Teaching Motion Graphs with Real-Time Notations
and Methods of Computer Graph Manipulation

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Abstract

Previous research suggest that real-time, graphical display of motion improves understanding. An instructional experiment assessed the effects of timing of presentation (real-time and delayed) across two methods of graph manipulation: 1) direct manipulation of graphs; and 2) the manipulation of numeric notations linked to graphs. Forty-eight eighth grade students were presented motion problems for two 40-minute sessions and tested one day and one week following instruction. The results showed that: 1) real-time presentation improved learning when used with direct graph manipulation; 2) real-time presentation with numeric manipulation improved learning for students with high mathematic skills, but delayed presentation improved learning for students with low mathematics skills; and 3) high interaction with direct graph manipulation was associated with improvements in learning, but high interaction with numeric manipulation was associated with decreases in learning.

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Students' Understanding of Motion Graphs

The concepts of velocity and acceleration are traditionally symbolized and taught with numeric values and mathematical functions like algebraic equations. Now, there is increasing recognition that other symbolic notations, such as graphs and tables, may be more appropriate for introducing these concepts. For example, graphs facilitate the communication of ideas and function as memory and thinking aids (Larkin & Simon, 1987; Phillips, 1986). Furthermore, the process of constructing and interpreting graphs with other notational systems, and connecting them to the events they represent, can provide the basis of conceptual understanding (Kaput, 1987a & b).

Unfortunately, notations like graphs are not often used because they are difficult to generate on a regular basis using traditional methods like pencil and paper. For example, students often cannot identify or construct a graph that accurately represents a motion event (Larkin, 1987; McDermott, Rosenquist & van Zee, 1987; Mokros & Tinker, 1987). These studies have found that with traditional teaching methods, many children and adults possess only a partial understanding of graphs and how they relate to motion events. Furthermore, the difficulties in generating graphs can make the use of graphs more a source of confusion than of help, particularly for students with poor mathematics skills and knowledge of graphs (Lesh, 1987). These problems have motivated the development of computer-based tools to help improve students' understanding.

Describing Motion with Real-Time Graphs

One such tool is the Microcomputer-Based Laboratory (MBL). MBLs instantaneously record and transform physical events, such as moving objects or the kinesthetic movements of students, into a variety of real-time graphs. The computer helps describe motion by plotting points onto the graphs step by step as events occur. The most noted advantage of real-time presentation is that it eliminates the time-intensive and often difficult task of constructing graphs with traditional media like paper and pencil (Trowbridge 1987; McDermott, 1982; Zeitsman & Hewson 1986). Secondly, real-time presentation allows students to observe their movements with the graphs simultaneously rather than sequentially (Brasell, 1987). Real-time presentation facilitates learning by reducing demands on working memory and processing capacity. As a result, real-time presentation helps students process information and form cognitive links between graph and motion events.

Producing Motion With Direct Graph Manipulation

An alternative to the MBL method, in which graphs are used to *describe* objects in motion, is a method in which graphs are directly manipulated to *produce* objects in motion (Trowbridge, 1987). That is, students construct and perform direct manipulations on graphs to produce motion events simulated on computer. To produce a motion event, students construct and manipulate lines plotted on graphs to specify initial values of motion parameters. Once values are specified, the computer translates the information into a computer-simulated motion event.

Producing Motion With Direct Manipulation of Linked Notations

To help students understand graphs and to facilitate the process of manipulating them, symbolic notations like numeric values, tables, and algebraic functions can be manipulated and presented simultaneously with graphs (Dickson, 1985; Lesh, 1987). Manipulating and linking symbolic notations to graphs provides yet another method of interacting and exploring motion graphs. Reference to notations linked to graphs serve as an important source of feedback and assist students in exploring possible relationships. For example, students that have very little understanding of graphs may find them to be more comprehensible when they are presented with numeric values of motion parameters. Furthermore, the simultaneous presentation of multiple notations can help students identify different relationships illustrated between notations. As a result, the presentation of linked notations serve as a scaffold to learning.

Purpose of Study

Although the real-time presentation of graphs with motion events has been found to improve learning when used with the MBL method, Beichner (1990) suggested that real-time presentation may not be a critical factor when applied to other methods. Beichner claimed that a fundamental feature of the MBL method was the immediate and direct control over physical motion events. It was the physical nature of the activity that helped students realize the relationships between events and motion graphs. This physical nature, however, is absent in methods in which graphs and notations are directly manipulated to produce motions simulated on computer. With these methods, physical actions are substituted with and mediated through the manipulation of symbols, creating an activity which is more abstract in nature. This abstract form of activity may hinder students' ability to recognize and learn the relationships between motion and graphs. Due to these differences, real-time presentation with the methods of direct manipulation may not necessarily improve learning as it has been found to do with the MBL method.

The purpose of this study was to test the effects of real-time versus delayed presentation across two methods: 1) direct graph manipulation; and 2) manipulation of graphs performed via the manipulation of numeric notations linked to graphs.

Direct graph manipulation . I conjectured that real-time presentation of graphs, motion events and numeric notations with direct graph manipulation improves learning based on findings from previous studies with the MBL method. This was hypothesized because of the following assumption: real-time presentation of numeric notations following each manipulation of the graphs, with the simultaneous presentation of motion events with graphs, cue students’ attention to the relationships between the graphs and motion events. The simultaneous and immediate presentation of numeric values allow students to refer to numerical values to help them interpret and evaluate each action they perform on the graphs.

Numeric manipulation. The importance of real-time presentation was also tested with the manipulation of numeric notations linked to graphs. With this method of graph manipulation, students performed actions on graphs indirectly by manipulating the numeric notations linked to the graphs. The hypothesis in this study was that learning with numeric manipulation also improves when graphs are immediately presented with each manipulation of the numeric notation. With real-time presentation, students are better able to learn the behaviors of the graphs and the relationships to motion events as represented by the numeric notation.

Contrasting methods. This study compared the effects of both methods of graph manipulation examined in this study. Although there are no reported studies on the differences between the methods, the hypothesis in this study was that direct graph manipulation improves understanding of motion graphs more than numeric manipulation. The assumption was that the actual manipulation of graphs would provide students a clearer and more direct illustration of the relationships between the graphs and motion events. The manipulation of the numeric notation, on the other hand, was thought to cue students to only a limited number of relationships given that students' focus was less on the graphs and more on the numeric notation itself.

Individual differences. Finally, this study tested the effects of real-time presentation across individual differences in mathematics skill and working memory capacity. The traditional view of individual differences is that variations in generic resources (e.g. working memory) and prior learning (e.g. mathematics skill) constrains learning by limiting available resources for higher-order processing (Anderson, 1989; Hulse, Egeth & Deese, 1980). According to Snow (1989), differences in learner attributes often result in aptitude-by-treatment effects, meaning that students respond differently to instruction depending on level of ability and prior knowledge. For example, individual differences in working memory capacity has been found to influence the effects of instruction (Carpenter & Just, 1992).

