GEOPHYSICAL RESEARCH LETTERS, VOL. 20, NO. 2, PAGES 213-216, FEBRUARY 5, 1993

NONLINEAR DYNAMICS ESTABLISHED IN THE ENSO

J. B. Elsner

Department of Meteorology, Florida State University

A. A. Tsonis

Department of Geosciences, University of Wisconsin-Milwaukee

Abstract. A time series describing the El-Nino - Southern Oscillation (ENSO) is analyzed using the latest techniques of chaos theory. The methods which rely on resampling statistics were developed to more finely distinguish between nonlinearity and linear correlated noise. From the results significant nonlinear structure arising from ENSO dynamics on the monthly time scale is established.

Introduction

Extended periods of anomalously warm sea surface temperatures occurring aperiodically off the coast of South America are called El Nino events. These changes are intimately linked to the atmospheric zonal circulation in this region called the Southern Oscillation [Barnett et al., 1988]. The zonal transport of atmospheric mass (Walker circulation) across the Pacific Ocean produces a dipole surface pressure fluctuation which is clearly identified by subtracting monthly averaged sea-level pressures at Darwin from similarly averaged values at Tahiti. Persistent negative values of this Southern Oscillation Index (SOI) correlate well with El Nino events. Normalized monthly values of SOI beginning in January of 1882 and continuing through April of 1992 (1324 consecutive months) are used in this study.

Recently, Bauer and Brown [1992] using the method of singular-spectrum analysis (SSA) have shown that the underlying dynamics of the ENSO system can be captured in a deterministic low-order model. Nonlinear deterministic structure in ENSO has been suggested in past studies. Vallis [1986] proposed a simplified theoretical model for ENSO which exhibits chaotic dynamics. Hense [1987] applied the Grassberger-Procaccia [Grassberger and Procaccia, 1983] technique to estimate the correlation dimension of a shorter segment of the above data and concluded the ENSO can be described by a system with a dimension between five and six. Lately, Tsonis and Elsner [1992] applying nonlinear prediction to distinguish between random fractal sequences and chaotic signals showed that, for the ENSO time series, the degradation of prediction error with time is exponential: a property which can aid in identifying chaotic dynamics.

The nonlinearity observed in the above studies, however, has never been tested against proper random processes, i.e., time-series data that preserve the observed autocorrelation structure, mainly because such a test has only lately been developed in the context of nonlinear prediction [Theiler et al., 1992, Smith, 1992, Kennel and Isabelle, 1992]. This is important since common randomizing procedures, like shuffling the data, can preserve the mean and variance but destroy temporal correlations present in the data, making it impossible to distinguish dynamics from autocorrelation structure in the data.

Paper number 93GL00046
0094-8534/93/93GL-00046$03.00

The Method of Surrogate Data

The procedure is called the method of surrogate data and it calls for the generation of a large number of random sequences of equal length as the time series to be tested. The idea is that the surrogate time series should be a non-deterministic record but similar in appearance to the original data. One method of surrogate generation is to preserve the amplitude spectrum of the raw data. First a Fourier transform of the raw data is computed; then each complex amplitude is multiplied by $e^{i\phi}$ where ϕ is independently chosen from the interval $[0, 2\pi]$. As long as $\phi(f) = \phi(-f)$ it is guaranteed that the inverse transform is real. Finally, the inverse Fourier transform is the surrogate time series. The raw data and a surrogate are shown in Figures 1 and 2 respectively along with their corresponding autocorrelation functions. Note the similarity in means and variances and note the autocorrelation is preserved in the surrogate data which is otherwise random.

