

# Daily tornado frequency distributions in the United States

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## Abstract

The authors examine daily tornado counts in the United States over the period 1994–2012 and find strong evidence for a power-law relationship in the distribution frequency. The scaling exponent is estimated at 1.64 (0.019 s.e.) giving a per tornado-day probability of 0.014% (return period of 71 years) that a tornado day produces 145 tornadoes as was observed on 27 April 2011. They also find that the total number of tornadoes by damage category on days with at least one violent tornado follows an exponential rule. On average, the daily number of tornadoes in the next lowest damage category is approximately twice the number in the current category. These findings are important and timely for tornado hazard models and for seasonal and sub-seasonal forecasts of tornado activity.

Keywords: tornado, climate, power law, statistics

## 1. Introduction

Statistics of past tornado occurrences are a guide to estimating future threats. For example, in the United States over the period 1994–2012 there was an average of 1265 tornadoes per year with an interquartile range of 256 tornadoes. The percentage of the annual count occurring on the day of the year with the most tornadoes ranged from a low of 3.2% in 1998 to a high of 12.2% in 2011. Of the 24 032 US tornadoes in the database over the 19-year period only 0.57% were violent (EF4 or EF5). Here we are motivated to better understand daily tornado statistics as foundation for building a catalog of future tornado events. The catalog can be used to assess the local risk of damage losses conditional on climate variations.

Statistical distributions are fit to tornado characteristics including intensity [1, 2] (Weibull), path length and width [3] (Weibull), and monthly frequency [4] (Poisson). Recently, we suggest a negative binomial distribution for the seasonal occurrence of tornadoes across the central Great Plains [5]. In this paper we are interested in daily occurrence. We describe

the distribution frequency of days with tornadoes as well as the frequency of tornadoes by damage category on days with violent tornadoes.

The distribution frequency of days with tornadoes can be described by a power-law distribution with a scaling exponent of 1.64. This indicates a long-tailed distribution with the potential for large outbreaks. Further, given at least one violent tornado (EF4 or EF5) the frequency by EF category follows an exponential law. On average, the daily number of tornadoes in the next highest damage category is approximately half the number in the current category. These findings are important and timely for developing a tornado hazard model [6] and for constructing models for seasonal and sub-seasonal tornado activity.

In section 2 we present our justification for the tornado data and study period. We also examine the aggregate distribution frequency by damage category. In section 3 we consider the distribution frequency of tornado days and find a power-law relationship. In section 4 we consider the distribution frequency by damage scale disaggregated by day and find an exponential rule. In section 5 we provide a brief summary and some concluding statements. The computer code to reproduce the analysis and results is available from <http://rpubs.com/jelsner/DailyCounts>.



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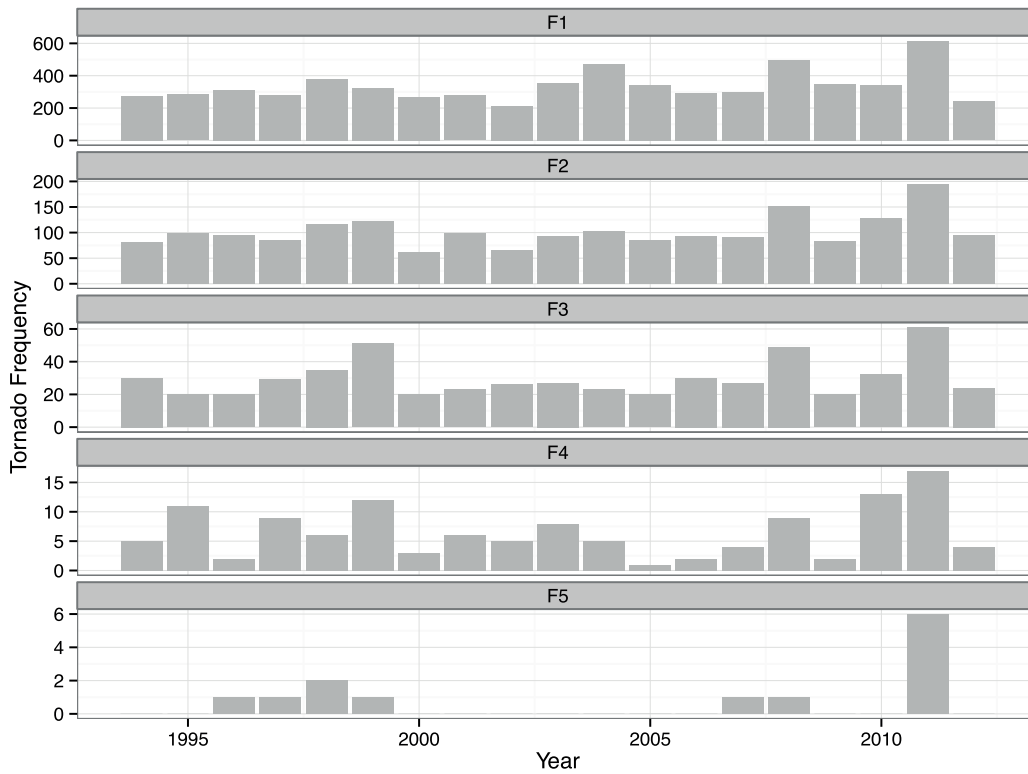