This study hypothesized that real-time presentation may be more beneficial for students with low or limited mathematics skill and working memory capacity than for students with high abilities. The assumption was that lower ability students are more likely to experience cognitive loads that exceed their cognitive capacity. Therefore, reducing cognitive demands with real-time presentation can mostly benefit students with lower abilities, whereas students with higher abilities may benefit little or none.

Methodology

Learning Environment

The computer software used to test the effects of real-time presentation with direct graph manipulation and numeric manipulation was designed and customized for this study (Rosenheck & Lehrer, 1990). Refer to Appendix A and B for screen examples. The software allowed students to pre-define motion events in which a student-controlled space ship (lower ship) and a computer-controlled space ship (upper ship) moved horizontally along a linear path for the duration of 0 to five seconds in time. The movements of the student-controlled ship were controlled by clicking on the numeric values (numeric manipulation) representing the parameters of the student-controlled ship, or by clicking on distance and velocity graphs (direct graph manipulation). The movements of a second ship (upper ship), however, were pre-determined and set by the computer for a given problem. The goal in each problem was to parallel the movements of the student-controlled ship with the movements of the computer-controlled by matching travel distance, velocity, acceleration, and travel time. To help students gauge their performance, the number of times a motion events were executed were displayed under the heading ‘trials’. When a match was achieved, students were congratulated by the computer.

When students clicked on the graphs, the actions were not only reflected in the motion events but also in the numeric notation. Likewise, when students clicked on the numeric notation, these actions were reflected in the graphs as well as the motion events. With real-time presentation, changes performed on one notation were simultaneously represented in the other notation. For example, a click on the distance graph to increase its slope simultaneously resulted in an increase in the value of velocity displayed in the numeric notation. Similarly, a click in the numeric notation to increase the value of velocity simultaneously resulted in an increase in the slope of the distance graph. With delayed presentation, on the other hand, changes made on one notation were not simultaneously represented in the other notation. Instead, all changes were represented following the presentation of a motion event upon students' request.

When executing a motion event, students viewed the motions of the ships by clicking on 'Go' to view them in real-time, or clicked repeatedly on ‘step’ to view the events at half-second time intervals. With real-time presentation, all graphs were plotted point by point with the presentation of a motion event. Refer to Appendix C. At the completion of the motion event, the computer connected the points on the graphs into a continuous line. With delayed presentation, on the other hand, the graphs were hidden from view during the presentation of a motion event. Following the completion of a motion event, complete graphs were presented simultaneously all at once rather than sequentially point by point.

Experimental Design

Using a 2x2 factorial randomized block design, a two-way analysis of covariance was performed on post test scores measuring learning one day after instruction (immediate learning) and one week after instruction (retention). Standardized and pretest scores on mathematics skill was used as the blocking variable. The dimensions of the analysis were type of presentation (real-time or delayed) and type of notation (direct graph manipulation or indirect manipulation via the manipulation of numeric notations linked to graphs). To test the effects of individual differences, post test scores were analyzed across individual differences in mathematics skill and working memory capacity. To balance for individual differences in mathematics skill between the four conditions, students were ranked by mathematics skill and then blocked (by alternate ranks) into groups of four students, starting from the lowest ranked and progressing sequentially up to the highest ranked. In each block of four students, students were systematically assigned to one of the four learning conditions such that the lowest ranked students within each group, for example, were evenly distributed across all four conditions.

Subjects

The participants were 48 eighth-graders from a Midwest suburban middle school, with 12 students for each learning condition. Students were selected from three science home groups, with 22 boys and 26 girls, ranging from 11 to 13 years of age, all volunteering to participate in this study. One of the home groups consisted of students with low mathematics abilities, and the remaining two home groups consisted of students with average and above average mathematics ability. For their participation, each student received laboratory credit from their science teacher.

Procedure

Students worked individually on a series of motion problems in 40-minute sessions on two consecutive days. Velocity problems were presented on the first day, and acceleration problems were presented on the second day. For each problem, students evaluated their performance by noting the number of times they had to run a motion event (or number of trials) before completing a problem. This performance criteria was used to discourage haphazard guessing and encourage the use of more effective problem-solving skills. When students arrived at what they believed was a correct solution, they executed a motion event to check and validate their responses. Students were instructed to work as many problems as possible during each session. Those who completed the assigned problems were instructed to repeat problems with the time remaining.

Three computers were used to present the motion problems. Each student was assigned to one of the three computers for both instructional sessions. The assignments were made so that each computer was used by an equal number of students from each condition. As the students worked on the computers, the experimenter maintained close observation and was present to answer questions to resolve difficulties that students had in learning to operate the computer software. However, no explicit instruction concerning the graphs and their relationships to motion parameters was given by the experimenter.

Motion Problem Sets

On the first day of instruction, students were presented with up to 20 velocity problems. In these problems, the goal was to match travel distance, time, and velocity between the student and computer-controlled space ships. In a typical velocity problem, the values for both space ships were predetermined at the beginning of each problem. The parameters of the student-controlled ship were set at the initial values: distance = 100, time = 5 seconds, velocity = 20, and acceleration = 0. The parameters of the computer-controlled ship were set at the specific values: distance = 50, time = 5, velocity = 10, and acceleration = 0. The values of the computer-controlled ship were fixed and remained the same through the duration of the problem. To match the motion of the student-controlled ship to the computer-controlled ship, distance was decreased from 100 to 50, and velocity is decreased from 20 to 10, as time and acceleration were held constant at their current and correct values. The values of time and acceleration were held constant in value by clicking on the titles identifying each parameter displayed in the numeric notation. With time and acceleration held at constant values, the values of distance and velocity were free to covary. As a result, decreasing the value of distance from 100 to 50 (by direct manipulation of the graphs or numeric notation) simultaneously decreased the value of velocity from 20 to 10, and vice versa.

On the second day of instruction, students were presented with up to 15 acceleration problems in which students were given the additional task of matching acceleration between the ships, in addition to distance, time, and velocity. These problems were structured like the velocity problems. As a typical example, the parameters of the student-controlled ship were set at initial values (distance = 100, time = 5, velocity =20, and acceleration = 0. The parameters for the computer-controlled ships were set at fixed values (distance = 100, time = 2.5, initial velocity = 20, and acceleration = 16). With positive acceleration, velocity had to increase over time. In this condition, the velocity displayed in the numeric notation represented the initial velocity. Given that the initial values of distance and velocity already match, their values were set to remain constant by clicking on their identifying titles displayed in the numeric notation. As a result, the values of time and acceleration were free to covary. The problem was solved by either decreasing the value of time from 5 to 2.5 (by direct manipulation of the graphs or the numeric notation) or increasing the value of acceleration from 0 to 16.

