will then receive information about the choices of participant X and participant Y.*** *The choices of the remaining 4 participants A will neither be announced to other participants A nor to participants B.*]

*Distributions X and Y in the previous round*

The "X" and/or "Y" on the buttons indicate that these participants *B* were selected in the *previous round* by participant *X* and/or participant *Y*. If the button is blank, this participant was neither selected by participant *X* nor by participant *Y*.

*Earnings participants A*

The *round earnings* of each participant *A* for the respective round will be determined in the following way. One of the two participants *X* and *Y* receives **1 point** in the current round. Each round-point *may* be worth **20 tokens**. Who (participant *X* or participant *Y*) will receive the round-point depends on the choices of participants *B* in part *B* of the current round. How this works precisely will be explained below. At the end of the experiment, the computer program will randomly determine *17 of the 51 rounds*. For each round-point of these rounds a participant *A* will receive 20 tokens. The *total earnings* of each participant *A* is the sum of all her or his round-points in the 17 rounds determined multiplied by 20 tokens.

[*In VS: The round earnings of each participant A for the respective round will be determined in the following way. Only one of the two participants X and Y receives* **20 tokens** *in the current round. Who (participant X or participant Y) will receive these 20 tokens depends on the choices of participants B in part B of the current round. How this works precisely will be explained below. The remaining participants A will earn nothing (***0 tokens***) in the current round. The total earnings of each participant A is the sum of all her or his round earnings.*]

*<u>Choices and earnings participants B</u>*

In *<u>each round</u>* each participant *B* faces an identical choice problem. Each participant *B* will be asked to make *<u>one choice</u>* in each round. Participants *B* can choose between the following three alternatives:

- '*<u>Choice 0</u>*': no costs involved (**0 tokens**). [*In CP: costs are **1 token**.*]
- '*<u>Choice X</u>*': costs are **1 token**.
- '*<u>Choice Y</u>*': costs are **1 token**.

When participant *B* makes a choice, no other participant (neither *B* nor *A*) knows this choice. Only after *all* participants *B* have made their choices, the computer program will count the number of *X*-choices and the number of *Y*-choices and will compare both numbers. There are **3** possible outcomes that are relevant for the *revenues* of participants *B* *<u>and</u>* for the earnings of participants *X* and *Y*. Each participant *B* will receive her or his revenue irrespective of the choice she or he made.

(1) <u>The number of *X*-choices **exceeds** the number of *Y*-choices</u>:

- *Each* participant *B* who is selected by participant *X* will get revenue of **($b_X$ + 1) tokens** [see table page 2] and *each* participant *B* who is not selected by participant *X* will get **1 token**.
- Participant *X* will get **1 round-point** [*in VS: **20 tokens***] and participant *Y* will get *nothing* (**0 round-points [***in VS: **0 tokens***]).

(2) <u>The number of *Y*-choices **exceeds** the number of *X*-choices</u>:

- *Each* participant *B* who is selected by participant *Y* will get revenue of **($b_Y$ + 1) tokens** [see table page 2] and *each* participant *B* who is not selected by participant *Y* will get **1 token**.
- Participant *Y* will get **1 round-point** [*in VS: **20 tokens***] and participant *X* will get *nothing* (**0 round-points [***in VS: **0 tokens***]).

(3) <u>The number of *X*-choices **is equal to** the number of *Y*-choices</u>:

The computer program will randomly choose which distribution (*X* or *Y*) will determine the revenues (each distribution has the same chance of 50% of being chosen).

- *Each* participant *B* who is selected by the chosen distribution will get revenue of **($b_X$ + 1)** *<u>or</u>* **($b_Y$ + 1) tokens**. *Each* participant *B* who is not selected by the chosen distribution will get **1 token**.
- The chosen participant *X* or *Y* will get **1 round-point** [*in VS: **20 tokens***] and the participant *X* or *Y* who is not chosen will get *nothing* (**0 round-points [***in VS: **0 tokens***]).

