Löschian Competition under Alternative Demand Conditions

By BRUCE L. BENSON*

Theoretical examination of the pricing behavior of firms competing in a spatial setting indicates that many of the conclusions of classical price theory are altered as the cost of distance becomes significant.[1] An assumption about entrepreneurial behavior is an integral component of these models of spatial competition (see Greenhut, Hwang, and Ohta). The traditional spatial model follows the Löschian assumption in location theory—that entrepreneurs price as if they are monopolists within their market areas.[2] Recently however, this Löschian assumption has been questioned.[3] For example, Greenhut, Hwang, and Ohta noted that unexpected conclusions derived in spatial price theory stem, in part, from this behavioral reaction assumption. Capozza and Van Order further contended that the Löschian assumption does not accurately depict entrepreneurial behavior. They *concluded* that another assumption concerning entrepreneurial behavior "... is closer to the way firms behave in most real world situations" (p. 896).

Capozza and Van Order proposed to investigate the extent to which spatial price theory replicates the results of spaceless price theory with free entry. To this end, they offered five "... intuitively appealing properties [or characteristics] of nonspatial competitive theory..."(p. 897) which they felt a reasonable model of spatial competition should obey:

> 1) As transport costs approach zero, perfect competition should be approached and price should approach marginal cost.
> 2) As fixed costs approach zero, concentrated production is less essential; spatial monopoly power is diminished; and again price should approach marginal cost... .
> 3) As costs (fixed, marginal, or transportation) rise, price should rise.
> 4) As demand density rises, firms should be able to take advantage of economies of scale. Price should fall in the long run.
> 5) As more firms enter the industry, there should be increased competition and price should fall. [p. 897]

Capozza and Van Order analyzed three models of spatial competition with these characteristics as the criteria for acceptance or rejection. They apparently felt that entrepreneurial behavior was *the crucial assumption* in a spatial competition model, since, while developing their models, they proposed: "The final assumption, *upon which our results depend*, concerns the reaction of a firm to a change in a competitor's price" (p. 898, emphasis added). In fact, the only difference between their three models was the entrepreneurial behavior assumption employed in each. Capozza and Van Order claimed that Löschian competition violates all the above characteristics![4] Actually, however, the characteristics can be derived

*Assistant professor of economics, Pennsylvania State University. I wish to thank M. L. Greenhut and an anonymous referee for helpful comments and suggestions.

[1]For example, see Melvin Greenhut, Ming Jeng Hwang and Hiroshi Ohta; Dennis Capozza and Kazem Attaran; Capozza and Robert Van Order; and my forthcoming paper.

[2]For example, see Edwin Mills and Michael Lav; Greenhut (1971); John Hartwick; Nicholas Stern; among others.

[3]For example, see Greenhut, Hwang, and Ohta; Capozza and Van Order.

[4]Capozza and Van Order argued that the appropriate assumption is that each entrepreneur believes that all other entrepreneurs will leave their own location and price unchanged—the Hotelling-Smithies assumption. They rejected both Löschian and Greenhut-Ohta behavior. The Greenhut-Ohta assumption is that each entrepreneur assumes the price at the edge of his market area is fixed (see Greenhut, Hwang, and Ohta).

for a Löschian competitor simply by altering another assumption used by Capozza and Van Order (C-V).

The purposes of the following presentation are to demonstrate that: 1) the C-V rejection of Löschian competition is not warranted, at least for the reasons they presented; and 2) the entrepreneurial behavior assumption is not the only crucial assumption in a model of spatial competition. The Löschian assumption produces results which violate the C-V characteristics, only when used with the assumption they made concerning individual demand. Löschian competition does not violate the characteristics when the shape of individual consumer demand functions is altered. Note the following discussion is *not* designed to change the C-V assumptions. Rather, it is presented to challenge one of their main conclusions—the rejection of Löschian competition. Furthermore, the amendment below does not imply agreement that the C-V properties are necessarily the only ones a spatial model should obey, nor does it imply agreement that a spatial model should obey all five of them.[5]

I. Assumptions for Two Models of Löschian Competition

Capozza and Van Order proposed several assumptions which are followed here, including:

ASSUMPTION 1: *A single commodity is produced by all firms and all firms have identical cost functions.*

ASSUMPTION 2: *The cost function is*

$$C=f+cQ \tag{1}$$

where Q is output, C is total production cost, f is fixed cost, and c is marginal cost.