Across each set of problems from each session, the initial parameter values were configured so that initial matching parameters occurred in different combinations. The purpose of this design was to focus attention onto the different relationships between the motion parameters. Furthermore, the number of initial parameters with matching values were gradually decreased in number to increase the level of difficulty. See Appendix D for a sample of problems with their assigned initial values. Noted here is that the values for each parameter were restricted in range (distance = 1 to 100, time = 0 to 5, velocity = 0 to 20, and acceleration = -20 to 20).

Instructional Treatments

The study tested two dimensions -- type of notation (direct versus indirect graph manipulation), and type of presentation (real-time versus delayed). Crossing the two dimensions resulted in four learning conditions; a) direct graph manipulation with real-time presentation; and b) direct graph manipulation with delayed presentation; c) numeric manipulation with real-time presentation; and d) numeric manipulation with delayed presentation. The computer program used in the study was adapted for each of the conditions. The specific adaptations for each condition are described below.

Number manipulation with real-time presentation. Students defined the movements of the student-controlled space ship by performing manipulations on the numeric notation (see Appendix A). With real-time presentation, all manipulations performed on the numeric notation were immediately reflected in the distance and velocity graphs. Furthermore, the graphs were plotted in real-time as the motion events were presented on screen (see Appendix C). In the graphs, the motions of both the student and the computer's ship were represented. As a result, the graphs could be examined to determine the appropriate parameter values, and to immediately evaluate the results of actions performed on the numeric notation.

Number manipulation with delayed presentation. This condition was similar to the previous condition in terms of what was displayed on the computer screen. The difference was that actions performed on the numeric notation were not immediately reflected in the graphs. The numerical changes were presented in the graphs only after the completion of a motion event. Furthermore, the graphs were hidden from view during the presentation of a motion event. At the completion of a motion event, the graphs were presented instantaneously in their complete form rather than sequentially point by point.

Graph manipulation with real-time presentation. Students defined the movements of the student-controlled space ship by performing direct manipulations on the distance and velocity graphs. See Appendix B for a screen example. With real-time presentation, changes made on the graphs were immediately reflected in the parameter values displayed in the numeric notation. In addition, the graphs were plotted simultaneously with the presentation of the motion events. The parameter values of the computer-controlled space ship also were displayed in a numeric notation. With both sets of numeric notations in view, students could identify the discrepancies between the parameter values to evaluate their actions on the graphs.

Graph manipulation with delayed presentation. The screen display in this condition was similar to the condition above. With delayed presentation, however, changes made on the graphs were not reflected in the values displayed in the numeric notation until after the completion of a motion event. During the presentation of a motion event, the graphs were hidden from view. At the completion of a motion event, complete graphs were displayed instantaneously, rather than sequentially in real-time.

Measures

Mathematics skill. The measure of mathematics skill was based on a math pretest and national percentiles on the Basic Math and Visual Materials portions of the Iowa Test of Basic Skills. The pretest scores and percentiles were standardized, and their mean was used to represent mathematics skill. The mean national percentile on the test of Basic Math was 80.79 (SD = 21.31), ranging from 16 to 99, and the mean national percentile on the test of Visual Materials was 81.83 (SD = 15.73), ranging from 40 to 99.

Students were pretested on basic algebra and graphing skills with a 14-item test. Seven items, mostly multiple choice, tested algebra problems presented in the form of story problems and functions (i.e. evaluate $y = c + bx$ when $c = 5$ and $b = 20$). The remaining items tested graphing skills. Three of these items tested quantitative interpretation of graphs involving the interpretation of values from line graphs, and three items tested the ability to interpret qualitative relationships in graphs (e.g. "Which graph is best described by the following statement? As the pot size increases, the plant height decreases"). On the last problem, students graphed the function $y = 2x$. One point was awarded for plotting the correct points on the graph, and 2 points were awarded for a complete graph.

The mean score on the 15-point pretest was 9.48 (SD = 2.98), ranging from 4.0 to 15.0. The mean score on the seven items testing algebraic skills was 4.93 (SD = 1.67). On the three items testing quantitative graphing skills, 45 percent of students correctly answered all three items, 40 percent correctly answered 2 of the items, and 15 percent correctly answered one item. On qualitative questions, about 60 percent answered all three items correctly, 25 percent correctly answered two items correctly, and 15 percent correctly answered one item. Only 22 percent of the students were able to completely graph the function $y = 2x$, 11 percent plotted partially correct graphs, and the remaining students did not plot the correct graph. The correlation between the math pretest and ITBS scores was $r(44) = .67$.

Working memory. A computer-based test was used to measure spatial working-memory capacity one week prior to instruction (Case, personal communication; Lehrer & Littlefield, 1993). Students were presented a 4x4 matrix of squares on a computer screen (see Appendix E). In the matrix, a specific number of squares were flashed at one second intervals one after another in a predetermined sequence. Following a single presentation of a sequence plus a 2-second delay, students had to identify (in no specific order) the squares presented in the sequence by selecting them from the matrix. The number of squares presented in a sequence ranged from 1 to 7. The sequences began with one square and then gradually progressed up to 7 squares. For each sequence length, four different sequences were presented. When a student performed perfect recall on at least 3 of the 4 sequences for a given length, they progressed to the next and longer sequence. The longest sequence achieved by a student was equated as the measure the student's working-memory capacity.

Immediate learning. A post test was given one day following instruction to measure understanding of graphs and concepts of motion. The post test was presented on computer and consisted of 35 multiple choice items in four different formats: qualitative interpretation of graphs, quantitative interpretation of graphs, interpreting relationships between distance and velocity graphs, and qualitative understanding of the relationships between distance, time, velocity, and acceleration.

Skills in quantitative interpretation of graphs were tested in 7 test items. In one of these items (Appendix F), a distance graph was presented and used to answer the question "How far did the spaceship travel after 2 seconds?" In another item (Appendix G), a distance-time graph representing the motions of two space ships had to be matched to one of five verbal interpretations of the graph.

The ability to perform qualitative interpretations of graphs were tested in 16 items. For example (Appendix H), one of four distance graphs were chosen to answer the question, "Which distance-time graph represents a spaceship moving with an increasing velocity?" Of these 16 test items, 10 items used distance-time graphs and six items used velocity-time graphs.

Qualitative interpretation skills also were tested with three test items that required the matching of a given distance-time graph to one of four velocity-time graphs. See Appendix I for an example. Three additional test items tested the reverse by matching a given velocity-time graph to one of four distance-time graphs.

Students' qualitative understanding of the relationships between distance, time, velocity, and acceleration were tested with six test items. In these test items, there were questions like: "If your spaceship travels at a constant velocity, how will increasing the distance affect travel time?" (see Appendix J); and "If your spaceship travels at a constant time, how will decreasing the velocity affect travel distance?" Students had to choose one out of five possible responses presented in each item.

