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Judgments of distance between trichords

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ABSTRACT

Music theorists have typically measured the distance between chords simply by summing the displacements of each voice (the "taxicab metric"). Our research tested this assumption of linearity and the influence of other factors such as tuning environment and direction of motion. In two related experiments, participants listened to pairs of trichords and rated their perceived musical distance. As predicted, increasing the total sum of motion generally created a sense of greater distance. However, other factors such as the number of common tones, the direction of motion, and the tuning environment were also shown to have significant effects. Overall, our results imply that the taxicab metric, while reasonable, underemphasizes common tones in standard tuning and displacement size in microtonal tunings, suggesting that displacements do not necessarily combine in a static, linear manner.

Keywords

Distance, trichord, voice leading, tuning

INTRODUCTION

Much recent research in music theory, including Roeder (1984, 1987), Cohn (1998), Lewin (1998), Straus (2003), Callender (2004, 2005), Tymozcko (2005a and 2005b), Callender, Quinn, and Tymozcko (2005), and nearly all of neo-Riemannian theory has focused on parsimonious, or smooth, voice leading. All of this work either implicitly or explicitly adopts measures of distance between chords; otherwise, it would not be possible to distinguish one voice leading as smaller than another. However, it is unclear how intuitions of distance are formed and which factors are most influential. Indeed, given the frequency with which music theorists appeal to intuitions about distance between chords, it is surprising how little is known about our perception of musical distance. Our research aims to provide some empirical grounding to these appeals and to determine those factors that are most important in our judgments of distance.

Most of the work mentioned above adopts the so-called "taxicab metric," where the distance between two chords is measured by simply summing the displacements of each voice. This approach assumes that displacements sum in a linear manner, but this may not be the case. Perhaps the Euclidean metric, where displacements are summed in a nonlinear manner, or some other metric would be more appropriate (Callender 2004, 2005). Relative to the taxicab metric, the Euclidean metric privileges minimal displacements over common-tone retention, whereas the converse is true for other metrics. One of the aims of our study is to test the assumption of linearity and the relative contributions of common tones and displacement size.

The established measures of chordal distance also ignore many important musical factors that may influence our judgment. These include the direction of motion (ascending or descending), the relationship of moving voices (similar, parallel, or contrary motion), the tuning environment (standard or microtonal), and — in the case of familiar structures such as the major triad — tonal implications. We must also consider the possibility of interaction between some of these variables. For instance, do listeners become more attuned to common-tone retention as the interval traversed by the moving voice increases? Similarly, is the direction of motion equally influential in both standard and microtonal tuning environments?

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METHOD

We conducted two related experiments, employing the same essential format but with different stimuli and subjects. For both experiments, each stimulus consisted of two trichords (chords with three pitch classes) presented in Shepard tones to eliminate the effects of register and spacing. Participants listened to pairs of chords and then rated their perceived distance on a ten-point scale (where 1 indicated the smallest distance and 10 indicated the greatest distance). Each trichord sounded for 1.5 seconds, with pairs heard in immediate succession and followed by nine seconds of silence. Participants responded by circling a number on a printed response sheet during this brief silent period.

Experiment 1

Nineteen graduate students and one faculty member from the Florida State University College of Music participated in this experiment. Thirteen subjects were music theorists and seven were composers; eleven were male and nine were female. Only one participant claimed to possess absolute pitch (AP); the other 19 indicated that they did not. The average age was 26.5 and the median age was 26.

This experiment employed a 3∗4 repeated measures design. The primary independent variables were the number of moving voices (1, 2, or 3) and the interval of voice-leading motion (whole step, half step, quarter step, or eighth step). Several other variables were also controlled: the direction of motion (ascending or descending), the relationship of moving voices (parallel or contrary), and the relative stability of paired sonorities (major/minor triad vs. other structure, and standard twelve-tone equal temperament vs. microtonal tuning — in other words, "in-tune" and "out-of-tune"). As an added precaution, stimuli presented in the first half of the experiment were played with their trichordal pairs exchanged during the second half of the experiment (i.e., trichord pair *XY* was later heard as *YX*).