**Note that all participants *A* and participants *B* will only get information about the number of *X*-choices and the number of *Y*-choices, but no information who of the participants *B* specifically has made which choice X, Y or 0.**

The <u>*round earnings*</u> of a participant *B* for a respective round are calculated in the following way: *round earnings = round revenue – round costs*. The <u>*total earnings*</u> of a participant *B* are the sum of all her or his round earnings.

The following tables give all your possible round earnings:

**<u>Possible round earnings participants *B*:</u>**

<u>*Case 1*</u>: You are selected **only** by distribution *X*:

| *Your choice* | *<u>More</u> X-choices than Y-choices* | *<u>Less</u> X-choices than Y-choices* | *<u>Equal</u> number X- and Y-choices* |
|---|---|---|---|
| Choice *0* | **($b_X$ + 1) tokens** | **1 token** | **($b_X$ + 1)** *or* **1 token** (50% chance each) |
| Choice *X* of *Y* | **($b_X$ + 0) tokens** | **0 token** | **($b_X$ + 1)** *or* **0 token** (50% chance each) |

<u>*Case 2*</u>: You are selected by distribution *X* **and** distribution *Y*:

| *Your choice* | *<u>More</u> X-choices than Y-choices* | *<u>Less</u> X-choices than Y-choices* | *<u>Equal</u> number X- and Y-choices* |
|---|---|---|---|
| Choice *0* | **($b_X$ + 1) tokens** | **($b_Y$ + 1) tokens** | **($b_X$ + 1)** *or* **($b_Y$ + 1) tokens** (50% chance each) |
| Choice *X* of *Y* | **($b_X$ + 0) tokens** | **($b_Y$ + 0) tokens** | **($b_X$ + 0)** *or* **($b_Y$ + 0) tokens** (50% chance each) |

<u>*Case 3*</u>: You are selected **only** by distribution *Y*:

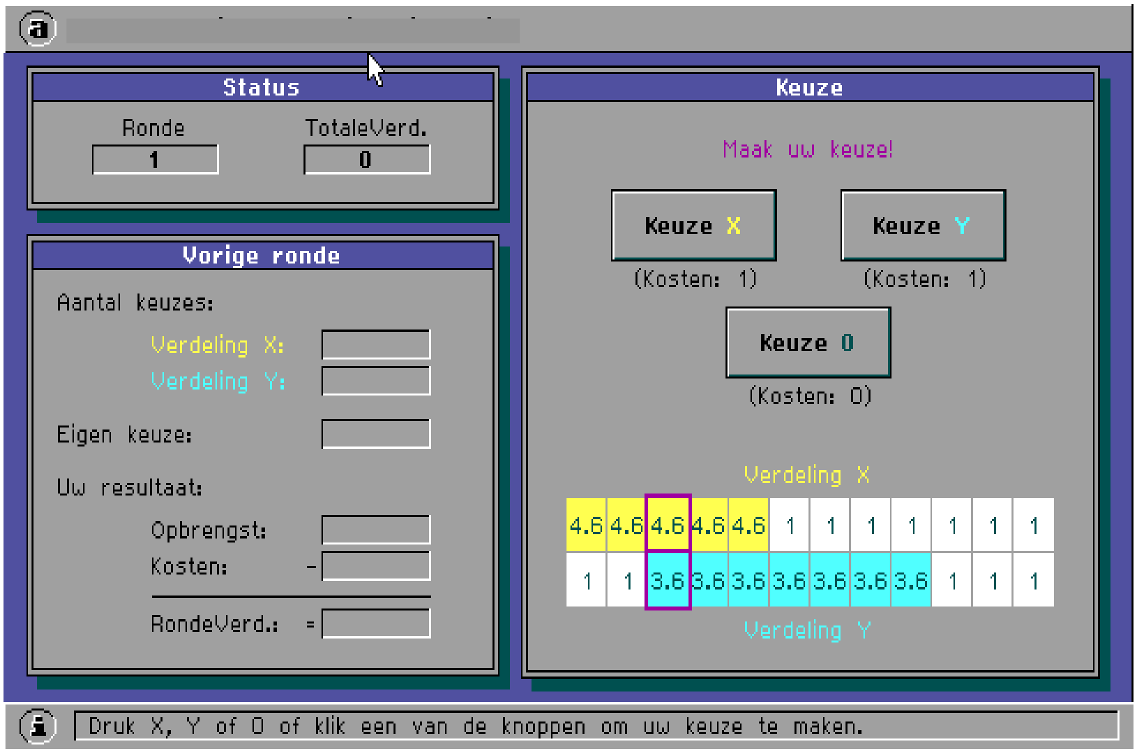
| *Your choice* | *<u>More</u> X-choices than Y-choices* | *<u>Less</u> X-choices than Y-choices* | *<u>Equal</u> number X- and Y-choices* |
|---|---|---|---|
| Choice *0* | **1 token** | **($b_Y$ + 1) tokens** | **($b_Y$ + 1)** *or* **1 token** (50% chance each) |
| Choice *X* of *Y* | **0 token** | **($b_Y$ + 0) tokens** | **($b_Y$ + 1)** *or* **0 token** (50% chance each) |