Retention. One week after instruction, students were retested for retention with a 32-item test almost identical to the first post test. The difference was that the specific values in the quantitative problems were varied. In addition, three items in the first post test that were found to be redundant were not included in this second post test. Two of the omitted items were pulled from the qualitative interpretation of graphs, and one item was pulled from the quantitative interpretation of graphs.

Results

An analysis of covariance (ANCOVA) was used to test for differences between treatments on immediate learning and retention. See Table 1 for group means and data sets on each of the post measures. Mathematics skill was used as a covariate. Working memory was omitted from the analysis because its partial correlation with learning and retention showed that working memory did not contribute to predicting performance independent of the contributions of mathematics skill. Mathematics skill, on the other hand, was found to contribute to post test performance independent of the contributions of working memory. These results showed that the effects of working memory were largely reflected in the effects of mathematics skill. In essence, working memory was omitted from the analysis to minimize redundancies in the results.

Insert Table 1 about here.

Forty-four scores on the test of immediate learning and 44 scores on the test of retention were used in the analysis. Because there were large deviations in individual test scores from three students in the delayed-numbers, real time-and-graphs, and delayed-and-graphs conditions, the student with the highest combined score on the post tests in each condition was omitted from analysis. These scores were omitted from the analysis in order to maintain normal distributions in the data as is required in an ANCOVA. A student absence in the delayed-and-graphs condition resulted in a missing score on the test of retention.

Post Test Performance

The analysis on immediate learning revealed no significant differences between the effects of real-time versus delayed presentation and the effects of graph versus numeric manipulation (see Table 2). This result did not support the findings of Brasell (1987) which suggested that learning is greater with real-time presentation than with delayed presentation. Furthermore, no significant interactions were found between the treatments and mathematics skill to suggest that students' response to the treatments depended on individual differences in mathematics skill.

Insert Table 2 about here.

Results from the analysis on retention, on the other hand, suggested that the effects of real-time presentation depended on individual differences in mathematics skill. A graph displaying the relationship between retention and mathematics skill (see Figure 1) suggested that students with high mathematics skill scored higher in retention with real-time presentation than with delayed presentation. Students with low mathematics skill, on the other hand, scored lower in retention with real-time presentation than with delayed presentation. Note however that the statistical analysis did not show the interaction to be statistically significant, $F(1, 36) = 3.22$, $p > .05$) (see Table 3).

Insert Figure 1 & Table 3 about here.

A significant interaction was found between the treatments on the test of retention. The effects of graph manipulation was found to depend on the type of presentation, $F(1, 35) = 5.51$, $p < .05$ (see Table 3). Figure 2 shows that the effects of direct graph manipulation resulted in lower retention than with numeric manipulation (or indirect graph manipulation) when screen presentations were delayed. When the presentations were real-time, on the other hand, the effects were reversed; direct manipulation of graphs was more effective than indirect manipulation with the numeric notation.

Insert Figure 2 about here.

Student Performance

Some performance data were collected from informal observations. Data was also collected from system-generated computer logs recording the activities of each individual student. The logs of seven students were selected from each of the four conditions and matched according to individual differences in mathematics skill. The following are some of the findings based on the observations and an exploratory analysis of the performance data.

The majority of students were able to complete all the assigned problems. Observations also showed that successful completion of many problems could be attributed to a trial-and-error strategy. With trial and error, students guided their actions solely on the visual feedback presented in notations and motion events. These actions did not appear to involve careful planning and reflection based on lessons learned from previous errors. These behaviors were mechanistic and automatic in appearance, and were similar to behaviors observed with video-arcade games. Students manipulated parameters in random directions with no evidence of planned action nor reflection on errors. For example, most evident was that students appeared to be more inclined to use trial and error with the use of real-time presentation, and less inclined when presentations were delayed.

In an attempt to determine if trial and error was used more often with real-time presentation, a random sampling of computer logs from individual students and performances was examined and analyzed. In a preliminary analysis, the assumption was made that trial and error would produce more errors on average than higher-level strategies. As a result, the total number of manipulations performed on the graphs and numeric notations during instruction were computed from the computer logs. At the time, it was hypothesized that higher numbers of manipulations, associated with the use of trial and error, would also be associated with lower post test scores.

With this performance measure, learning and retention scores were tested against the treatments using ANCOVA and the number of manipulations as the covariate. The results did not support our hypothesis that high numbers of manipulations resulted in lower post test scores. The results also did not provide any indication that real-time presentation was contributing to high numbers of manipulations or errors. However, this analysis did reveal other findings. The results showed that the effects of graph and numeric manipulation depended on the number of manipulations. Figure 3 shows that a high number of direct manipulations on the graphs was associated with high levels of learning, but high numbers of manipulations on the numeric notation was associated with low levels of learning. The same results were found with retention. Figure 4 shows that high numbers of direct manipulations on the graphs was associated with high levels of retention, whereas high numbers of manipulations on the numbers were associated with low levels of retention.

Insert Figures 3 to 4 about here.

Discussion

Real-time Versus Delayed Presentation

The results showed no significant differences between real-time versus delayed presentation of motion graphs with computer-simulated motion events and numeric notations, when using direct graph manipulation. Furthermore, no main differences were found with numeric manipulation. These findings failed to support the hypotheses in the study that real-time presentation was expected to improve learning.

The type of presentation, real-time or delayed, did make a significant difference when the results of both methods were examined together. The results showed that when presentations were in real-time, no difference in learning resulted from the use of either method. Delaying the presentations, on the other hand, resulted in significantly lower learning with direct graph manipulation than with numeric manipulation. These findings did not support the hypothesis that direct graph manipulation would improve learning more than numeric manipulation.

What the findings show is that real-time presentation is important when learning with direct graph manipulation, and not necessarily important with the use of numeric manipulation. These findings can be attributed to the differences in demands between the manipulation of graphs versus the manipulation of numeric notations. Observations indicated that students experienced more difficulties with graph manipulation than with numeric manipulation. Such an observation was consistent with the assumption that manipulating the graphs would be a more complicated task than manipulating the numeric notation. These findings and observations suggest that the potential difficulties with direct graph manipulation can be overcome with the real-time presentation of numeric values.

Dependence on Individual Differences & Problem-Solving Strategies

No significant differences were found between the effects of real-time and delayed presentations because their effects depended on individual differences in mathematics skill. The results showed that with both methods, real-time presentation significantly increased learning for students with high mathematics skill, whereas delayed presentation helped students with low mathematics skill. These results were found in students' retention scores one week after the instruction, with similar trends found in the scores on the post test that immediately followed the instruction. These findings suggest that the importance of real-time presentation depends on individual differences in mathematics skill, specifically with the methods of direct graph manipulation combined with real-time presentations of notations and computer-simulated motion events .