For every stimulus in which a single voice moved by interval *x*, there was another stimulus in which two voices moved by interval *x*/2, and yet another in which all three voices moved by *x*/2. The value of *x* ranged from a whole step to a quarter step, so it was possible to compare the ratings for trichord pairs with different sums of motion but the same number of common tones as well as the ratings for trichord pairs with the same sum of motion but different numbers of common tones (see Figure 1).

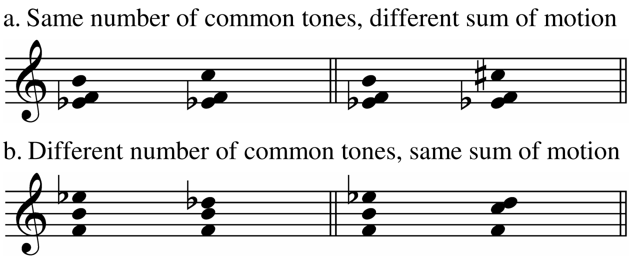

Figure 1. Four representative stimuli from Experiment 1 (presented in Shepard tones)

Experiment 2

Twenty-three undergraduate music majors from Florida State University participated in this experiment. Twelve were female and eleven were male. Two participants claimed to possess absolute pitch (AP), 20 indicated that they did not, and one did not provide this information. The average age was 18.7 and the median age was 19.

This experiment employed a 3∗3∗2 repeated measures design. The primary independent variables were the number of moving voices (1, 2, or 3), the tuning environment (standard twelve-tone equal temperament, microtonal with one voice detuned by a quarter step, and exclusively major/minor triads), and the interval of voice-leading motion (whole step or half step). Two other variables were also controlled: the direction of motion (ascending or descending) and the relationship of moving voices (parallel/similar or contrary). As an added precaution, every stimulus was balanced by another inversionally equivalent stimulus. (See Figure 2. Arrows attached to accidentals indicate that the associated pitch is to be altered by one quarter step in the direction of the arrow. For example, in the top microtonal motion of Figure 2, the uppermost voice moves from the pitch halfway between D and D-flat to the pitch halfway between D-flat and C.)

This experiment separated the intervals of voice leading from those of chord structure. In Experiment 1, all "out-of-tune" structures involved microtonal voice leading, and voice leading by larger intervals involved only "in-tune" structures. Experiment 2 used identical combinations of traditional whole-step and half-step voice-leading intervals in all three tuning environments (as shown in Figure 2).

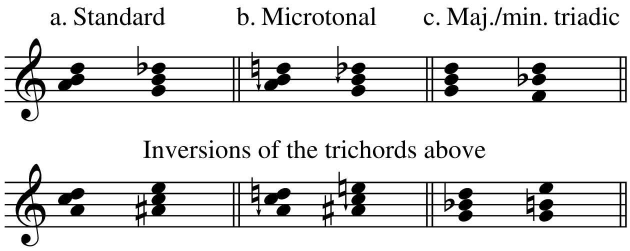

Figure 2. Six representative stimuli from Experiment 2 (presented in Shepard tones): voice leading by whole step, half step, and common tone

RESULTS

Our results support several common assumptions from the music theoretical literature:

- The total sum of voice-leading motion correlated with ratings of distance.
- Increasing the number of common tones reduced listeners' sense of distance.
- The major/minor triadic relationships known as L, P, and R were perceived as especially close.

Displacement Size vs. Common Tones

As expected, increasing the total voice-leading motion (i.e., the sum of the displacements for all three voices) between the two trichords led to a correlating perception of greater musical distance ($r = .251$ in Experiment 1, $r = .353$ in Experiment 2; $p < .001$). Obviously, the total voice-leading

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motion is a function of both the size and number of displacements. Increasing the number of moving voices in Experiment 2 produced a corresponding increase in the distance rating, and this was true in all three tuning environments ($r = .329$ for major/minor triads, $r = .320$ for other structures in standard tuning, $r = .367$ for structures with microtonal tuning; $p < .001$). The same correlation was observed in Experiment 1, but only in the standard tuning environment ($r = .210$, $p < .001$); this will be discussed in more depth shortly.