[5]All five characteristics are actually intuitively unappealing when put in a spatial context (see my 1979a paper). The characteristics are based upon spaceless competition where alteration of any one parameter effects only supply *or* demand. I have demonstrated elsewhere (1979a) that both costs (supply) *and* demand are impacted when these parameters change in spatial competition.

ASSUMPTION 3: *Transport cost per unit of distance is identical between any two points and equal to t dollars per unit.*

ASSUMPTION 4: *Firms can enter freely and will do so until all economic profits disappear.*

Two additional assumptions proposed by Capozza and Van Order will be slightly altered:

ASSUMPTION 5: *Potential consumers occupy a homogeneous unbounded linear space of uniform density D.*

ASSUMPTION 6: *All market areas consist of segments of this linear space.*

Capozza and Van Order chose to work with a plain and with circular markets. The use of linear markets alters the magnitude of the values that are obtained (i.e., profit-maximizing price) *but* the general conclusions derived below continue to hold under the C-V market shape assumptions. Working with the linear market, on the other hand, does substantially simplify the computations which follow.

Löschian behavior characterizes the spatial entrepreneur. That is:

ASSUMPTION 7: *Each entrepreneur assumes his market area is fixed and prices as a monopolist within his space.*

Capozza and Van Order assumed that individual demand was linear, but this need not be the case. Let us start with a general demand function given by

$$P+tu=a-\frac{b}{x}q^{x} \tag{2}$$

where P is mill price, q is demand per consumer, a and b are constants, and u is units of distance.[6] Now x can be varied to encom-

[6]This general demand function has been used in other spatial pricing models. Several implications of this general function are noted in John Greenhut, Greenhut, and William Kelly, for example, but Löschian competition was not assumed in that paper.

pass a wide range of possible demand functions. For example, if $x=1$, the demand function is linear and the C-V conclusions would follow. If $x>1$, the function is convex upwards. With $x<1$, the function is concave upwards. The function is extremely concave if $x<0$.[7] If $-1<x<0$ and $a=0$, the function describes a constant elasticity demand curve. So x is the vital parameter in determining the shape of the individual demand curve. I must express individual demand in terms of q rather than delivered price $(P+tu)$ in order to obtain the aggregate market demand. This yields

$$(3) \qquad q=\left[\frac{x}{b}(a-tu-P)\right]^{1/x}$$

where u can vary from zero to U, the distance to the boundary of a firm's market.

Equation (3) describes an extremely general relationship for individual demand, as x can vary anywhere from -1 to $+\infty$ while still yielding stable equilibrium results. It is possible, however, to depict the general conclusions for any Löschian type spatial firm with two values of x:

ASSUMPTION 8A: $x=1$ *and individual demand is*

$$(4a) \qquad q=\frac{1}{b}(a-tu-P)$$

ASSUMPTION 8B: $x=-1/2$ *and individual demand is*

$$(4b) \qquad q=\left[-\frac{1}{2b}(a-tu-P)\right]^{-2}$$

We can derive the C-V results with $x=1$ and the derived relationships are representative of all $x>0$ (positive exponential demands). The comparative statics using $x=-1/2$ are the same for all $-1<x<0$ (negative exponential demand).

[7]It must be assumed that $x>-1$. If $x<-1$, there will be no profit-maximizing equilibrium, given the assumption of constant marginal cost made earlier. Marginal revenue will be increasing if $x<-1$, and any adjustment will move production away from the $MR=MC$ level.

II. Löschian Reactions under Alternative Demand Conditions

For $x=1$, the aggregate spatial demand is

$$(5) \qquad Q_p=D\int_0^U \frac{1}{b}(a-tu-P)\,du$$

$$(6) \qquad Q_p=\frac{DU}{b}\left(a-P-\frac{tU}{2}\right)$$

where Q_p represents the aggregation of positive exponential demands. The aggregation of demand with $x=-1/2$ provides

$$(5') \quad Q_n=D\int_0^U\left[-\frac{1}{2b}(a-tu-P)\right]^{-2}du$$

$$(6') \quad Q_n=4b^2D\left(\frac{1}{P-a}-\frac{1}{P-a-tU}\right)$$

where Q_n represents the aggregation of negative exponential demands. Profits are zero in the long run, i.e.,