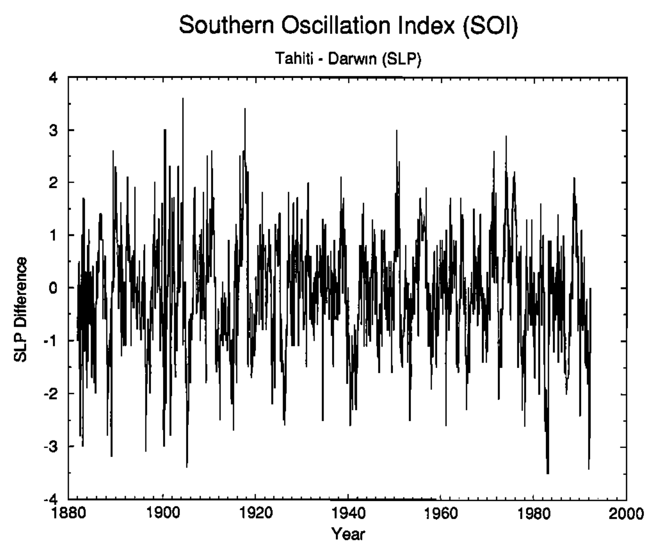

Fig. 1a. Time series of normalized monthly mean sea-level pressure differences between Tahiti and Darwin in standard deviations. The record reveals aperiodic fluctuations in the atmospheric Walker circulation over the tropical Pacific Ocean and is typically referred to as the Southern Oscillation Index (SOI). The record used for analysis and prediction in the present study runs from January, 1882 through April, 1992 for a total of 1324 consecutive months.

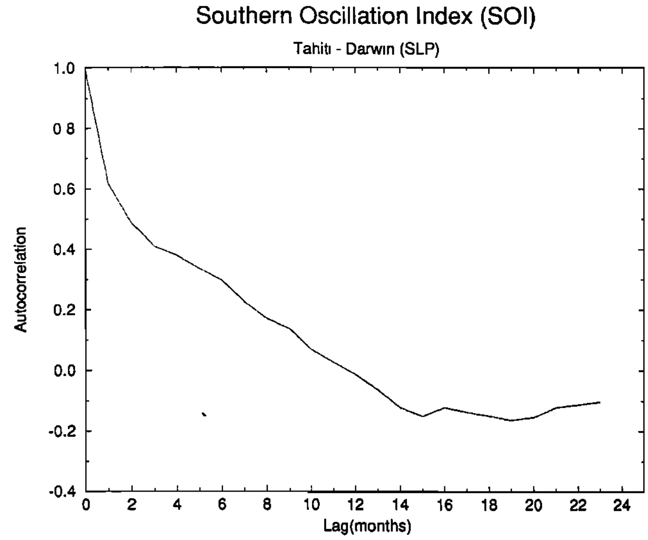

Fig. 1b. Autocorrelation function of the SOI with lag in months. Serial correlation exists in the record for several months with the first zero-crossing occurring between eleven and twelve months.

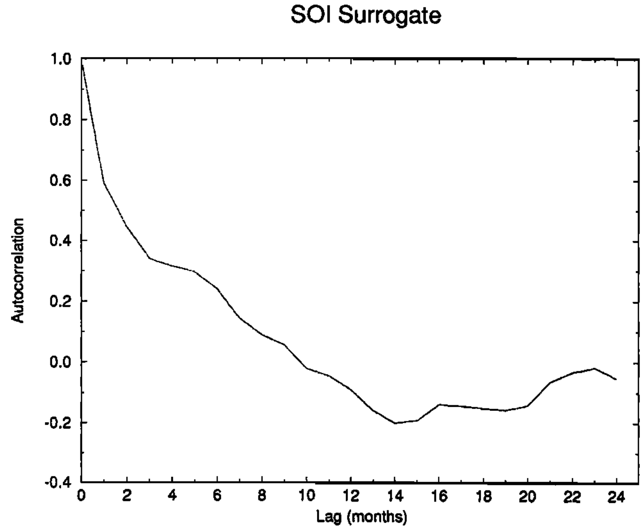

Fig. 2b. Autocorrelation function of the surrogate SOI with lag in months. Note the near identical nature of the autocorrelation function compared to the autocorrelation function of the actual SOI (Figure 1b). Minor differences are attributed to short-comings of the numerical algorithm.

Once we have a way to produce such random processes we can define a null hypothesis against which the raw data can be tested using a discriminating statistic. According to the above algorithm for generating surrogate records, the null hypothesis is that the raw data come from a linear autocorrelated gaussian process. The discriminating statistic (e.g., Lyapunov spectrum, correlation dimension, etc.) is computed for each surrogate time series and its distribution approximated. If the discriminating statistic for the real data is significantly outside the range of the distribution based on the surrogates, then the null hypothesis of linearly correlated noise is rejected. It can therefore be concluded that significant nonlinear structure is present in the record.