The general assumption in the music theoretical literature is that voice-leading motion sums in a linear fashion (e.g., two voices moving by half step is equivalent to one voice moving by whole step). However, the results of both experiments indicate that the taxicab metric does not always correspond to listeners' judgments of distance, suggesting that interval displacements are not necessarily perceived as combining in a static, linear manner.[1]

In Experiment 1, the effect of common tones apparently interacted with the effect of individual voice-leading distances and listeners reacted inconsistently to both variables. A single voice moving by a quarter step (abbreviated QCC for the motions in all three voices: quarter step, common tone, common tone) produced a greater sense of distance than did two voices moving by eighth step (EEC, $p = .02$); indeed, *three* voices moving by eighth step (EEE) created less sense of distance than did a single voice moving by quarter step, although the difference fell short of statistical significance. Relative to the taxicab metric, listeners in these cases privileged displacement size over common-tone retention. On the other hand, a single voice moving by whole step (WCC) created *less* sense of distance than did two voices moving by half step (HHC, $p = .003$). In this case subjects privileged common-tone retention over displacement size (again relative to the taxicab metric). In between these two extremes, there were no significant differences in the ratings for a single voice moving by half step (HCC), two voices moving by quarter step (QQC), and three voices moving by quarter step (QQQ). Overall, we found that the strength of displacement size relative to the number of moving voices in judgments of distance is inversely proportional to the size of the displacements. More complete results for Experiment 1 are depicted in Figure 3.

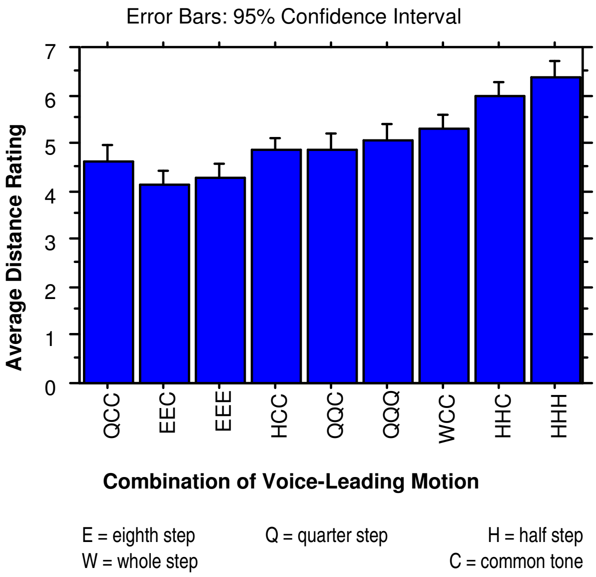

Figure 3. Average distance ratings for each voice-leading combination from Experiment 1

Experiment 2 involved microtonal tuning but not microtonal voice leading; that is, individual voices were always displaced either by half step or whole step. As predicted by the results for half-step and whole-step motions in the first experiment, participants appeared to prioritize common tones over the sum of motion. Listeners rated a single whole-step motion as closer than two half-step motions ($p < .001$); they also rated two whole-step motions as closer than one whole-step motion plus two half-step motions (i.e., one common tone vs. no common tones, $p < .001$). The distinction between two and three moving voices was apparently less important: overall, ratings for stimuli with a 1.5-step interval sum did not differ significantly when a single common tone was retained (WHC vs. HHH). However, judgments of distance in this case were influenced by the tuning environment: WHC was perceived as a significantly larger motion than HHH in the standard tuning environment ($p = .02$), while the opposite was true in the microtonal environment, though the latter fell somewhat short of statistical significance ($p = .09$). More complete results for Experiment 2 are depicted in Figures 4 and 5.