Although it was expected that individual differences in mathematics skill would influence the effects of real-time presentation, the observed trend was opposite of what was hypothesized. These findings did not support the hypothesis that anticipated that students with low abilities would benefit more than students with higher abilities from real-time presentation, and not from delayed presentations. Furthermore, the results differ from studies that tested real-time presentation with the MBL method, when the simultaneous presentation of graphs with physical motion events resulted in clear differences in learning (Brasell 1987).

What may have contributed to these outcomes were the problem-solving strategies used by students and how the use of particular strategies tended to be associated with the type of presentation received. There is evidence that delayed presentation can benefit students because a delay reduces the incidence of trial-and-error problem-solving strategies (Cope & Simmons, 1994). In this study, the delay was found to benefit students with low math ability. Restricting access to feedback helped to induce these students to plan their actions and to interpret the results of their actions. In other words, the delayed presentations helped encourage students to practice effective problem-solving strategies that support learning and understanding.

Observations indicated that real-time presentation, on the other hand, allowed students the option of using trial-and-error. This low-level strategy tended to prohibit rather than support high-level learning. With this low-level strategy, actions were guided by visual feedback from the presented notations, and not by careful observation and deliberate application of what students learn from their observations. As a result, this use of low-level procedural knowledge diverted students' attention away from the relevant information and relationships (Olive, 1991; Olive & Scally, 1987). The interaction between type of presentation and mathematic skills, suggested in Figure 1, suggest that students with low mathematics skill were more vulnerable to the negative effects of real-time presentation and the use of trial-and-error, particularly because these students lacked alternative problem-solving skills and strategies. Due to a lack of alternative strategies, students with lower ability were more likely to adopt the trial-and-error strategy than students with higher ability.

Direct Graph Manipulation Versus Numeric Manipulation

The contrast between both methods revealed no significant differences in learning. This finding failed to support the hypothesis that direct graph manipulation may be more effective than mediating students' interaction with graphs through the manipulation of a numeric notation. However, further exploratory analysis showed that the more students manipulated the graphs, the higher students scored on learning and retention. In contrast, high numbers of manipulations on the numeric notation were associated with lower scores in learning and retention.

High numbers of interactions with direct graph manipulation were associated with better learning because direct interaction with the lines in the graphs explicitly cued students' attention to the relationships between the graphs and motion events. Furthermore, the process of manipulating the lines in the graphs helped to illustrate the dynamic relationships between motion parameters. With the numeric notation, on the other hand, no visible nor explicit representations of the relationships were presented without directing attention to the graphs. Instead, the manipulation of the numeric notation drew students' immediate attention to the numeric notation and not to the graphs. Furthermore, it is possible that some students that used numeric manipulation did not use nor rely on the graphs to perform the problem tasks. Nevertheless, a reduction in overall attention to the graphs may explain the negative relationship found between learning and amount of interaction.

Research Implications

Potential differences between techniques. The observations on the influences of real-time presentation on the types of problem-solving strategies students use hint at possible differences between the methods of computer manipulation (presenting graphs with computer-simulated events) and physical manipulation with MBL (graphs are presented with concrete and physical motion events). With the MBL method, students form cognitive links between graphs and motion events because the direct, immediate and kinesthetic control of motion graphs brings these relationships directly to attention (Beichner, 1990; Mokros & Tinker, 1987). When looking at computer-based manipulations with computer-simulated events, on the other hand, students control events by centering actions on computer-based notations rather than on the physical event itself. These types of actions may be perceived procedurally rather than conceptually by students as a series of symbol manipulations. As a result, it may be more difficult for students to make meaningful cognitive links between their actions and motion events without the presence of direct and physical control. Under such circumstances, one might examine how the effects of real-time presentations are influenced with and without the availability of direct control. One might also examine how direct control influences the use of problem-solving strategies and interact with individual differences in mathematics skill. The potential differences between computer versus physical manipulation with the use of real-time presentations will require further examination and research, and can be tested by directly contrasting the methods with matching applications and post measures.

Instructional Implications

This study demonstrated that real-time presentation is not as clearly beneficial as previous studies indicate (Brasell, 1987). This study showed that real-time presentation as well as delayed presentations of motion events and notations can improve or inhibit learning depending on what computer techniques are used and the abilities of the student. Effective pedagogical use of computer-based manipulation of notations and motion events depends largely on how instructional tasks are structurally designed and how these designs induce students to practice appropriate problem-solving and learning strategies. This study demonstrated that such structural designs can include the use of delayed presentations as well as real-time presentation, and that decisions to use these methods must take individual differences in abilities, including level of task difficulty, into consideration.