Figure 1. Annual tornado frequency by damage category.

2. Tornado data and study period

The US Storm Prediction Center (SPC) maintains the best available record of tornadoes in the United States compiled from National Weather Service (NWS) *Storm Data* publications and reviewed by the US National Climate Data Center [7]. We obtain the dataset containing all reported tornadoes over the period 1950–2012 from [www.pmarshwx.com/gis/torn.zip](http://www.pmarshwx.com/gis/torn.zip). According to a report by the Pacific Northwest National Laboratory for the US Nuclear Regulatory Commission [8], the SPC database is in reasonably good condition and acceptable for use in this type of climatology.

Since 1994, the United States has been almost completely covered by NOAA's Doppler weather radars. Even if a tornado is not directly observed, radar signatures and modern damage assessments by NWS personnel can discern tornado occurrence and perhaps tornado strength. Thus we restrict our study to the period 1994–2012, inclusive. This is in contrast to past work looking at tornado intensity distributions (e.g., [1]) that explore the entire database, most of which is pre-Doppler. Improved tornado observation practices have led to an increase in the number of reported weaker tornadoes [9]. Even today some smaller tornadoes may go undocumented in places with few people or limited communication infrastructure. Thus we further restrict our study to tornadoes rated EF1 or higher consistent with advice given by the SPC.

Figure 1 shows data used in this study as time series of annual tornado frequency by damage category. There are no large upward or downward trends. Years with relatively high numbers of weaker tornadoes tend to be years with relatively

Table 1. Distribution of tornadoes by EF category. (Note: Ratio is defined as  $EF_k/EF_{k-1}$ , where  $k$  is the EF damage rating category. Data are over the period 1994–2012, inclusive.)

Category	Count	Per cent of total	Ratio
EF1	6420	70.8	3.31
EF2	1938	21.4	3.42
EF3	567	6.26	4.57
EF4	124	1.37	9.54
EF5	13	0.14	

high numbers of strong and violent tornadoes and vice versa. Rank correlations range from a low of 0.27 between EF1 and EF4 counts to a high of 0.66 between EF2 and EF4 counts (1994–2012).

There are 9062 tornado reports (EF1 or higher) over the period 1994–2012 inclusive. Table 1 gives the distribution of the reports by EF rating by count and by percentage of the total. It also gives the factor by which the frequency in the category exceeds the frequency in the next highest category.

Total tornado frequency is dominated by the numbers in the EF1 and EF2 categories. More than 70% of all the tornadoes are EF1 and more than 20% are EF2. There are 124 EF4 and only 13 EF5 tornadoes over the period of study accounting for 1.37 and 0.14% of the total respectively.