[1] Certainly there are models of distance between *major and minor triads* that take into account much more than displacements, such as root motion reckoned along the circle of perfect fifths and differential structural levels of pitches in a tonal context. (See Krumhansl 1990 and, in particular, Lerdahl 2001.) However, our work is focused on judgments of distances in a *non*-tonal context, for which the taxicab metric is the dominant model. One exception is Parncutt's (1989) model of pitch commonality.

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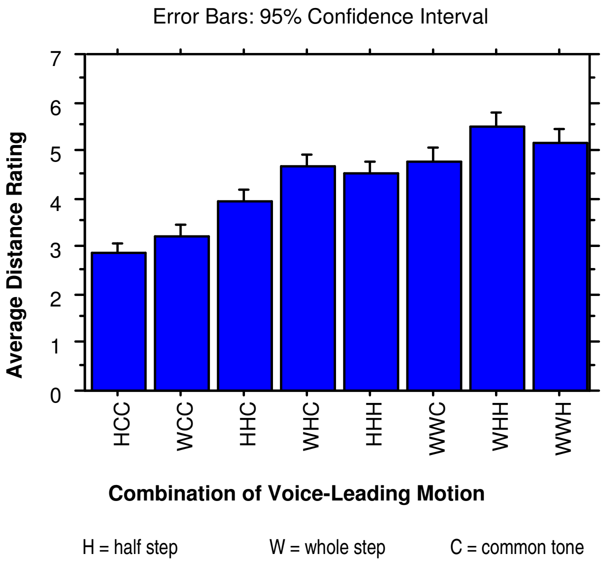


Figure 4. Average distance ratings for each voice-leading combination from Experiment 2

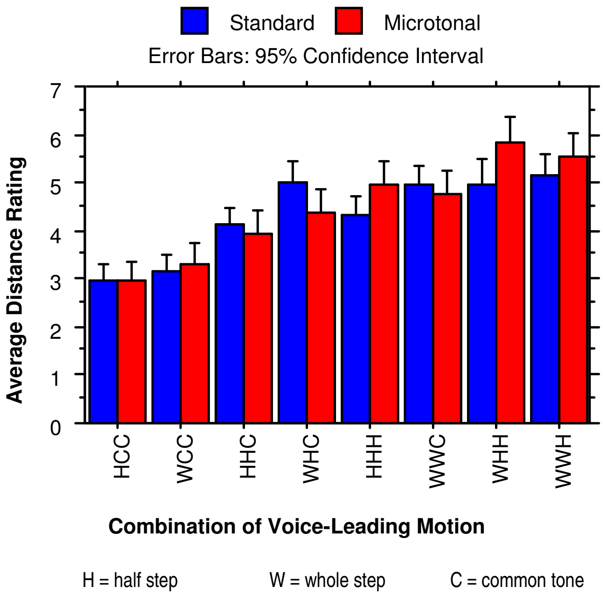


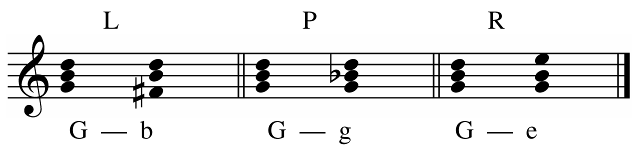
Figure 5. Average distance ratings in the standard and microtonal tuning environments for each non-triadic voice-leading combination from Experiment 2

It appears that the taxicab metric may underemphasize the importance of interval size when individual voices move microtonally (i.e., by intervals smaller than a half step) while underemphasizing the importance of common tones when individual voices move by traditional intervals (half steps and whole steps).[2] Our conjecture is that microtonal displacements may be perceived as alterations of a single pitch rather than as motions from one pitch to another. Thus, the number of moving voices becomes less consequential for very small displacements. This was particularly noticeable in Experiment 1, where there was no significant effect of the number of voices for displacements by eighth or quarter step. In contrast, there was a significant difference between one and two or three voices by half-step ($p < .0001$ in both cases), though the difference between two and three voices by half step fell just shy of statistical significance ($p = .0626$).