$$(7) \qquad Y=PQ-C=0$$

Substituting (1) and (6) into (7) yields

$$(8) \quad Y=\frac{DU}{b}\left(a-P-\frac{tU}{2}\right)(P-c)-f=0$$

while (1) and (6′) into (7) provides

$$(8') \quad Y=4b^2D\left(\frac{1}{P-a}-\frac{1}{P-a-tU}\right)(P-c)-f=0$$

The alternative Löschian profit-maximizing prices are obtained by maximizing (8) and (8′) with respect to price. Thus

$$(9) \qquad P_p=1/2(a+c)-\frac{tU}{4}$$

TABLE 1—COMPARATIVE STATICS FOR LÖSCHIAN COMPETITION UNDER ALTERNATIVE DEMAND CONDITIONS

	Löschian Reaction	
Parameter Change	C-V Results ($x=1$)	$x=-1/2$
Marginal Cost	±	+
Demand Density	+	−
Fixed Costs	−	+
Transport Cost per Unit of Geographic Distance	−	+
Entry of New Firms	+	−

when $x=1$, but

$$(9') \quad P_n=c+\left[(c-a)tU+(c-a)^2\right]^{1/2}$$

for $x=-1/2$.[8, 9]

Let us now examine Löschian competition to see whether it is always inconsistent with the characteristics (C1–C5) which Capozza and Van Order set forth (the comparative statics are summarized in Table 1):

C1) For $x=1$, the profit-maximizing price rises as transportation costs (t) decrease. This contradicts C1, as C-V noted. However, if $x=-1/2$, the profit-maximizing price falls towards marginal cost as transportation costs decline. Characteristic 1 is satisfied only in part, however, by Löschian competition and negative exponential demand. Price does move in the direction that C-V argue it should as parameter t changes, but price does not approach marginal cost as t approaches zero.

Examination of the C-V model reveals perfect competition need not be the structure of their industry even in a spaceless setting. Let $t=0$. Then the cost function (C-V's as well as my equation (1)) results in spaceless imperfect competition or monopoly.[10] The large number of firms required for perfect competition necessitates each producing at a relatively small scale. Thus, price must be greater than marginal cost for survival (unless we assume each of a large number of firms produce and sell an infinite quantity). This long-run cost function has economies of scale (actually, increasing returns) over the entire range so we might expect some firms to expand the scale of their output and lower price. Other firms will therefore be forced out of the industry, and market structure approaches oligopoly or monopoly in this spaceless setting. Free entry prevents profit taking so one firm (or a few firms) may charge a price near marginal cost. Price falls because of scale economies, not because of perfect competition. Furthermore, it is very unlikely that price would equal marginal cost. Also note that as long as $t>0$, a single firm (or a few firms) cannot crowd out all the others. As transport costs fall, profits are available and there is entry. Therefore it is conceivable that spatial competitors operate at smaller and smaller scales as transport costs fall and

[8]The behavioral assumption of Löschian competition implies that $\partial U/\partial P=0$ (see C-V). Similar prices were derived in Greenhut, Greenhut, and Kelly.

[9]It should be pointed out that a in equations (4b), (5′), (6′), (8′), and (9′) is not a price intercept as it is in (4a), (5), (6), (8), and (9). As $P\rightarrow\infty$ the function approaches the price axis. The variable a is the asymptote of the curve, or the limit the function approaches as $q\rightarrow\infty$. Also note that in order to have a positive value under the square root $(c-a)>0$.

[10]Another specification of costs has been suggested which results in spatial firms producing efficiently in the long run (see Greenhut, 1971; Ohta, 1976). Justice cannot be done to their interesting arguments here. Briefly however, the spatial entrepreneur has a required return or opportunity cost because he conceives of only a few nearby rivals as his primary competition. As in classical oligopoly theory, he must take the uncertain reaction of these rivals into account when making decisions. It follows that price may exceed marginal production costs (where production costs do not include the subjective required return to the entrepreneur), but production is efficient when the entrepreneur's opportunity costs are taken into account.

market shares shrink. A rise in price should not be surprising when there are economies of scale over the entire range of output.[11] However, the pricing reaction to changing transportation costs is in the direction that Capozza and Van Order argue it should be when $x=-1/2$.