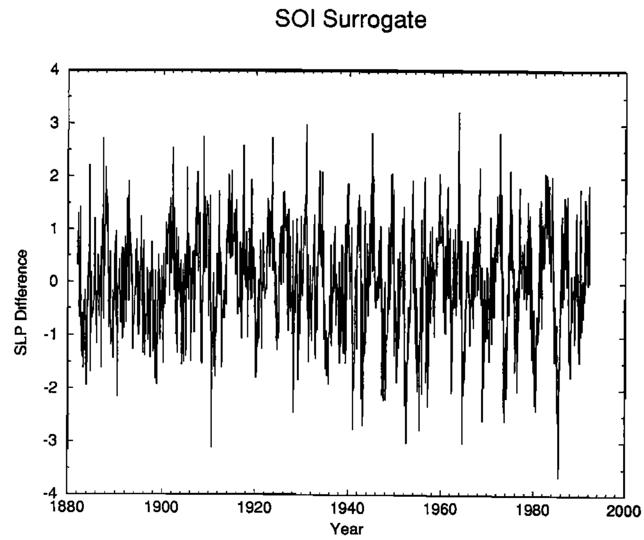

Fig. 2a. Time series generated by randomizing the phases of the SOI Fourier spectrum and then taking the inverse spectrum. The record serves as a surrogate for the original SOI. Means, variances and autocorrelation from the original record are preserved in the surrogates.

Procedures that are commonly used to identify or obtain evidence about the existence of nonlinear deterministic dynamics in time series can be divided into, a) dimension estimates, b) estimates of Lyapunov exponents, and c) nonlinear prediction. In our case we choose the prediction error as the discriminating statistic. We found that choosing the correlation dimension, or the highest Lyapunov exponent leads to an inadequate estimation of the statistic due to the small sample size. Nonlinear prediction, unlike other methods for identifying chaos, make use of more information in the available data and thus often works well with small data sets [Casdagli, 1989, Elsner and Tsonis, 1992]. Calculation of the correlation dimension, for example, is based on the estimation of the scaling region which is typically small thus exploiting only a small subset of the available points in the phase space.

Results and Conclusions

The nonlinear prediction model used here is a version of the Farmer and Sidorowich [1987] and Sugihara and May [1990] interpolative algorithms used in Wales [1991] and based on the simplex method. An embedding dimension (E) is chosen and the phase space is constructed using time delays of the time series. For each point in phase space the nearby points are located and a minimal neighborhood is defined to be such that the subsequence containing the point we wish to predict is located within the simplex with a minimum diameter formed from the E+1 closest neighbors. The prediction is made by keeping track of where the points in the simplex end up after a number of time steps in to the future. For the SOI we used an embedding dimension of six and a time delay of eleven months. Embedding dimension was determined by the dimension which results in the highest correlation coefficient between one-step predicted and actual values [Sugihara and May, 1990]. The delay time was determined by the first zero-crossing of the autocorrelation function. The prediction algorithm is trained on the first 1200 months and predictions are made for the remaining 134