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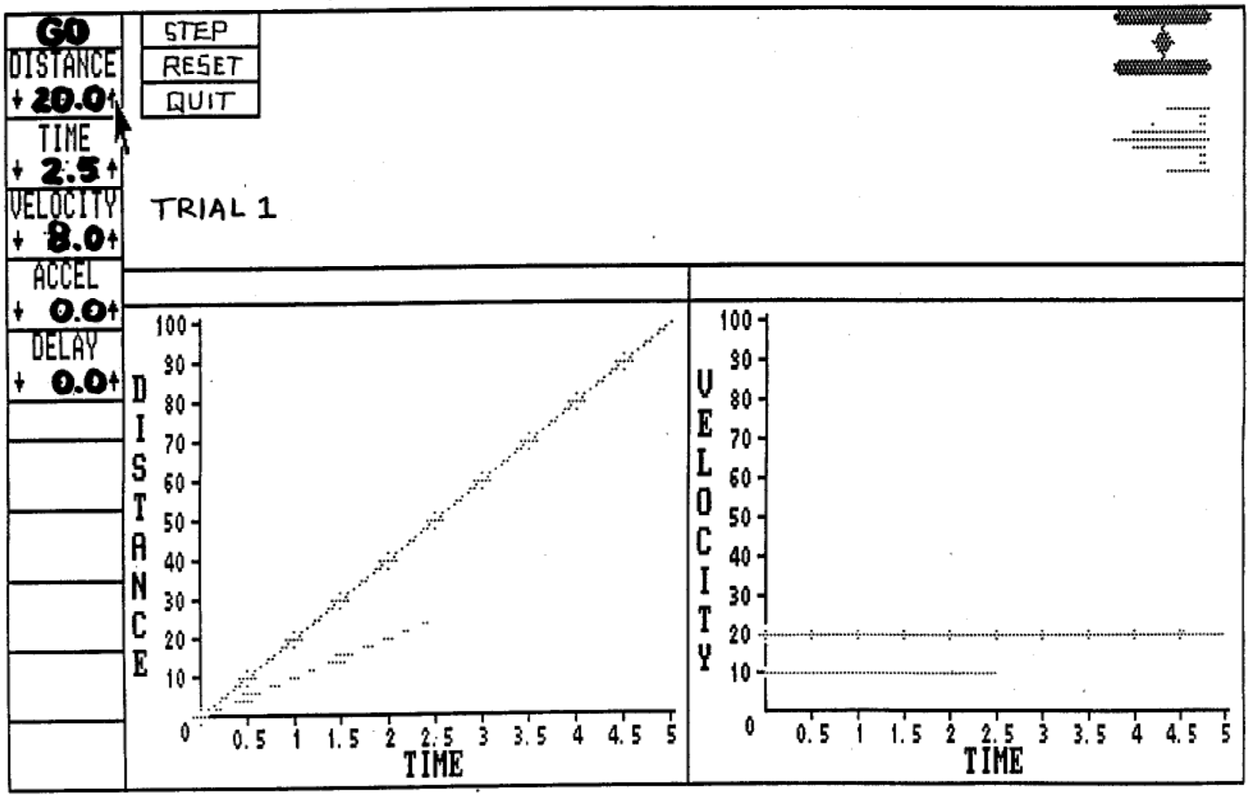
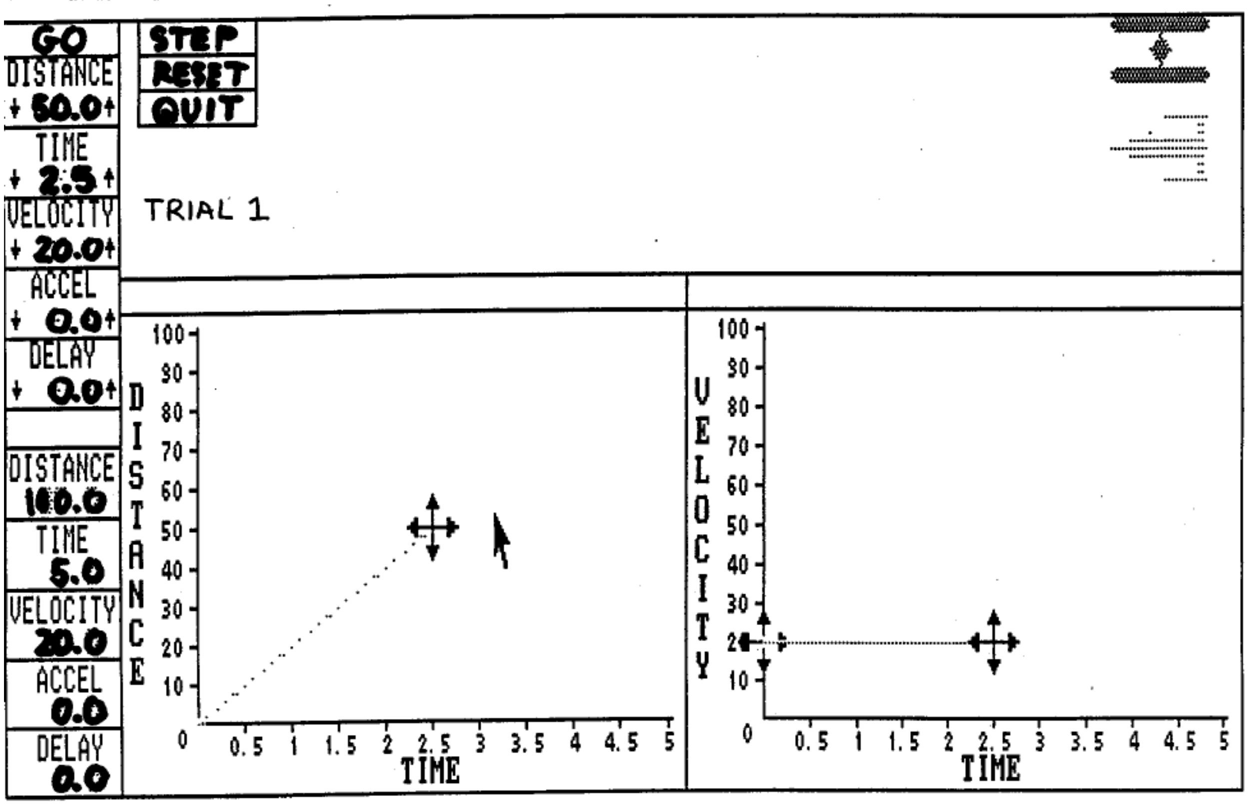
Appendix A

Computer display in the numeric condition



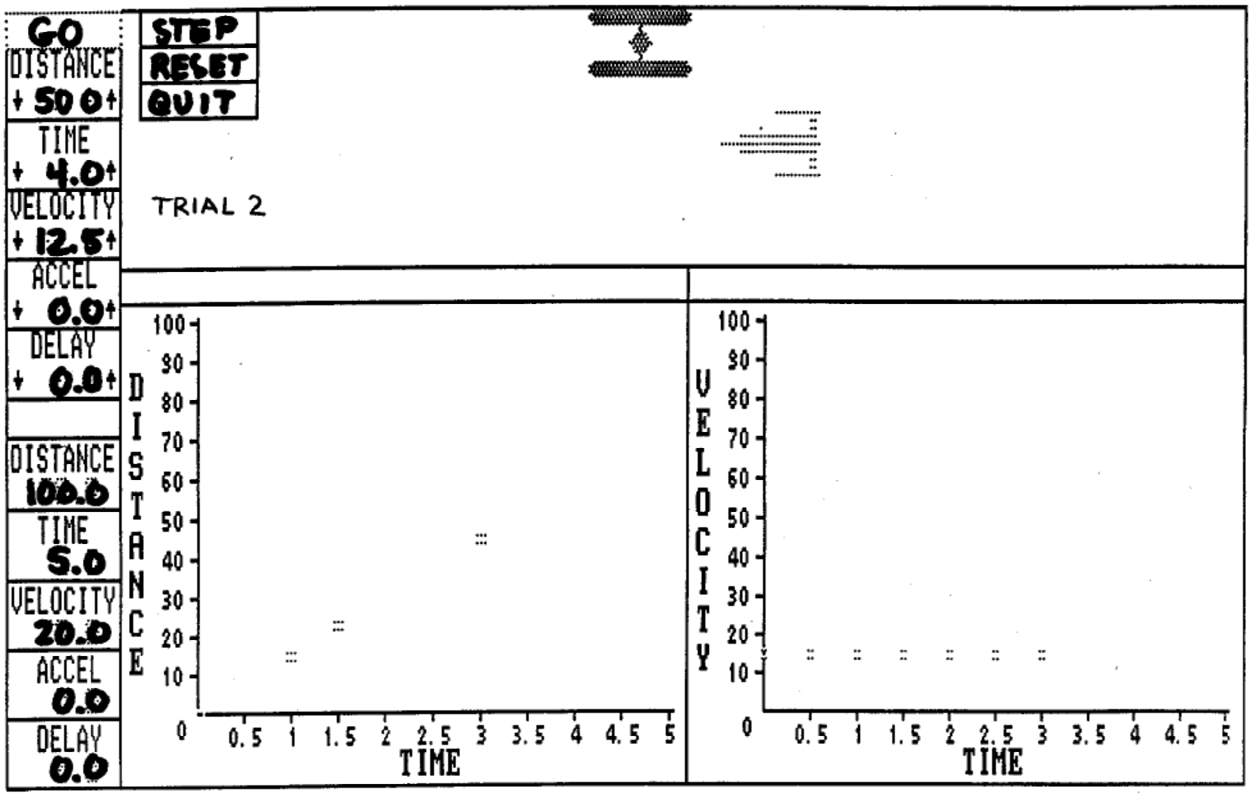
Appendix B

Computer display in the graph condition



Appendix C

Real-time presentation of graphs during a motion simulation



Appendix D

The initial setting of parameter values of the student and computer-controlled space ships in problem samples