<u>*Case 4*</u>: You are selected by **neither** distribution:

| *Your choice* | *<u>More</u> X-choices than Y-choices* | *<u>Less</u> X-choices than Y-choices* | *<u>Equal</u> number X- and Y-choices* |
|---|---|---|---|
| Choice *0* | **1 token** | **1 token** | **1 token** |
| Choice *X* of *Y* | **0 token** | **0 token** | **0 token** |

*Example participants B*

Below you can see a figure that participants *B* will also encounter on the computer screen.




Translation from Dutch: Round = 'Ronde'; Total earnings = 'TotaleVerd.'; Previous round = 'Vorige ronde'; Number of choices = 'Aantal keuzes'; Distribution = 'Verdeling'; Own choice = 'Eigen keuze'; Your result = 'Uw resultaat'; Revenue = 'Opbrengst'; Costs = 'Kosten'; Round earnings = 'RondeVerd.'; Choice = 'Keuze'; Make your choice! = 'Maak uw Keuze!'; Press X, Y, or 0 or click one of the buttons to maker your choice = 'Druk X, Y of 0 of klik een van de knoppen om uw keuze te maken'.

In this example participant *X* has selected five participants *B* and participant *Y* has selected seven participants *B*. In case the number of *X*-choices exceeds the number of *Y*-choices, each participant *B* selected by participant *X* will get revenue of 4.6 tokens and each non-selected participant *B* 1 token. In case the number of *Y*-choices exceeds the number of *X*-choices, each participant *B* selected by participant *Y* will get revenue of 3.6 tokens and each not selected participant *B* 1 token. In case the number of *X*-choices is equal to the number of *Y*-choices, one of both distributions will be randomly chosen to determine the revenue for each participant. Note that there are two participants *B* who are selected only by distribution *X*, three participants by both distributions, four participants only by distribution *Y*, and three participants by neither distribution. The *purple* frame identifies you as one of the participants *B* in the figure. In the example, you are selected by both participants *X* and *Y*. There is no fixed ordering of participants *B* in this figure. In each round, the positions of participants will be ordered according to the distributions *X* and *Y*.

*Computer screens*

*Computer screen participants A:* [only given to participants *A*]

The computer screen has four main windows:

(1) The ***Status*** window shows the current *round number* and the *total points* [*in VS: total earnings*] up to the previous round.

(2) The ***Previous round*** window depicts the following information about the previous round:

 (a) The number of *X*-choices.

 (b) The number of *Y*-choices.

 (c) Your distribution ("X" or "Y") [*in VS: ("Yes – X", "Yes – Y" or "No")*].

 (d) Your *round-points* [*in VS: round earnings*].