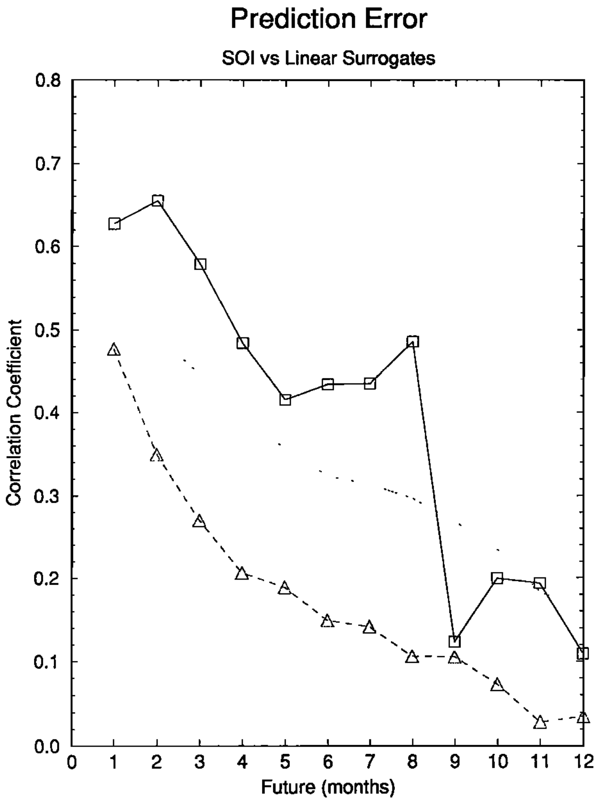

Fig. 3. Prediction error as a function of months into the future using a nonlinear prediction algorithm. Solid line with squares is the error function of the original SOI and the dashed line with triangles the mean error function of fifty surrogates. The one-standard deviation level is the dotted line. Based on the significantly better short-term forecasts of the SOI compared to the surrogates, the null hypothesis of a linearly correlated process for the SOI is rejected at a 95% confidence level. Correlation coefficients are based on a sample size of 134.

month at a time using an embedding dimension of six. This is done for both the SOI and fifty surrogates. Results are shown in Figure 3. Prediction error for the SOI is near two standard deviations greater than the mean prediction error of the linear gaussian surrogates. The null hypothesis of linear gaussian process for the SOI is thus rejected at a confidence level of 95%. It is therefore concluded that the actual SOI has nonlinear structure which can be detected and exploited by strictly nonlinear prediction models. Of course a comparison with the surrogates provides only a necessary, not sufficient, condition for the detection of nonlinearity, since rejection of the null hypothesis only determines what the system in not and not what it is.

Evidence for nonlinear structure is further corroborated by repeating the method using a autoregressive model for predictions. As expected, the linear autoregressive forecast skill on the SOI is not significantly different from the mean autoregressive forecast skill on the surrogates.

The evidence presented above for nonlinearity in the SOI does not imediately imply nonlinear dynamics of ENSO. Nonlinearity in a time series may arise due to nonlinear amplitude distributions rather than nonlinear dynamics [Theiler et al., 1992]. For example, suppose that although the dynamics is linear, the observation function is nonlinear. In this case the null hypothesis should be that the data come values. Correlation coefficients between the predicted and

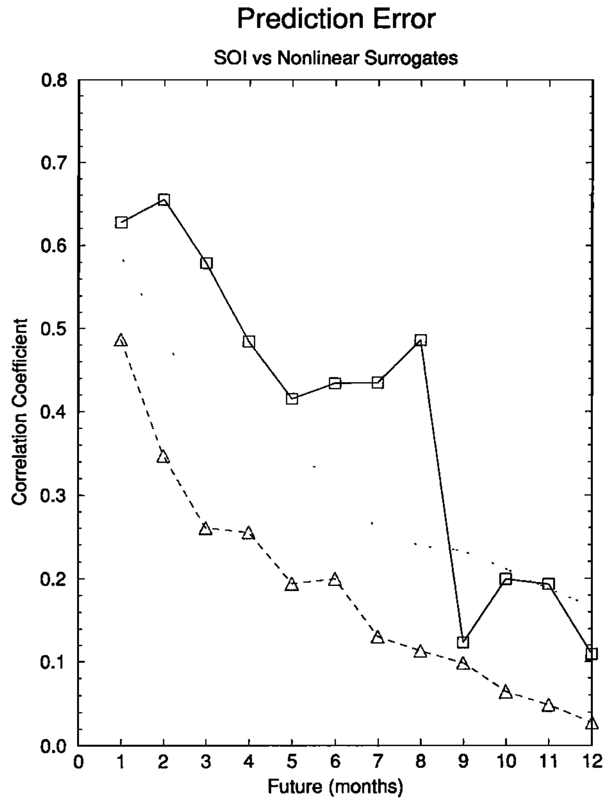

Fig. 4. Comparison of nonlinear prediction error of the SOI and the mean of fifty amplitude adjusted surrogates. The null hypothesis of nonlinear transformation of a linear gaussian process for the SOI is rejected at about the 95% confidence level.

actual values are thus based on a sample size of about 134.

The next step is to make twelve month predictions one from a monotonic nonlinear transformation of a linear gaussian process.

In such a case the surrogate data are generated differently. One algorithm requires generating a gaussian random time series and re-ordering this record according to the rank of the original series. If x_t is the n-th smallest of all the x's in the SOI then y_t will be the n-th smallest of all the y's (i.e. y_t follows x_t). Taking the Fourier transform of y_t, randomizing its phases and taking the inverse transform produces a record, call it y_t'. Then, if the original SOI (x_t) is time re-ordered so that it follows y_t', the time re-ordered time series provides a surrogate having not only the same mean, variance and autocorrelation, but also the same amplitude distribution as the SOI (previously only the mean variance and autocorrelation, not the amplitude distribution was preserved in the surrogates). In this case, if the null hypothesis is rejected then evidence points toward nonlinearity as a result of dynamics.