The frequency of EF1 tornadoes exceeds the frequency of EF2 tornadoes by a ratio of 3.31, which is similar to the ratio by which EF2 frequency exceeds EF3 frequency. However, the ratio by which EF3 tornado frequency exceeds EF4 frequency is 4.57, which is less than half the ratio by which EF4 tornado

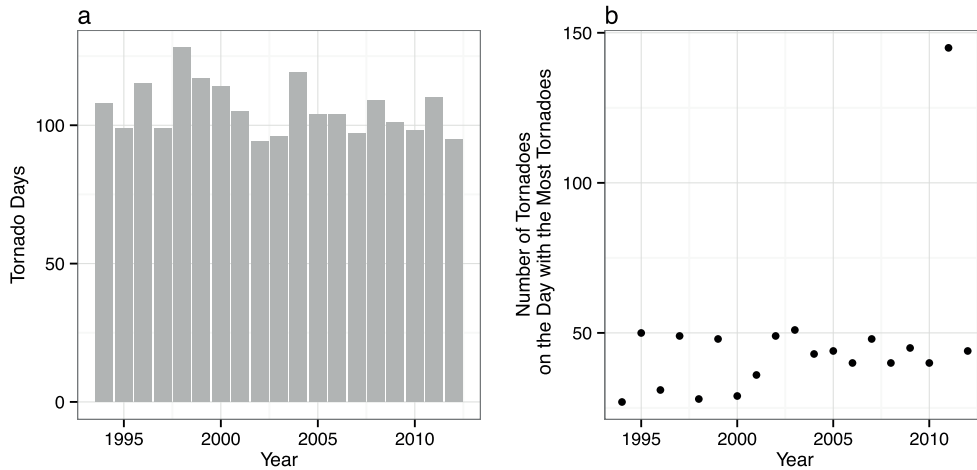


Figure 2. Annual number of tornado days (a) and the annual count of tornadoes on the tornado day with the most tornadoes (b).

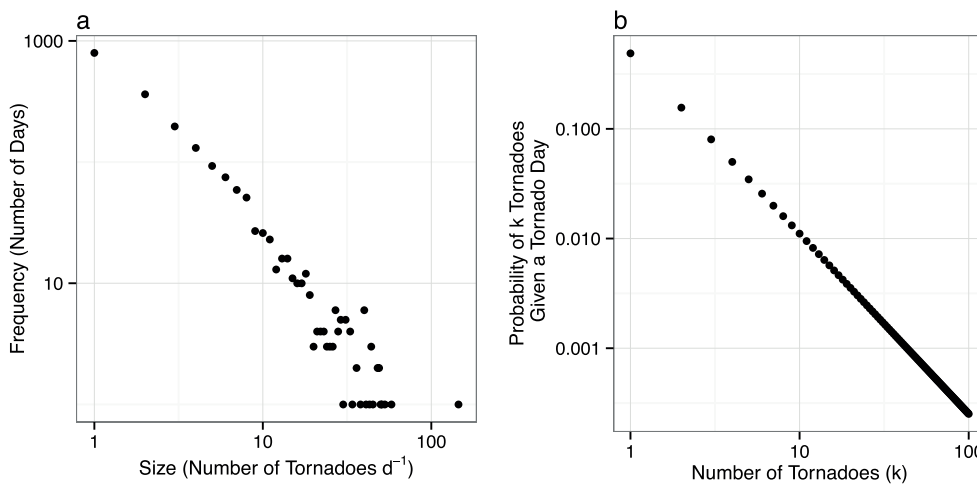


Figure 3. Power law for the daily tornado frequency. (a) The observed distribution frequency by size (number of tornadoes per day). (b) The probability distribution of the number of tornadoes on a random tornado day assuming a power law with a scaling exponent of 1.64.

frequency exceeds EF5 frequency. Thus, *in aggregate*, there does not appear to be a simple relationship between frequency and EF category (intensity). That is, the ratio of the frequency from one category to the next depends on the category.

### 3. Distribution frequency of tornado days

The rather complicated distribution frequency for the aggregate set of tornadoes [10] arises because of the combination of tornado-day distribution and the per day damage category distribution. A majority of days have no tornadoes. Over the 6939 days from 1 January 1994 through 31 December 2012, only 2012 (29%) had at least one EF1 or stronger tornado.

Figure 2 shows the number of tornado days by year as well as the number of tornadoes on the day with the most tornadoes by year. The time series indicate no upward or downward trends although the 145 tornadoes on 27 April 2011 stands out as very unusual.