(3) In the ***Choice*** window you will find *twelve buttons*. Press the buttons of the participants *B* who you want to select. When you have chosen you will have to wait until the other participant *A* has made his or her choice [*in VS: until all participants A have made their choices*].

(4) The ***Result*** window shows the results of the *current* round, hence, after each participant has made a choice. Each *yellow* rectangle shown represents one *X*-choice and each *blue* rectangle represents one *Y*-choice. After a few seconds the result will also appear in numbers.

At the lower bound of your screen the *Information bar* is located. There you are told the current status of the experiment.

*Computer screens participants B*: [only given to participants *B*]

The computer screen has four main windows.

(1) The ***Status*** window shows the current *round number* and the *total earnings* up to the previous round.

(2) The ***Previous round*** window depicts the following information about the previous round:

 (a) The number of *X*-choices.

 (b) The number of *Y*-choices.

 (c) Your *choice*.

 (d) Your *revenue*.

 (e) Your *costs*.

 (f) Your *round earnings*.

(3) In the *Choice* window you will find three *buttons*. Press the button "Choice *X*", the button "Choice *Y*", or the button "Choice *0*" with the mouse, or press the key "*X*", "*Y*", or "*0*". When you have chosen you will have to wait until all participants have made their choices. In this window you will also be informed about the distribution *X* and the distribution *Y* at the beginning of each round.

(4) The *Result* window shows the results of the *current* round, hence, after each participant has made a choice. Each *yellow* rectangle shown represents one *X*-choice and each *blue* rectangle represents one *Y*-choice. After a few seconds the result will also appear in numbers.

At the lower bound of your screen the *Information bar* is located. There you are told the current status of the experiment.

*Further procedures*

Before the 51 rounds of the experiment that are relevant for your earnings start, we will ask you to participate in *three training-rounds*. You will have to answer questions in order to proceed further in these training-rounds. In the training-rounds you are not matched with other participants but with the computer program. **You cannot draw conclusions about choices of other participants *A* or participants *B* based on the results in the training-rounds.** When you are ready with the training-rounds, we will ask you to answer more questions.

We will now start with the three training-rounds. If you have any questions, please raise your hand. One of us will come to you to answer them.

# Appendix D – Procedures

**TABLE D.1:** SEQUENCE OF 'ACUTAL' CANDIDATES DRAWN FROM THE POOL OF POTENTIAL CANDIDATES 1-6 IN VS

| Round | 'Actual' candidates | Round | 'Actual' candidates | Round | 'Actual' candidates |
|---|---|---|---|---|---|
| 1 | *3-1* | 21 | *1-4* | 41 | *2-3* |
| 2 | *4-6* | 22 | *5-6* | 42 | *2-6* |
| 3 | *3-5* | 23 | *1-2* | 43 | *5-1* |
| 4 | *1-2* | 24 | *4-2* | 44 | *1-2* |
| 5 | *4-6* | 25 | *5-3* | 45 | *6-5* |
| 6 | *5-1* | 26 | *5-3* | 46 | *2-1* |
| 7 | *2-3* | 27 | *4-6* | 47 | *3-6* |
| 8 | *6-5* | 28 | *3-5* | 48 | *6-3* |
| 9 | *3-5* | 29 | *2-6* | 49 | *1-4* |
| 10 | *4-2* | 30 | *2-4* | 50 | *4-2* |
| 11 | *1-5* | 31 | *3-1* | 51 | *5-1* |
| 12 | *6-3* | 32 | *1-4* | | |
| 13 | *4-2* | 33 | *3-5* | | |
| 14 | *1-3* | 34 | *5-4* | | |
| 15 | *6-3* | 35 | *3-5* | | |
| 16 | *6-4* | 36 | *1-6* | | |
| 17 | *2-5* | 37 | *6-1* | | |
| 18 | *4-2* | 38 | *4-2* | | |
| 19 | *1-2* | 39 | *1-6* | | |
| 20 | *6-3* | 40 | *1-5* | | |