C2) The reactions to changes in fixed costs are also quite different when $x=-1/2$ instead of unity. For $x=1$, price does rise as fixed costs fall. However, if $x=-1/2$, price falls as fixed cost decreases ($\partial P_n/\partial f>0$ in equation (8′)). The C-V directional requirement is fulfilled under this demand condition. The limit requirement is not however, since price does not approach marginal cost as $f\rightarrow 0$.

As fixed costs fall, scale economies become less significant. However, as long as fixed costs are positive, one firm (or a few firms) can produce more efficiently than a large number of firms. Thus, in a spaceless setting we would expect imperfect competition or monopoly as long as $f>0$. Of course, an attempt to reap all monopoly profits leads to entry, but it is doubtful that price would approach marginal cost, given $f>0$. In the limit, of course, with $f=0$, free entry implies price equals marginal production cost in a spaceless setting. Thus, this criticism of Löschian competition based on C2 appears to remain valid in part. Even though the comparative statics of Löschian competition with $x=-1/2$ match those desired by Capozza and Van Order, price does not equal marginal cost in the $f=0$ limit. However, I have demonstrated elsewhere (1979b) that price should not equal site-specific marginal production costs as long as $t>0$. Price should exceed plant marginal costs in order to cover society's costs which result from the availability of multiple locations. Multiple locations are desirable, of course, because society's transport costs are reduced with each additional location and effective (or net) demand increases. If price equals site-specific marginal production costs, society's transport costs are relatively high because a relatively small number of locations are available.[12] In other words, an individual firm's costs and revenues do not account for all the social costs and benefits resulting from spatial competition. So price should equal plant marginal production costs only when f *and* t are *both* zero; that is, when space is irrelevant.

C3) As marginal costs rise, the Löschian competitor will raise his price just as C-V argue. Note $\partial P_p/\partial c$ in equation (9), and $\partial P_n/\partial c$ in equation (9′) are obviously both positive.[13]

C4) The Löschian price reaction to a change in density (D) also depends on the shape of the individual demand curve. When demand is linear ($x=1$) price will increase with an increase in density. A higher density permits smaller market areas and the aggregate demand becomes more inelastic when the market area shrinks under an $x=1$. However, when demand is the negative exponential (i.e., $x=-1/2$), price falls as density increases. The aggregate demand becomes more elastic when the extent of the market is reduced in size and $x=-1/2$ (see Greenhut, Greenhut, and Kelly).[14]

C5) As firms enter the industry, the market area shrinks (U becomes smaller). As U becomes smaller, the mill price rises if $x=1$, but falls when x is $-1/2$.

How is it that the shape of the individual demand function can cause such different results in a spatial world? The individual buyer's demand certainly does not play a significant role in spaceless economics. Yet apparently it does in spatial microeconomics.

[11] Actually, of course, things are not quite this simple. Shares of existing demand shrink, but as transport costs fall, effective demand rises. Whether firms operate at a larger or smaller scale depends upon the elasticity of firm demand. This point is discussed later. It is also examined in more detail in my 1979a paper.

[12] In fact, free entry and Löschian behavior lead to the socially efficient price and number of plants (see my 1979b paper). Price equals the full marginal cost of production including plant-specific costs and the increase in costs due to multiple locations.

[13] Capozza and Van Order argued that $\partial P_p/\partial c$ is ambiguous. This is true when there is a positive fixed cost (i.e., when we use equation (8)). The relationship is not ambiguous, however, when the assumption of zero fixed cost in the long run is made.

[14] The elasticity of spatial aggregate demand under alternative demand conditions is discussed below, in the context of the relationships of price to density, transport costs, and entry of new firms.