Figure 3(a) shows the frequency distribution of tornado days by size on a log–log graph. Size refers to the number

of tornadoes occurring during a single calendar day. The straight-line appearance of the points suggests that the size of a tornado day follows a power-law relationship (Pareto distribution). Formally, the probability that a random tornado day has  $x$  tornadoes is given by

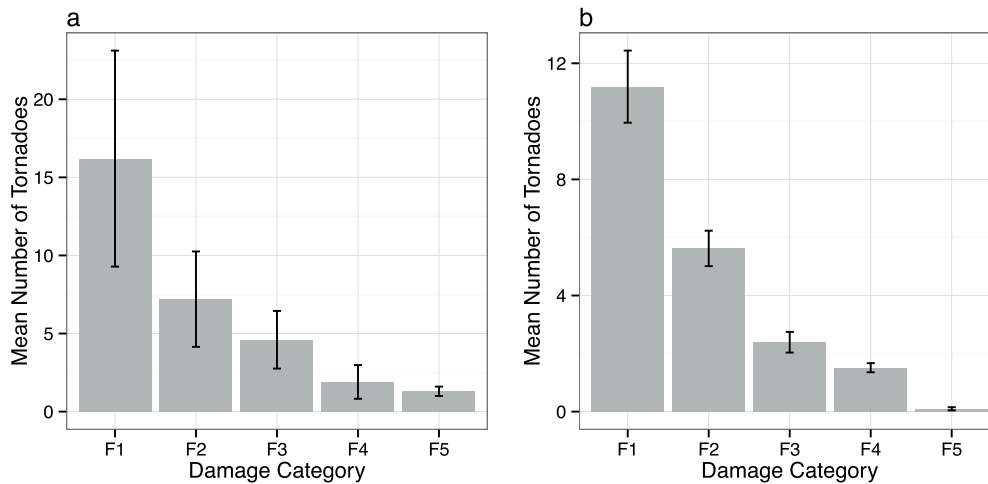
$$P(X = x) = 1/\zeta(s) \cdot 1/x^s$$

where  $s$  is the scaling exponent and where

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

is Reimann’s zeta function. The rank  $n$  ranges from 1 to the number of unique tornado-day sizes in the record (47).

The points on the right half of the plot in figure 3(a) tend to fall slightly below the line connecting the points in the left half of the plot. This might be due to under-reporting on days with a large number of tornadoes. Limited resources could force the attention on the most significant damage on days with more than 10 or so tornadoes. In other words, historically on days



**Figure 4.** Average number of tornadoes by EF damage category. (a) On days with at least one EF5 tornado and (b) on days with at least one EF4 tornado. The vertical lines are one standard error.

with many tornadoes the size of the outbreak (total number of tornadoes) might be biased low compared to the size on days with relatively fewer tornadoes.

We estimate the exponent  $s$  using the `vglm()` function in R to be 1.64 with a standard error of 0.019. This result is consistent with the results of Malamud and Turcotte [11] who find a power-law distribution with a scaling exponent of 1.8 for the frequency density of tornado path length per day. Such power laws indicate scale invariance, a symmetry property indicating predictability across different outbreak sizes. This property can be exploited to efficiently model outbreaks.

Since the scaling exponent is less than two the distribution has no mean and no variance. That is, the first and second moments of the probability density function diverge. However, the expected value of the square root of the outbreak size  $X$  (number of tornadoes) is given by

$$E(X^{1/2}) = \zeta(s - 1/2) / \zeta(s) = 3.55.$$

So the typical outbreak size as defined by the square-root norm is between 12 and 13 tornadoes per tornado day.

The power-law distribution helps explain why the aggregate distribution frequency discussed in the previous section is not simple. We find that the average number of tornadoes on tornado days without a violent tornado (EF4 or EF5) is 4 which compares to an average of 20 tornadoes on days with at least one violent tornado. Since most tornado days have less than a handful of tornadoes and most violent tornadoes occur with larger outbreaks (days with more tornadoes), there is an excess accumulation in the aggregate of weaker EF1 and EF2 tornadoes relative to violent tornadoes.