Another (not necessarily incompatible) explanation is that the connection between pitches a half step apart is stronger than that between pitches a whole step apart. If this is true, then half-step motion should exert a greater influence on judgments of distance, relative to the displacement size, than should whole-step motion. In Experiment 2, a multiple regression analysis with the number of half-step and whole-step motions as the independent variables yields coefficients of 1.307 for the former and 1.784 for the latter. Setting whole steps equal to 1 and half steps equal to the ratio of these coefficients, 0.733, we can derive an adjusted interval sum for each combination of motions. For instance, three voices moving by half step (HHH) has an adjusted interval sum of $.733 + .733 + .733 = 2.199$, which is slightly *larger* than that for two voices moving by whole step (WWC). The adjusted interval sums correlate slightly better than the original interval sums with the trichord distance ratings from Experiment 2 ($r = .366$, $p < .0001$).

Triad Distance and Neo-Riemannian Theory

Neo-Riemannian theorists privilege the major/minor triadic relationships designated L, P, and R (see Figure 6); these are the only possible major/minor triadic combinations that retain two common tones. Of the 22 major/minor triadic stimuli presented in Experiment 2, listeners rated the six possible L, P, and R relationships as closest. Not surprisingly, the two closest triadic relationships other than L, P, and R involved traditional root motion by perfect fourth/fifth (e.g., C major to F minor). However, not all root motions by perfect fourth\fifth were interpreted as especially close. Chord progressions that might be represented as i – V and V – i were rated as close, but chord progressions that might be represented as I – v or i – IV were rated as distant, presumably reflecting the less common use of these chord successions in the tonal repertoire. As a group, the L/P/R progressions were judged to be significantly closer than the group involving root motion by perfect fourth/fifth, which in turn was significantly closer than the group of all other triadic successions ($p < .04$). The latter category received ratings that were virtually identical to those for the group of non-triadic stimuli in this experiment.



[2] One consequence of the latter is that any judgment of voice-leading distance that consistently places more emphasis on common-tone retention than the taxicab metric will violate the triangle inequality (Callender 2005).

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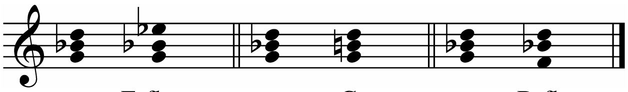

Figure 6. The L, P, and R relationships shown from a major triad (upper line) and a minor triad (lower line)

Triadic successions that conformed to a single key (e.g., C major and D minor) were perceived as closer than triadic successions that did not (e.g., C major and D-flat minor, $p = .003$).[3] Major and minor triads with the same root were heard as closer than triads whose roots were related by second, third, or fourth/fifth ($p < .0001$). Chords whose roots were separated by a second were rated as significantly more distant than chords whose roots were separated by either a third ($p = .0125$) or a fourth/fifth ($p = .0443$). Root motions by third and fourth/fifth did not receive significantly different ratings.

The foregoing compares quite well with distances between major and minor triads in the four-dimensional space derived by Krumhansl and Kessler (1982) from their probe-tone studies. Of the 22 major/minor triadic stimuli in Experiment 2, the group of L/P/R motions were rated significantly closer in this four-dimensional space than were root motions by perfect fourth/fifth, which in turn were significantly closer than the group of all other triadic successions ($p < .0001$). Diatonic successions were rated as closer than non-diatonic successions ($p < .0001$), and triads with the same root were perceived as significantly closer than triads whose roots were separated by either a third or a fourth/fifth, which in turn were heard as significantly closer than triads whose roots were separated by a second ($p < .0001$). While root motions by fourth/fifth are generally smaller than those by third in the Krumhansl and Kessler space, the differences fall short of statistical significance ($p = .1$).