Prediction errors as a function of future time are accumulated for fifty surrogates under this hypothesis. Averages and one sigmas are plotted in Figure 4. Again here, the null hypothesis is rejected at a confidence level of about 95%. Therefore, not only is there evidence of low-dimensional nonlinear structure in ENSO but the nonlinearity is more likely a result of dynamics rather than due to a nonlinear amplitude distribution.

Summary

To summarize, the SOI provides a useful time series of ENSO dynamics. Coupled with the fact that reliable values are available for better than 110 consecutive years allows for a strictly empirical investigation of intrinsic nonlinearity within this important atmosphere/ocean system. The present paper attacked this problem utilizing a nonlinear time-series prediction algorithm in tandem with the method of surrogate data. From the results, nonlinear dynamics in ENSO are established at a high confidence level.

In terms of actually forecasting ENSO, the prediction skill demonstrated here, even with the nonlinear model is quite modest. However, by incorporating time series of wind stress and sea surface temperatures, for example, from different locations over the tropical Pacific basin or by using principal components of spatial empirical orthogonal functions considerable improvement in forecast skill can be expected.

Acknowledgements. Thanks are extended to Scott Applequist and Mike Kirby for running the autoregressive model. The first author would like to thank Prof. Ahlquist for his constructive comments and deepest gratitudes are extended to Prof. Magnan for many stimulating lunch-time conversations which have contributed to the content of this paper. A portion of this research was supported by NOAA's TOGA Program on Prediction (T-POP) Grant NA16RC 0454-01.

References

Barnett, T., N. Graham, M. Cane, S. Zebiak, S. Dolan, J. O'Brien, D. Legler, On the prediction of the El Nino of 1986-1987, Science, 241, 192-196, 1988.

Bauer, S. T., and M. B. Brown, Empirical low-order ENSO dynamics, Geophys. Res. Lett., 19, 2055-2058, 1992.

Casdagli, M. C., Nonlinear prediction of time series, Physica D, 35, 335-356, 1989.

Elsner, J. B. and A. A. Tsonis, Nonlinear prediction, chaos and noise, Bull. Amer. Meteor. Soc., 73, 49-60, 1992.

Farmer, J. D. and J. J. Sidorowich, Predicting chaotic time series, Phys. Rev. Lett., 59, 845-848, 1987.

Grassberger, P. and I. Procaccia, Measuring the strangeness of strange attractors, Physica D, 9, 189-208, 1983.

Hense, A., On the possible existence of a strange attractor for the Southern Oscillation, Beitr. Phys. Atmosph., 60, 34-47, 1987.

Kennel, M. B. and S. Isabelle, A method to distinguish possible chaos from colored noise and determine embedding parameters, Phys. Rev. A, preprint, 1992.

Smith, L. A., Identification and prediction of low dimensional dynamics, Physica D, 58, 50-76, 1992.

Sugihara, G. and R. M. May, Nonlinear forecasting as a way of distinguishing chaos from measurment error in time series, Nature, 344, 734-741, 1990.

Theiler, J., S. Eubank, A. Longtin, B. Galdrikian and J. D. Farmer, Testing for nonlinearity in time series: the method of surrogate data, Physica D, 58, 77-94, 1992

Tsonis, A. A. and J. B. Elsner, Nonlinear prediction as a way of distinguishing chaos rom random fractal sequences, Nature, 358, 217-220, 1992.

Vallis, G. K., El-Nino: A chaotic dynamical system? Science, 232, 243-245, 1986.

Wales, D. J., Calculating the rate of loss of information from chaotic time series by forecasting, Nature, 350, 485-488, 1991.

J. B. Elsner, Department of Meteorology, B-161, Florida State University, Tallahassee, FL 32306.

A. A. Tsonis, Department of Geosciences, P. O Box 413, University of Wisconsin-Milwaukee, Milwaukee, WI 53201.

(Received October 1, 1992;
Accepted December 28, 1992)