Example velocity problems with zero acceleration:

		Distance	Time	Velocity	Acceleration
1)	Control	50	2.5	20	0
	Target	100	5	20	0
3)	Control	75	1	75	0
	Target	75	5	15	0
5)	Control	80	4	20	0
	Target	20	4	5	0
12)	Control	80*	4*	20	0
	Target	40	1	40	0
14)	Control	70*	5*	15	0
	Target	70	1	70	0

Example problems with acceleration:

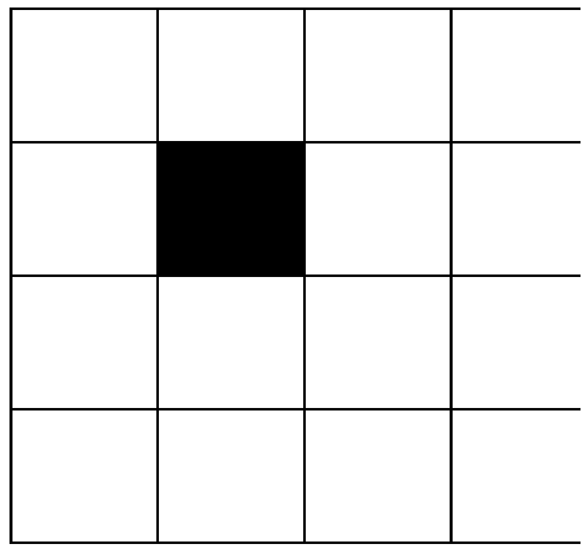
		Distance	Time	Velocity	Acceleration
1)	Control	100	5	20	0
	Target	100	2.5	20	16
3)	Control	80	4	20	0
	Target	50	5	30	-4
6)	Control	80	2	60	-20
	Target	100	4	5	10
12)	Control	90*	2*	65	-20
	Target	65	3	50	-20
15)	Control	80*	4*	20	0
	Target	50	5	30	-8

* Numeric values were hidden and masked with ?? in the number notation to increase the level of difficulty and to induce students to look to the graphs to determine the hidden values.

Note: The target parameter values were predetermined by the computer and remained constant throughout a given problem.

Appendix E

<u>Computer display on the working memory test</u>



Appendix F

Posttest item to test quantitative interpretation of graphs

The motion of an spaceship is represented by the distance-time graph. How far did the spaceship travel after 2 seconds?

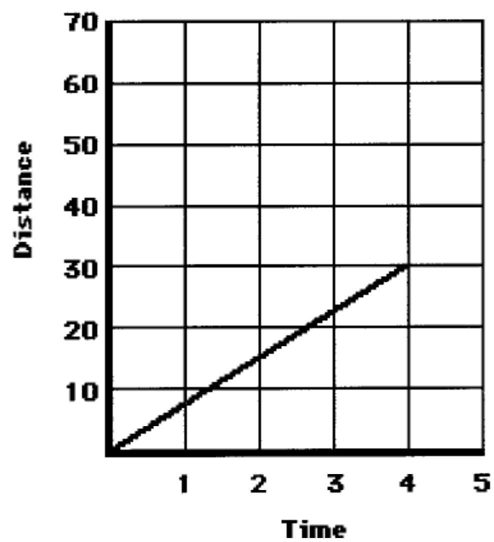
(A) Distance = 150 miles

(B) Distance = 30 miles

(C) Distance = 75 miles

(D) Distance = 15 miles

(E) The distance travelled cannot be determined from this graph.



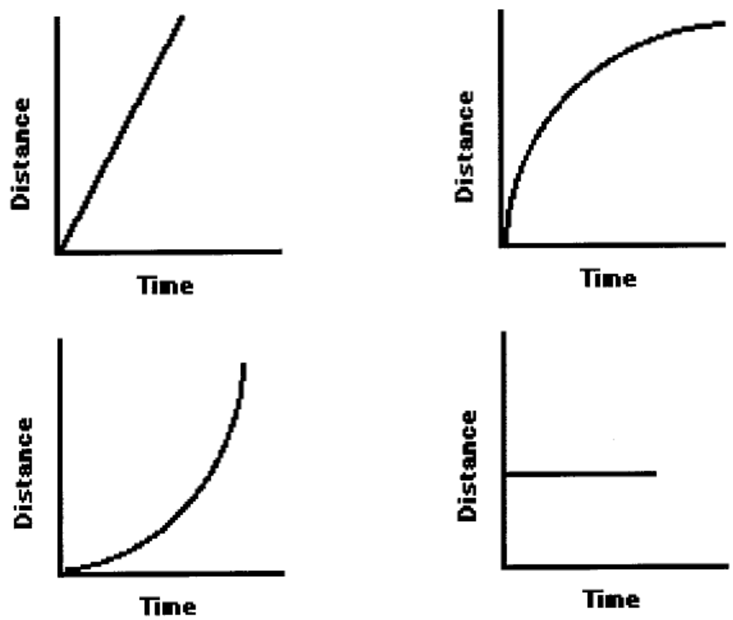
Appendix G

Posttest item to test quantitative interpretation of graphs

Appendix H

Posttest item to test qualitative interpretation of graphs

Which distance-time graph represents a spaceship moving with an increasing velocity?



Appendix I

Posttest item to test ability to identify qualitative relationships

between distance-time and velocity-time graphs

Appendix J

Posttest item to test qualitative understanding of kinematic concepts

If your spaceship travels at a constant velocity, how will increasing the distance affect travel time?

(A) Time increases.

(B) Time decreases.

(C) Time stays about the same.

(D) Time stays exactly the same.

(E) None of the above.