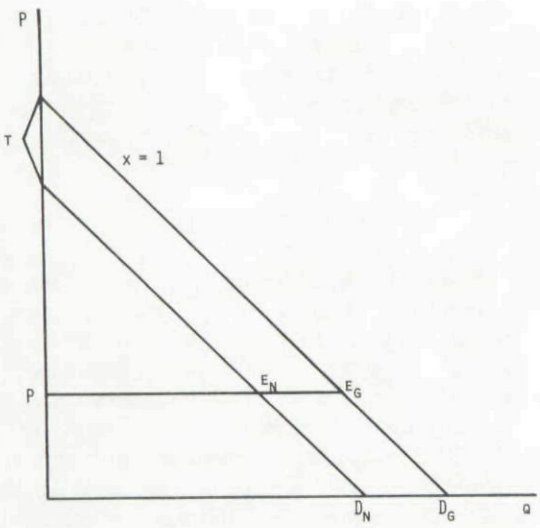

FIGURE 1. NET INDIVIDUAL DEMANDS AND CHANGING ELASTICITY FOR $x=1$

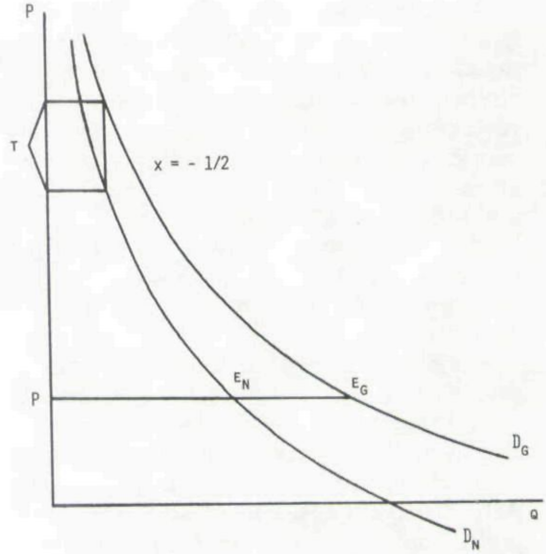

FIGURE 2. NET INDIVIDUAL DEMANDS AND CHANGING ELASTICITY FOR $x=-1/2$

The underlying reason for the different results observed here is the difference between spatial and spaceless aggregate demand. The price received by the firm in spaceless economics equals the price paid by the consumer. Spatial consumers, however, incur buying costs in addition to the price paid to the seller.[15] Producers are therefore subject to demand elasticity that relates to the net market price (net of transportation costs), not the total price paid by consumers (delivered price). The price paid by consumers differs from the price received by producers.

Elasticity at any given price increases when transportation costs are subtracted from a linear gross individual demand curve to obtain net demand (the relevant demand from the seller's point of view). The elasticity of gross demand (D_G) at price P in Figure 1 is less than the elasticity at price P of the demand that is net of transportation costs (D_N) ($E_G<E_N$). These relations apply to any gross demand with $x>0$. However, the elasticity at a given price decreases as transportation costs are subtracted from any gross demand of the concavity given by $-1<x<0$ (as $E_G>E_N$ at price P in Figure 2).

Elasticity of aggregate demand increases at any price as more distant consumers are added and $x>0$. Elasticity of aggregate demand falls, however, as more distant consumers are added and $-1<x<0$, because successively more and more inelastic demands at a given common price are being added to the total.

Anything which causes a spatial firm to lose or gain distant consumers changes the elasticity of the aggregate demand faced by the firm. For example, the impact of entry of a new competitor at a distance is to take away the most elastic demand for the representative seller when $x>0$. The aggregate demand left for that seller is less elastic than was the original demand. The resulting price effect is upward as Capozza and Van Order noted. However, when $-1<x<0$ applies, distant competition removes the least elastic demands from the representative seller's market space. The remaining aggregate demand is then more elastic than was the original aggregate demand. The price effect in this case, contrary to Capozza and Van Order, *is downward*.

[15]For a discussion of the difference between spatial and spaceless aggregate demand, see Greenhut (1978).

We can now see more clearly why changes in costs or demand density have the effects noted in Table 1. If transportation costs fall, elasticity of net demand for consumers at each point in the geographic space will decrease when $x>0$. The resulting aggregate demand is less elastic and the price effect is upward. The opposite is true if $-1<x<0$. In that event, the elasticity of individual net demands increases, the aggregate demand becomes more elastic, and the price effect is downward.

A similar argument applies to changes in density. Capozza and Van Order observed that "higher density permits smaller market areas" and "the spatial demand curve is more elastic for smaller market radii" (p. 902). A higher density will lead to short run profits and entry. The market areas of existing firms shrink and the spatial aggregate demand is indeed more inelastic for smaller markets *if* $x>0$. However, if $-1<x<0$, demand becomes more elastic as markets become smaller. Therefore, price rises when $x=1$, but falls when $-1<x<0$.