The power-law distribution for the size of a tornado day implies that although a day with 100 tornadoes is rare, it is not astronomically rare. Figure 3(b) shows the probability distribution of tornado-day sizes out to an outbreak with 100 tornadoes assuming a power law with a scaling exponent of 1.64. Using this relationship, we find a per tornado-day probability of 0.014% that a tornado day has 145 tornadoes. This is the conditional probability given a tornado day. Since

there are approximately 100 tornado days per year (see figure 2(a)) the annual return period can be estimated as  $1/0.014$  (71 years).

#### 4. Disaggregated damage scale frequency distribution

We return to the damage scale frequency distribution, but this time we *disaggregate* the frequency by tornado days. Disaggregation allows us to better understand the distribution of tornadoes by damage category by removing the excess of weaker tornadoes (from days that do not experience strong tornadoes). Here we are interested in whether there is a pattern to the distribution of the number of tornadoes by damage rating. That is we ask, on days with at least one EF5 tornado, what is the frequency distribution by damage category? The question is answered in figure 4.

Here the total number of tornadoes by damage category is shown as a bar plot. Given at least one EF5 tornado the daily frequency by EF category appears to follow an exponential law. The daily number of tornadoes in the next lower damage category is approximately twice the number in the current category (table 2). Since there are only 10 days with at least one EF5 tornado during our study period, we repeat the analysis using a threshold of at least one EF4 tornado and find a similar result.

The number–intensity ratio (the ratio of frequencies by successive EF category) appears as an exponential rule. This is consistent with the fact that it is more likely to get an EF5 tornado on a day with a large outbreak of tornadoes. A random sample from a day with a violent tornado has probability  $P(\text{EF}k \geq \text{EF}k-1) = e^{-a \cdot k}$ , where  $a = 0.693$ , for the distribution of the number of tornadoes by EF rating. The rule implies the distribution of tornado intensities within a given outbreak on average is fixed and scales with outbreak size, such that violent tornadoes occur more likely as the size of the outbreak increases. The exponential behavior accords with the theoretical number–intensity (energy of displacement)

**Table 2.** Distribution of tornadoes by EF category on days with an EF5 tornado. (Note: ratio is defined as  $EF_k/EF_{k-1}$ , where  $k$  is the EF damage rating category. The values in parentheses are the ratio on days with at least one EF4 tornado. Data are over the period 1994–2012, inclusive.)

Category	Count	Per cent of total	Ratio
EF1	162	51.9	2.25 (1.99)
EF2	72	23.1	1.57 (2.35)
EF3	46	14.7	2.42 (1.58)
EF4	19	6.1	1.46
EF5	13	4.2	

distribution of low pressure systems [12]. Accordingly if tornado intensity is expressed as lifetime minimum pressure, the theoretical number–intensity (pressure ratio) distributions will be a power law.

## 5. Summary and conclusions

Distributions describing tornado occurrence characteristics provide a framework for modeling outbreaks for the purpose of estimating damage losses. Here we examine daily tornado occurrences by damage category using the best available database of historical tornadoes. We use data on tornado occurrence going back to 1994 with the start year determined by the advent of comprehensive NWS operational Doppler radar coverage. We ignore EF0 tornadoes because it is generally assumed that some weak tornadoes remain undocumented in places with few people or limited communication infrastructure.

Consistent with earlier studies on tornado path length, we find a power-law scaling relationship with an exponent of 1.64 (0.019 s.e.) for the distribution frequency of tornado days. This indicates a long-tailed distribution with the potential for ‘unusually’ large outbreaks as was observed on April 27, 2011. It also suggests that it might be possible to create an integrated total energy scale for tornadoes (E-scale, [13]) similar to the Power Dissipation Index for hurricanes [14]. Such an energy scale might help provide a constraint that governs the distribution of tornado intensities within an outbreak.

We also find the total number of tornadoes by damage category on days with at least one violent tornado follows an exponential rule. On average, the daily number of tornadoes in the next highest damage category is about half the number in the current category. It seems reasonable to suppose then that

this exponential parameter is linearly related (in a statistical sense) to the total destructive energy of the outbreak. These findings are important and timely for the development of tornado hazard catalogs and for seasonal and sub-seasonal forecasts of tornado activity.

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