Table 1

Descriptive statistics on posttests by treatment condition

	Immediate learning[a]			
	N	Mean	SD	Adjusted*
Numeric Condition				
Real-time Presentation	11	17.09	4.25	17.16
Delayed Presentation	11	16.27	3.44	16.28
Graph Condition				
Real-time Presentation	11	17.64	3.04	17.77
Delayed Presentation	11	17.09	4.48	16.81

	Retention[b]			
	N	Mean	SD	Adjusted*
Numeric Condition				
Real-time Presentation	11	13.64	3.70	12.82
Delayed Presentation	11	15.18	3.03	14.99
Graph Condition				
Real-time Presentation	11	14.36	4.84	14.65
Delayed Presentation	10	12.10	3.63	12.87

* Means adjusted to individual differences in mathematics skill.

[a] Maximum possible score on learning = 35.

[b] Maximum possible score on retention = 32.

Table 2

ANCOVA analysis on learning covaried with mathematics skill

DEP VAR: LEARN N: 44 MULTIPLE R: 0.606 SQUARED MULTIPLE R: 0.367

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO
Notation[a]	3.83	1	3.83	0.36
Presentation[b]	9.59	1	9.59	0.90
Math	200.22	1	200.22	18.88*
Notation*Presentation	0.06	1	0.06	0.01
Notation*Math	0.50	1	0.50	0.05
Presentation*Math	0.34	1	0.34	0.03
Notation*Presentation*Math	8.11	1	8.11	0.76
Error	381.69	36	10.60	

*$p < .05$

[a] Notation (Numeric, Graphical)

[b] Presentation (Real-time, Delayed)

Table 3

ANCOVA analysis on retention covaried with mathematics skill

DEP VAR: RETENT N: 43 MULTIPLE R: 0.745 SQUARED MULTIPLE R: 0.555

ANALYSIS OF VARIANCE

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO
Notation[a]	11.96	1	11.96	1.48
Presentation[b]	5.86	1	5.86	0.73
Math	243.49	1	243.49	30.14*
Notation*Presentation	44.49	1	44.49	5.51*
Notation*Math	1.85	1	1.85	0.23
Presentation*Math	26.00	1	26.00	3.22
Notation*Presentation*Math	0.99	1	0.99	0.12
Error	282.73	35	8.08	

*$p < .05$

[a] Notation (Numeric, Graphical)

[b] Presentation (Real-time, Delayed)

Figure 1. The interaction between type of presentation and mathematics skill on the measure of retention.

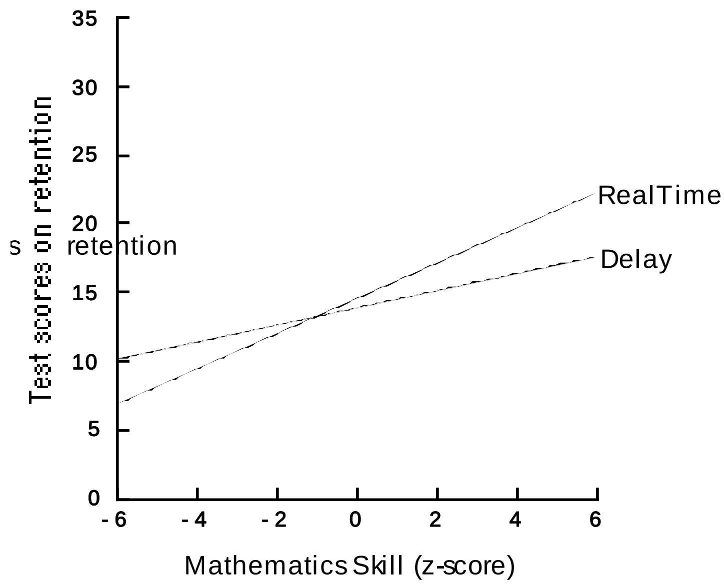


Figure 2. The interaction between notation and presentation when covaried with mathematics skill on the measure of retention.

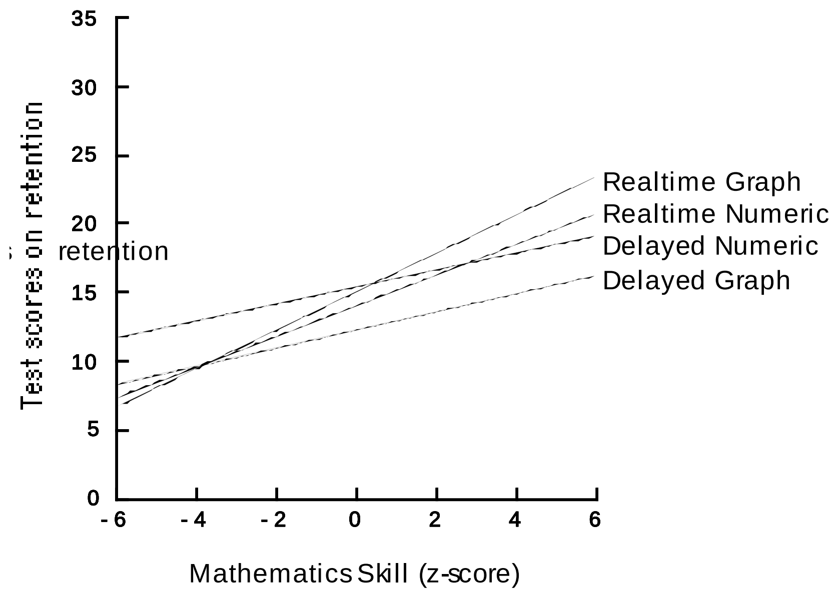


Figure 3. The interaction between the method of graph manipulation (or notation) and number of manipulations on students' learning.

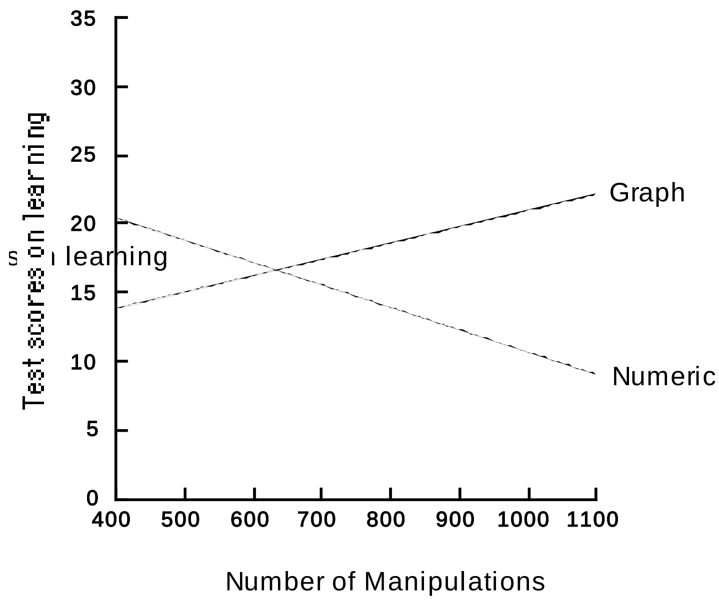


Figure 4. The interaction between the method of graph manipulation (or notation) and number of manipulations on students' retention.