When fixed costs rise, each firm faces losses. There is exit and the market areas of remaining firms expand. Elasticity of aggregate demand increases as distant buyers are added and $x>0$. The price effect is downward. The opposite is true, however, for $-1<x<0$. Elasticity decreases as distant buyers are added in this case, and the price effect is upward.

Alternative specifications of demand have an impact on theoretical pricing policies because of the effect on the elasticity of *actual* aggregate demand for spatial competitors. All that the alternative behavior assumptions really do is change the elasticity of each firm's *perceived* demand (see Greenhut, Hwang, and Ohta). A Löschian competitor views his demand as more inelastic than does the Hotelling-Smithies entrepreneur that Capozza and Van Order maintain is "...closer to the way most firms behave in most real world situations" (p. 896).[16] When we realize that alternative demand assumptions and alternative behavioral assumptions are just altering the demand elasticities to which a firm reacts, it is not surprising to find that we can obtain similar results by varying either assumption.

III. Conclusions

Each of the C-V "intuitively appealing properties" apply to Löschian competition when demand is negative exponential in form.[17] It follows that one cannot reject Löschian competition for the reasons given by Capozza and Van Order. Perhaps one should reject linear (or positive exponential) demand instead? Of course not. At least three models predict similar results. Capozza and Van Order prefer linear demand and Hotelling-Smithies behavior on the part of entrepreneurs. Greenhut, Hwang, and Ohta proposed positive exponential demand and Greenhut-Ohta behavior. Here we assume negative exponential demand and Löschian competition. To accept one and reject the others requires detailed analysis of all the consequences of each model, (as well as any

[16]In fact, as Ohta (1980) has noted, the Löschian competitor's perceived demand is his actual demand following entry. The Hotelling-Smithies (and Greenhut-Ohta) demands are perceived demands which are believed to be more elastic than the actual Löschian demand. Thus the Löschian assumption might be restated as, "...firms recognize the actual demand they face and profit maximize accordingly" (my 1979a paper, p. 24). Hotelling-Smithies (and Greenhut-Ohta) competition imply that entrepreneurs are ignorant of their actual demand situation and never learn, even with continued entry and many observations.

[17]Actually, of course, characteristics 1 and 2 are only met in part. The comparative statics are those desired by C-V. However, the limiting requirements (that price approach marginal cost as t or f approach zero) are not met. I maintain that these limiting requirements need not be met in a spaceless world either, however, because the cost function specified by C-V has economies of scale over the entire range of output. Market structure is more likely to be monopoly or oligopoly than perfect competition under such cost conditions. Thus, C-V try to force a spatial model to match the results of spaceless perfect competition when their assumptions would prevent perfect competition in a spaceless setting. For more on this argument see my 1979a paper. Furthermore, I argue elsewhere that price should exceed plant-specific marginal production costs as long as $t>0$ (see my 1979b paper). This results because of additional social costs and benefits that occur in spatial competition which are not accounted for by individual firms. So, even though the limit requirements are not met for characteristics 1 and 2, I maintain that they should not be met.

others that might be proposed) including empirical testing.[18] Until this is effected, one cannot accept any particular assumption concerning entrepreneurial behavior (or individual spatial demand) as being superior to any other assumption.[19] One can only stipulate that there are at least two crucial assumptions in a model of spatial competition, and probably many more.[20]

[18]The Greenhut-Ohta assumption of entrepreneurial reactions was not discussed here. This does not mean that C-V's questioning of the Greenhut-Ohta model is accepted. Rather, their model has many desired characteristics which require separate examination.

[19]Neither C-V's preferred model nor the model discussed here will conform completely with spaceless economic conclusions. Inclusion of costly distance still results in many conclusions which contradict classical spaceless economic predictions. We have developed spatial models which conform in some ways with spaceless economics then, but this does not imply that space is not deserving of considerable attention by microeconomists.

[20]For example, both this model and the C-V model assumed constant marginal costs. However, allowing for increasing (and/or decreasing) marginal costs will also affect price reactions (see Greenhut, Greenhut, and Kelly).

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