Direction and Relationship of Moving Voices

It is somewhat unclear under what circumstances and to what extent the direction of motion affects our perception of musical distance. In Experiment 1, descending motions created a greater sense of distance than did ascending motions of the same size ($p < .0001$). Significant differences were observed when the voice leading was by eighth step, quarter step, and half step; these results are shown in Figure 7. Notice that whole-step motions show the opposite trend: whole steps were heard as more distant in their ascending form, although the difference was not significant.

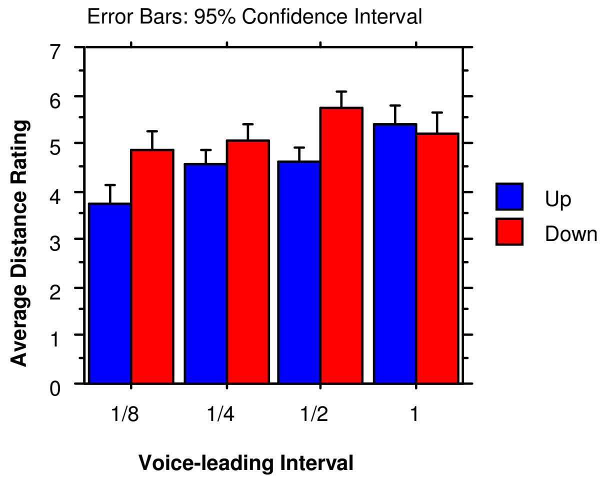

Figure 7. Average distance ratings for ascending and descending voice-leading motion in Experiment 1

The direction of voice-leading motion was not shown to have any significant effects in Experiment 2, which excluded microtonal voice-leading intervals. However, similar trends that fell short of statistical significance were observed: half-step motions were heard as more distant in their descending form, while whole-step motions were heard as more distant in their ascending form. Although further study is needed, we speculate that relatively small voice-leading intervals (a half step or less) create less sense of distance when ascending, while relatively large voice-leading intervals (more than a half step) may create less sense of distance when descending.[4] A possible explanation for this result is that there are two competing factors on the effect of direction. First, since ascent in pitch generally leads to heightened tension, one might suppose that this increase in tension correlates with an increase in perceived distance. Second, it has been shown that in a tonal context the distance from the first to seventh degree in the diatonic scale (descending by half step) is perceived as larger than the distance when this motion is reversed (ascending by half step), since the latter motion is the resolution of the leading-tone. (See Krumhansl 1990, pp. 121-123, and the discussion of melodic tension in chapter 4 of Lerdahl 2001.) Thus, for Western listeners, it may be that the second factor is dominant for motions of a half step or less, while for larger motions the first factor is dominant.

When multiple voices move simultaneously, contrary motion may produce a slightly greater sense of distance than does parallel motion. This effect was observed overall in Experiment 1 ($p < .01$); it was entirely attributable to stimuli in which the voice-leading motion was by half step. The overall effect of contrary motion fell just short of statistical significance in Experiment 2 ($p = .0678$), but it was observed at significant levels under several circumstances:

- when all three voices moved, leaving no common tones ($p < .01$)
- in the microtonal tuning environment ($p = .04$)

[3] For the purposes of this categorization, triadic successions that suggested a natural or harmonic minor scale were considered to conform to a single key.

[4] Preliminary results from a follow-up experiment support this speculation.

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- when at least one voice moved by half step ($p = .04$); the effect was especially strong when voice leading was entirely by half step, $p = .001$)

CONCLUSIONS

Our experimental results provide support for a variety of common music theoretical assumptions. We observed, for example, a general correlation between the number of moving voices and a sense of distance; we also found evidence that the most privileged triadic relationships tend to be heard as closer than other triadic combinations. The sum of individual voice-leading motion was, indeed, shown to approximate listeners' overall perception of distance. At the same time, however, our results suggest that many factors (e.g., displacement size, tuning environment, direction of motion, and relationship of moving voices) interact with one another, contributing to our sense of musical distance in a more complex fashion than had been previously recognized. Although a great deal of further study is warranted, at this point we believe that an accurate model for perceived musical distance cannot rely on a straightforward metric that only combines features in a linear manner.

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